

Notes on the surface formalism for (d,p) reactions to bound and resonance states

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Reminder: Motivation

Present theories provide valuable information on angular momenta ...but have serious limitations in resonance cases

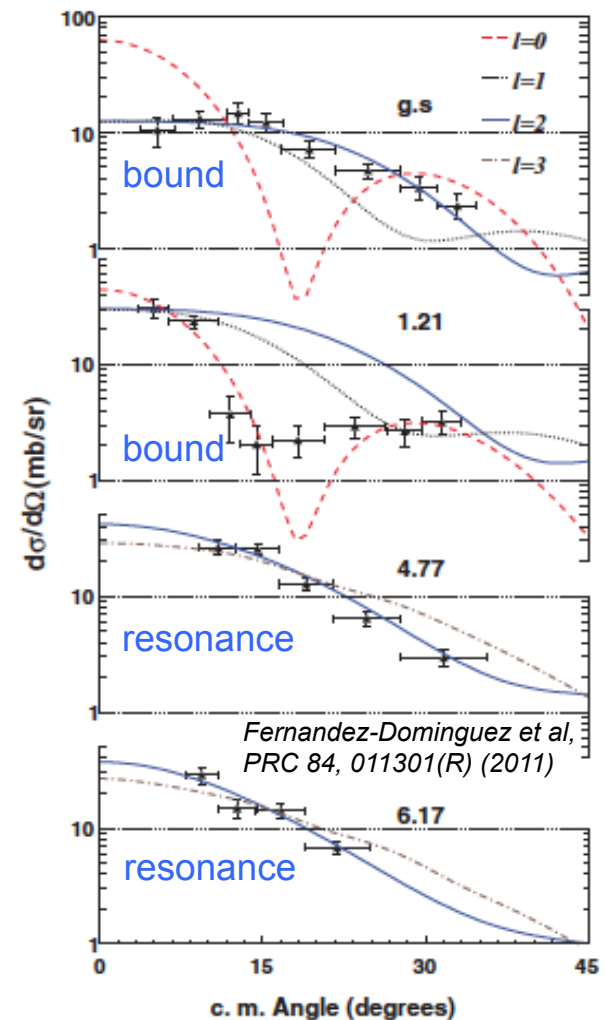
- Problem: present theories rely heavily on one-body overlap function of A and A+1 systems

$$I_A^F(r) = \langle \varphi_A | \varphi_F \rangle$$

- carries structure information
 - not well-known in nuclear interior
 - typically approximated by single-particle function
- Calculations converge very slowly
 - Not appropriate for describing reactions involving wide resonances
 - Desired resonance properties (energies and widths) cannot be reliably obtained

**Mukhamedzhanov's suggestion: Extend R-matrix description to transfer reactions
→ Surface Formalism, PRC (2011)**

Example: $^{20}\text{O}(d,p)^{21}\text{O}$ inverse-kinematics experiment - Interpreted the traditional way



Mukhamedzhanov's surface formalism for (d,p)

Features/Claims:

- Model dependence contained in interior term → Claim: term is small
- Claim: surface term is dominant
- Surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.
- Formalism valid for bound states and resonances
- Formalism derived for DWBA and CDCC
- Claim: Exterior term vanishes in CDCC implementation

Surface formulation

$$\mathbf{M} = \underbrace{\mathbf{M}^{(\text{post})}(\mathbf{0}, \mathbf{a})}_{\text{model dependence}} + \underbrace{\mathbf{M}_{\text{surf}}(\mathbf{a}) + \mathbf{M}^{(\text{prior})}(\mathbf{a}, \infty)}_{\text{asymptotic quantities}}$$

$M_{\text{surf}}(\mathbf{a}) = f(\mathbf{a}, C_A^F, B_{nA})$
 $B_{nA} = \log \text{ derivative of } I_A^F \text{ at surface radius } a$
 ANC: C_A^F defined through: $I_A^F(r_{nA}) \rightarrow C_A^F W(kr_{nA})$
 related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

TORUS: tests & implementation of surface formalism for (d,p)

Status:

- **Relative contributions of terms studied within DWBA → PRC 2014**
- **Implementation of M_{surf} within CDCC underway (Ian)**
- **TBD: study of relative contributions within CDCC (Year 5)**
- **TBD: using R-matrix-type parametrization and fit real data (Year 5)**

Mukhamedzhanov, PRC 84, 044616 (2011) - overview

- I. Introduction
- II. Formalism for bound states
 - A. Post-form DWBA
 - B. Prior-form DWBA
 - C. Post-form CDCC
 - D. Prior-form CDCC
- III. Formalism for resonances
 - A. Post-form DWBA
 - B. Prior-form DWBA
 - C. Post-form CDCC
- IV. Summary
- Appendix
 - A. $b+B$ scattering wave function
 - B. Matrix element M_S^{DW}
 - C. Matrix element M_S^{CDCC}

PHYSICAL REVIEW C 84, 044616 (2011)

Theory of deuteron stripping: From surface integrals to a generalized R -matrix approach

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(Received 23 August 2011; published 21 October 2011)

There are two main reasons for the absence of a practical theory of stripping to resonance states that could be used by experimental groups: The numerical problem of the convergence of the distorted-wave Born approximation (DWBA) matrix element when the full transition operator is included and the ambiguity over what spectroscopic information can be extracted from the analysis of transfer reactions populating the resonance states. The purpose of this paper is to address both questions. The theory of the deuteron stripping is developed, which is based on the post continuum discretized coupled channels (CDCCs) formalism going beyond of the DWBA and surface integral formulation of the reaction theory [A. S. Kadyrov *et al.*, *Ann. Phys.* **324**, 1516 (2009)]. First, the formalism is developed for the DWBA and then it is extended to the CDCC formalism, which is the ultimate goal of this work. The CDCC wave function takes into account not only the initial elastic $d + A$ channel but also its coupling to the deuteron breakup channel $p + n + A$ missing in the DWBA. Stripping to both bound states and resonances is included. The convergence problem for stripping to resonance states is solved in the post CDCC formalism. The reaction amplitude is parametrized in terms of the reduced width amplitudes (asymptotic normalization coefficients), inverse level matrix, boundary condition, and channel radius, which are the same parameters used in the conventional R -matrix method. For stripping to resonance states, many-level and one- and two-channel cases are considered. The theory provides a consistent tool to analyze both binary resonant reactions and deuteron stripping in terms of the same parameters.

DOI: [10.1103/PhysRevC.84.044616](https://doi.org/10.1103/PhysRevC.84.044616)

PACS number(s): 24.30.-v, 25.45.Hi, 24.10.-i

Reminder: (d,p) formalism

Transition matrix element – DWBA and CDCC approximations

Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$

DWBA

$$\langle \varphi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle$$

3-body

$$\langle \varphi_F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_A \Psi_i^{3B(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Psi_i^{3B(+)} \rangle$$

CDCC

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Psi_i^{\text{CDCC}(+)} \rangle$$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$I_A^F = \langle \varphi_A | \varphi_F \rangle$ one-body overlap

$$M^{(\text{prior})} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Surface formalism for DWBA

$$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$

DWBA

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle$$

$$I_A^F = \langle \varphi_A | \varphi_F \rangle = I_A^F(r_{nA})$$

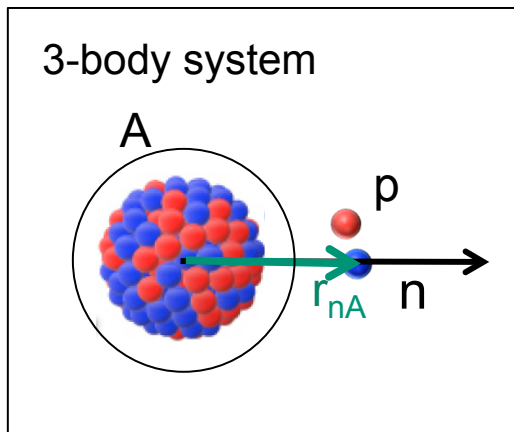
Mukhamed-zhanov

Interior + exterior

$$M^{(\text{post})} = M^{(\text{post})}(0, a) + M^{(\text{post})}(a, \infty)$$

$$M_{\text{surf}}(a) + M^{(\text{prior})}(a, \infty)$$

One-body overlap function of A and A+1 systems



Transition matrix element M:

- Cross section $\sigma \sim M^2$
- Is split into interior and exterior parts

Surface formulation

$$M = M^{(\text{post})}(0, a) + M_{\text{surf}}(a) + M^{(\text{prior})}(a, \infty)$$

model dependence asymptotic quantities

$$M_{\text{surf}}(a) = f(a, C_A^F, B_{nA})$$

B_{nA} = log derivative of I_A^F at surface radius a

ANC: C_A^F defined through: $I_A^F(r_{nA}) \rightarrow C_A^F W(kr_{nA})$
related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

Surface formalism for DWBA – bound states

DWBA

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle$$

$$I_A^F = \langle \varphi_A | \varphi_F \rangle = I_A^F(r_{nA})$$

Green's Theorem:

$$\begin{aligned} & \int_{r \leq R} d\mathbf{r} f(\mathbf{r}) [\overleftarrow{T} - \overrightarrow{T}] g(\mathbf{r}) \\ &= -\frac{1}{2\mu} \oint_{r=R} dS [g(\mathbf{r}) \nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r}) \nabla_{\mathbf{r}} g(\mathbf{r})] \\ &= -\frac{1}{2\mu} R^2 \int d\Omega_{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right]_{r=R} \end{aligned}$$

Surface term:

$$\begin{aligned} M_S^{\text{DW}}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) &= \frac{1}{2\mu_{nA}} R_{nA}^2 \int d\mathbf{r}_{pF} \chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{dA}} \\ &\times \left[\varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA}) \frac{\partial [I_A^F(\mathbf{r}_{nA})]^*}{\partial r_{nA}} - [I_A^F(\mathbf{r}_{nA})]^* \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA}=R_{nA}} \end{aligned}$$

$$\begin{aligned} \Delta \bar{V}_{pF} &= U_{pA} + V_{pn} - U_{pF} \\ &= [V_{pn} + U_{dA}] - [U_{pF}] + (U_{pA} - U_{dA}). \end{aligned} \quad (11)$$

$$\begin{aligned} M_{\text{ext}}^{\text{DW(post)}}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) &= M_S^{\text{DW}}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) \\ &+ M_{\text{ext}}^{\text{DW(prior)}}(\mathbf{k}_{pF}, \mathbf{k}_{dA}), \end{aligned} \quad (14)$$

$$M_S^{\text{DW}}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) = \langle \chi_{pF}^{(-)} I_A^F | \overleftarrow{T} - \overrightarrow{T} | \varphi_d \chi_{dA}^{(+)} \rangle \Big|_{r_{nA} > R_{nA}}. \quad (16)$$

Surface formalism for DWBA – bound states

$$M_S^{DW}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{j_{nA} m_{j_{nA}} m_{l_{nA}} M_n} \langle J_A M_A j_{nA} m_{j_{nA}} | J_F M_F \rangle \langle J_n M_n l_{nA} m_{l_{nA}} | j_{nA} m_{j_{nA}} \rangle \langle J_p M_p J_n M_n | J_d M_d \rangle \gamma_{nA j_{nA} l_{nA}} \\ \times \int d\mathbf{r}_{pF} \chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{nA}} Y_{l_{nA} m_{l_{nA}}}^*(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Big|_{r_{nA}=R_{nA}}$$

Asymptotic overlap:

$$I_{A j_{nA} l_{nA}}^F(R_{nA}) \stackrel{r_{nA} > R_{nA}}{\approx} C_{A j_{nA} l_{nA}}^F i^{l_{nA}+1} \kappa_{nA} h_{l_{nA}}^{(1)}(i\kappa_{nA} r_{nA})$$

ANC

Reduced-width amplitudes:

$$\gamma_{nA j_{nA} l_{nA}} = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} I_{A j_{nA} l_{nA}}^F(R_{nA}) \\ = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} i^{l_{nA}+1} \kappa_{nA} C_{A j_{nA} l_{nA}}^F h_{l_{nA}}^{(1)}(i\kappa_{nA} R_{nA})$$

Boundary condition:

$$B_{nA} = \frac{1}{h_{l_{nA}}^{(1)}(i\kappa_{nA} R_{nA})} \frac{d[r_{nA} h_{l_{nA}}^{(1)}(i\kappa_{nA} r_{nA})]}{dr} \Big|_{r_{nA}=R_{nA}}$$

$$M_{\text{ext}}^{DW(\text{prior})}(\mathbf{k}_{pF}, \mathbf{k}_{dA})$$

Suggestion: Parametrize in terms of ANC

Surface formulation

$$\mathbf{M} = \mathbf{M}^{(\text{post})}(\mathbf{0}, \mathbf{a})$$

$$+ \mathbf{M}_{\text{surf}}(\mathbf{a})$$

$$+ \mathbf{M}^{(\text{prior})}(\mathbf{a}, \infty)$$

Model dependent! Small?

Asymptotic quantities

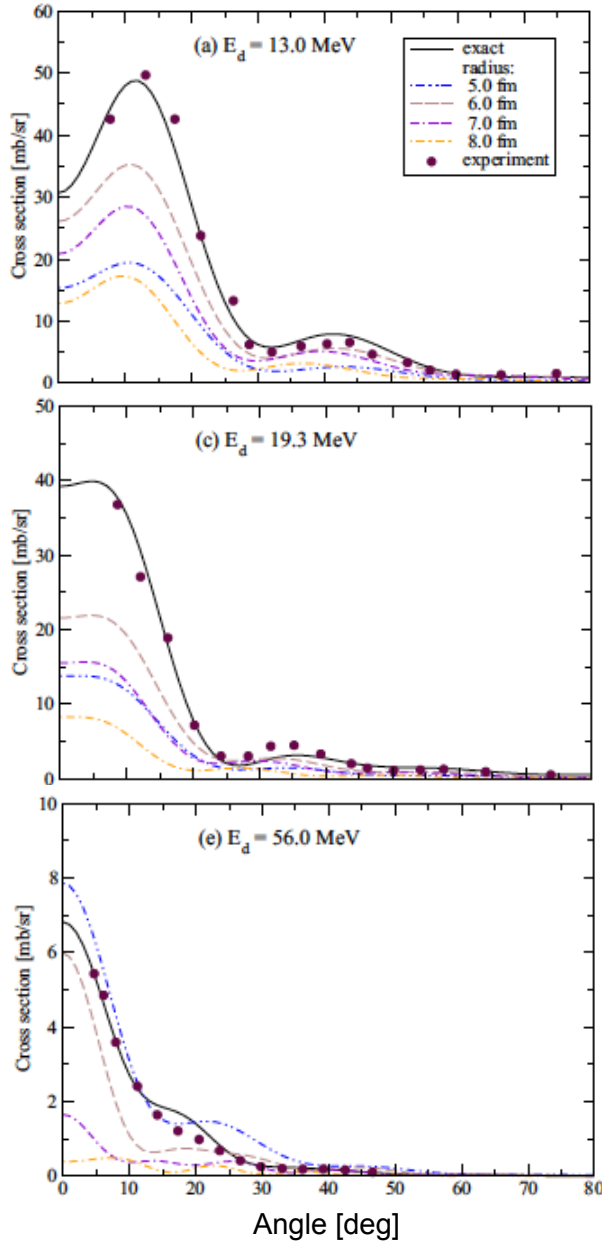
... and analogously for $\mathbf{M}^{\text{prior}}$

Also...

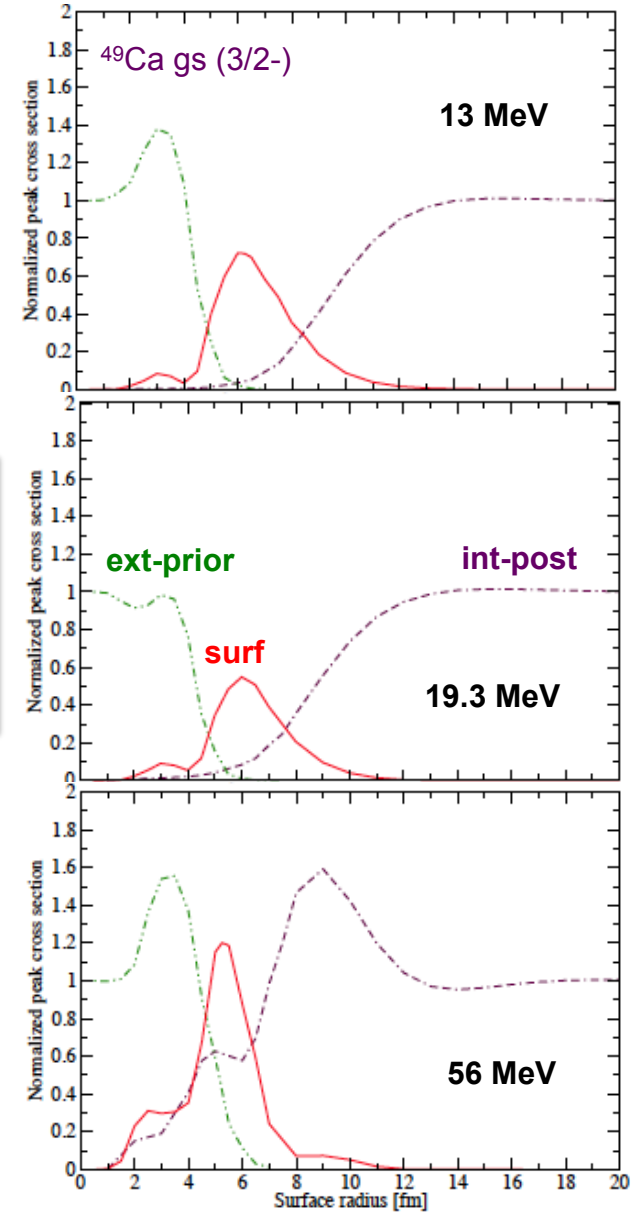
$$\mathbf{M}_{\text{surf}}(\mathbf{a}) = \mathbf{M}^{(\text{prior})}(\mathbf{0}, \mathbf{a}) - \mathbf{M}^{(\text{post})}(\mathbf{0}, \mathbf{a})$$

Numerical tests of the formalism (DWBA) – $^{48}\text{Ca}(d,p)$ at $E_d=13, 19.3, 56 \text{ MeV}$

Angular cross section – Surface term only



Peak cross section relative to full calculation



Surface term approximation improves with decreasing energy

Calculations for ^{49}Ca 1st (1/2-) give similar results

Surface formalism for CDCC – bound states

$$M^{(\text{post})} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$



CDCC

$$\langle I_A^F \chi_{pF}^{(-)} | \Delta V_{pF} | \Psi_i^{\text{CDCC}(+)} \rangle$$

$$\Delta \bar{V}_{pF}^{P_{pn}} = U_{pA}^{P_{pn}} + V_{pn} - U_{pF} = [-U_{pF}] + [U_{pA}^{P_{pn}} + V_{pn}]. \quad (58)$$

$$\begin{aligned} M_{\text{ext}}^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) &\equiv M_S^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) \\ &= \langle \chi_{pF}^{(-)} I_A^F | \overleftarrow{T} - \overrightarrow{T} | \Psi_i^{\text{CDCC}(+)} \rangle \Big|_{r_{nA} > R_{nA}}, \end{aligned} \quad (62)$$

Surface term:

$$\begin{aligned} &M_S^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) \\ &= \frac{R_{nA}^2}{2\mu_{nA}} \int d\mathbf{r}_{pF} \chi_{-k_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{r_{nA}} \left[\Psi_i^{\text{CDCC}(+)}(\mathbf{r}_{pF}, \mathbf{r}_{nA}) \frac{\partial [I_A^F(\mathbf{r}_{nA})]^*}{\partial r_{nA}} - [I_A^F(\mathbf{r}_{nA})]^* \frac{\partial \Psi_i^{\text{CDCC}(+)}(\mathbf{r}_{pF}, \mathbf{r}_{nA})}{\partial r_{nA}} \right] \Big|_{r_{nA}=R_{nA}}. \\ &= -M_{S_{R_{nA}}}^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{j_{nA} m_{j_{nA}} m_{l_{nA}} M_n} \langle J_A M_A j_{nA} m_{j_{nA}} | J_F M_F \rangle \\ &\quad \times \langle J_n M_n l_{nA} m_{l_{nA}} | j_{nA} m_{j_{nA}} \rangle \gamma_{nA j_{nA} l_{nA}} \int d\mathbf{r}_{pF} \chi_{-k_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{r_{nA}} Y_{l_{nA} m_{l_{nA}}}^*(\hat{\mathbf{r}}_{nA}) \\ &\quad \times \left[\Psi_i^{\text{CDCC}(+)}(\mathbf{r}_{pF}, \mathbf{r}_{nA}) (B_{nA} - 1) - R_{nA} \frac{\partial \Psi_i^{\text{CDCC}(+)}(\mathbf{r}_{pF}, \mathbf{r}_{nA})}{\partial r_{nA}} \right] \Big|_{r_{nA}=R_{nA}}. \end{aligned}$$

Note: CDCC post form has no prior-extrior contribution in PRC 84 (2011):

$$\begin{aligned} &M^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) \\ &= M_{\text{int}}^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}) \\ &\quad - M_{S_{R_{nA}}}^{\text{CDCC}(\text{post})}(\mathbf{k}_{pF}, \mathbf{k}_{dA}), \end{aligned}$$

Derivation for CDCC prior form does give a prior-extrior contribution! (???)

Surface formalism for DWBA – resonance states

d+A → p + (n+A)

↓
b+B

$$M^{DW(post)}(P, \mathbf{k}_{dA}) = \langle \chi_{pF}^{(-)} \Psi_{bB}^{(int)(-)} | \Delta \bar{V}_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle, \quad (94)$$

Surface formulation

M = M^(post)(0, a)

+ M_{surf}(a)

+ M^(prior)(a, ∞)

f(Γ^{1/2}, [A⁻¹], I_A^F):

contribution hopefully small

b + B = n + A

b + B ≠ n + A

b + B = n + A

b + B ≠ n + A

Total post matrix element for b + B ≠ n + A example:

$$\begin{aligned} M^{DW(post)}(P, \mathbf{k}_{dA}) = & 2\pi \sqrt{\frac{1}{\mu_{bB} k_{bB}}} \sum_{J_F M_F s' l' m'_l m_l M_n} i^l \langle s m_s l m_l | J_F M_F \rangle \langle s' m_{s'} l' m'_l | J_F M_F \rangle \langle J_n M_n J_A M_A | s' m_{s'} \rangle \langle J_n M_n J_p M_p | J_d M_d \rangle \\ & \times e^{-i\delta_{bB}^{nl}} Y_{lm_l}^*(-\hat{\mathbf{k}}_{bB}) \sum_{\nu, \tau=1}^N [\Gamma_{\nu b B s l J_F}(E_{bB})]^{1/2} [A^{-1}]_{\nu\tau} \left\{ \langle \chi_{pF}^{(-)} I_{A s' l' J_F}^F | \Delta \bar{V}_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle \Big|_{r_{nA} \leq R_{nA}} \right. \\ & + \sqrt{\frac{2\mu_{nA}}{R_{nA}}} \gamma_{\tau n A s' l' J_F} \left\langle \chi_{pF}^{(-)} \frac{O_{l'}^*(k_{nA}, r_{nA})}{r_{nA}} \frac{R_{nA}}{O_{l'}^*(k_{nA}, R_{nA})} Y_{l' m_{l'}}^*(\hat{\mathbf{r}}_{nA}) \Big| \Delta \bar{V}_{dA} \Big| \varphi_d \chi_{dA}^{(+)} \right\rangle \Big|_{r_{nA} > R_{nA}} + \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \gamma_{\tau n A s' l' J_F} \\ & \times \left. \int d\mathbf{r}_{pF} \chi_{-k_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{\tau n A} Y_{l' m_{l'}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{p n}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{p n}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Big|_{r_{nA} = R_{nA}} \right\}. \end{aligned} \quad (117)$$

Analogously for CDCC resonance case

Concluding Notes

Surface formalism for studying resonances with (d,p):

- Uses successful R-matrix ideas to emphasize **asymptotic properties** of the wave function
- Separation into interior and exterior leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing **meaningful spectroscopic information**
- Our studies withing DWBA implementation show that the surface term is dominant; dependence on model for nuclear interior is reduced
- Including breakup via CDCC removes is being implemented and tested
- **More checks** of the formalism itself are required
- Formalism paves way to move beyond conceptual and practical problems

The Surface formalism is a promising approach for transfer reactions with unstable isotopes.

Study: testing the claims

The surface formalism: a closer look

Features:

- Model dependence contained in interior term
- Surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.
- Exterior term vanishes in CDCC implementation

Questions:

- **Is the surface term dominant?
Where?**
- **Do we get reduced dependence on model for interior?
Under which circumstances?**
- **Can we extract useful spectroscopic quantities from comparison to experiment?**

Surface formulation

$$\mathbf{M} = \underbrace{\mathbf{M}^{(\text{post})}(\mathbf{0}, \mathbf{a})}_{\text{model dependence}} + \underbrace{\mathbf{M}_{\text{surf}}(\mathbf{a}) + \mathbf{M}^{(\text{prior})}(\mathbf{a}, \infty)}_{\text{asymptotic quantities}}$$

$$M_{\text{surf}}(\mathbf{a}) = f(\mathbf{a}, C_A^F, B_{nA})$$

$$B_{nA} = \log \text{ derivative of } I_A^F \text{ at surface radius } a$$

$$\text{ANC: } C_A^F \text{ defined through: } I_A^F(r_{nA}) \rightarrow C_A^F W(kr_{nA}) \\ \text{related to reduced width amplitude } C_A^F \sim \gamma_{nA}$$

Tests completed:

- $^{90}\text{Zr}(d,p)$ for $E_d=11$ MeV
 - ^{91}Zr gs, 1st excited state, **2f_{7/2} resonance**
- $^{48}\text{Ca}(d,p)$ for $E_d=13, 19.3, 56$ MeV
 - ^{49}Ca gs, 1st excited state
- $^{20}\text{O}(d,p)$ for $E_d=21$ MeV
 - ^{21}O gs, 1st excited state, **1d_{3/2} and 1f_{7/2} resonances**
- Akram:
 - $^{12}\text{C}(d,p)$ for $E_d=30$ MeV
 - $^{40}\text{Ca}(d,p)$ for $E_d=34.4$ MeV
 - $^{209}\text{Pb}(d,p)$ for $E_d=52$ MeV

Internal, surface, external contributions – $^{90}\text{Zr}(d,p)$ at $E_d=11$ MeV

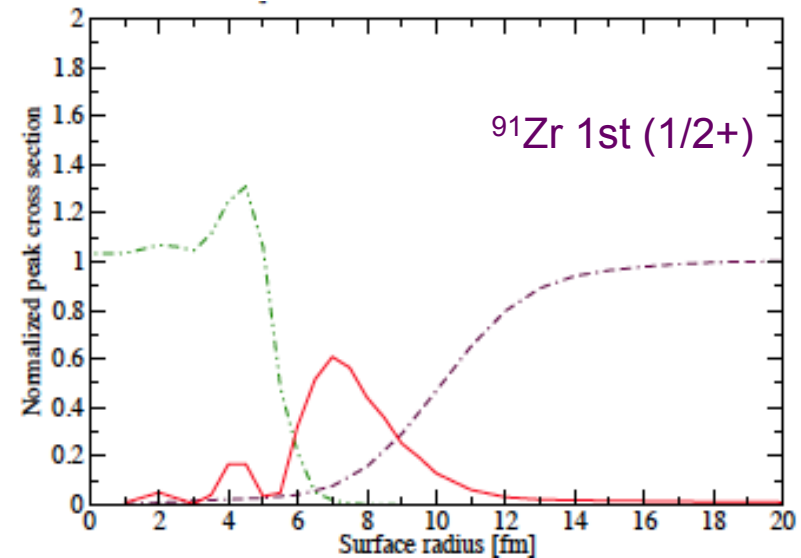
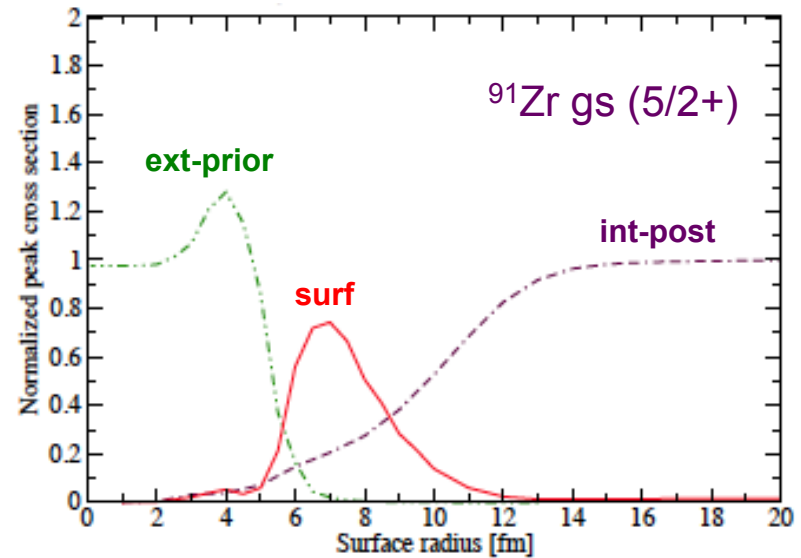
$$M = \underbrace{M^{(\text{post})}(0, a)}_{\text{model dependence}} + \underbrace{M_{(\text{surf})}(a) + M^{(\text{prior})}(a, \infty)}_{\text{asymptotic quantities}}$$

Observations

- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

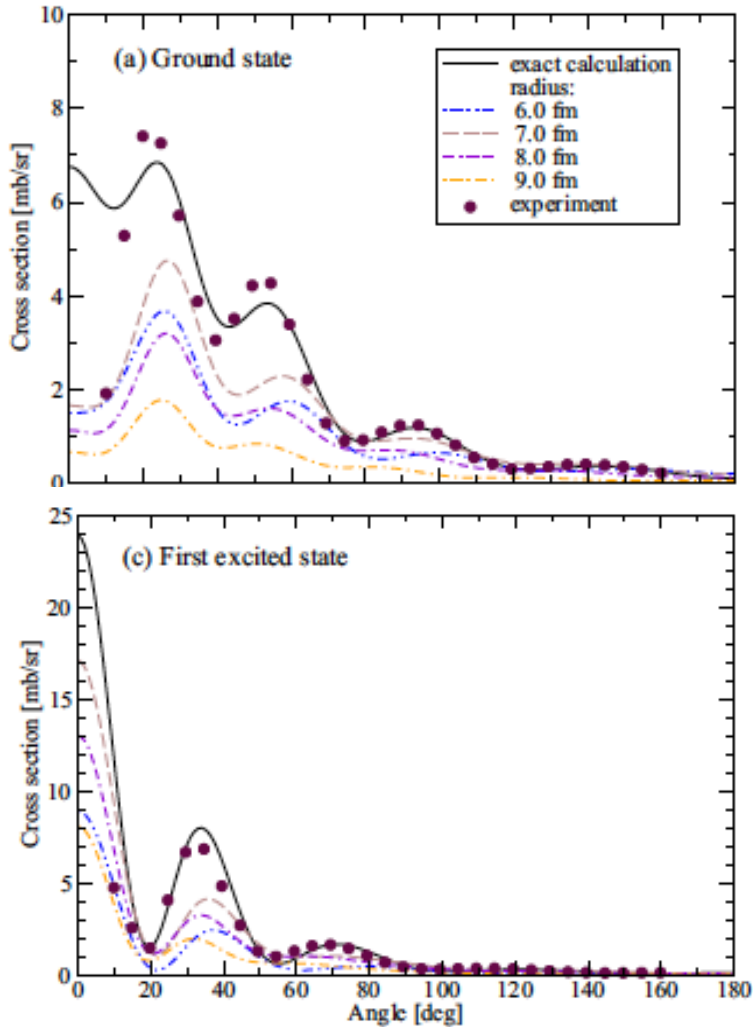
Peak cross section relative to full calculation



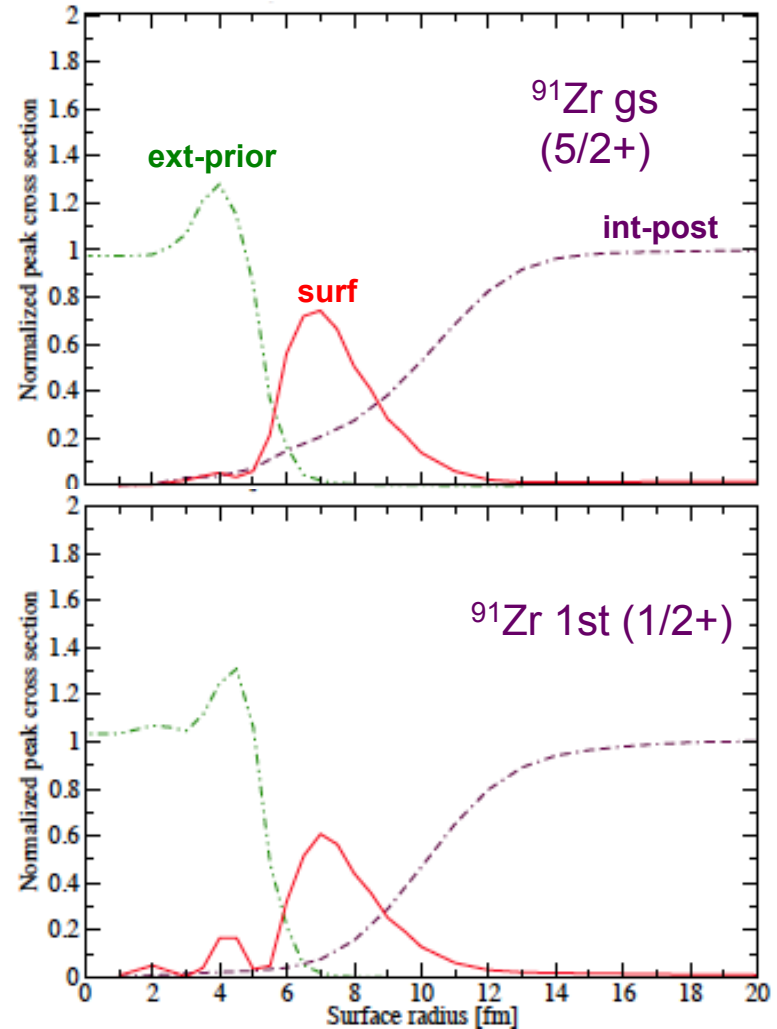
Escher, Thompson, Mukhamedzhanov, JPCS (2012).

The surface contribution – $^{90}\text{Zr}(d,p)$ at $E_d=11$ MeV

Angular cross section – Surface term only



Peak cross section relative to full calculation

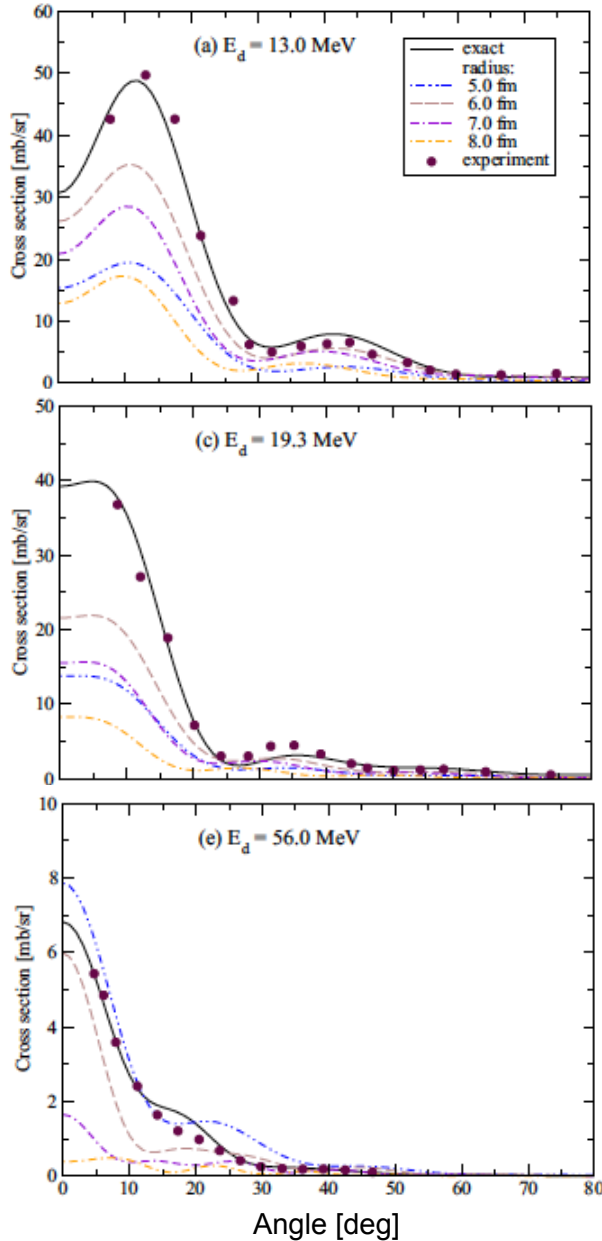


Escher et al, PRC (2014)

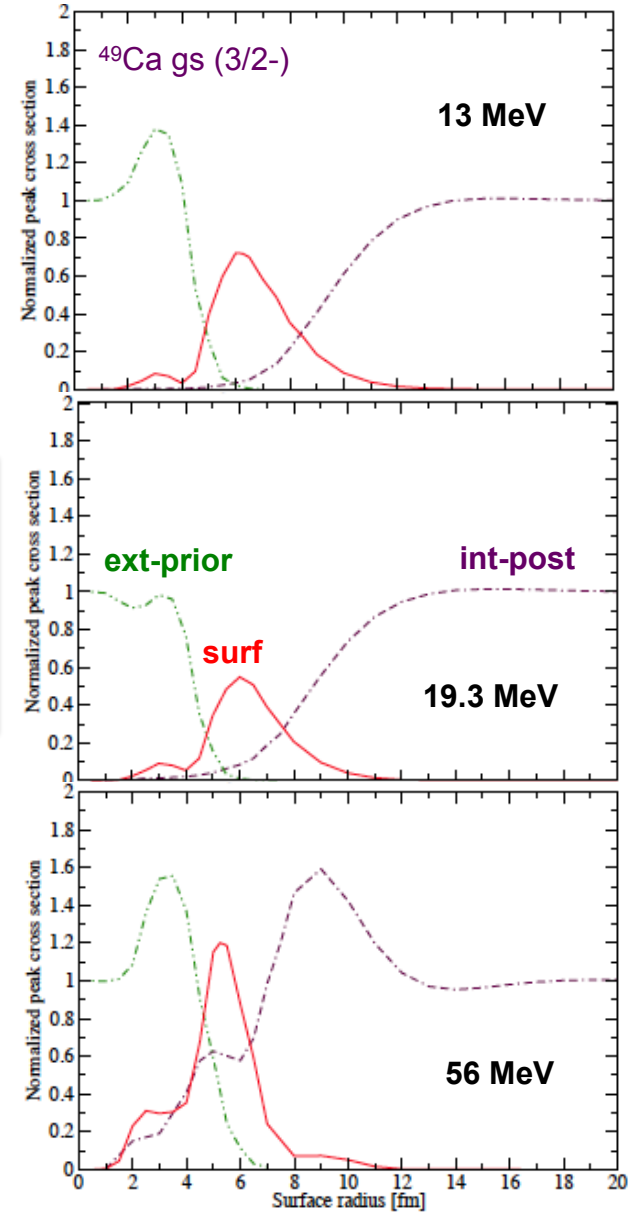
- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain

Effect of varying the beam energy – $^{48}\text{Ca}(d,p)$ at $E_d=13, 19.3, 56 \text{ MeV}$

Angular cross section – Surface term only



Peak cross section relative to full calculation

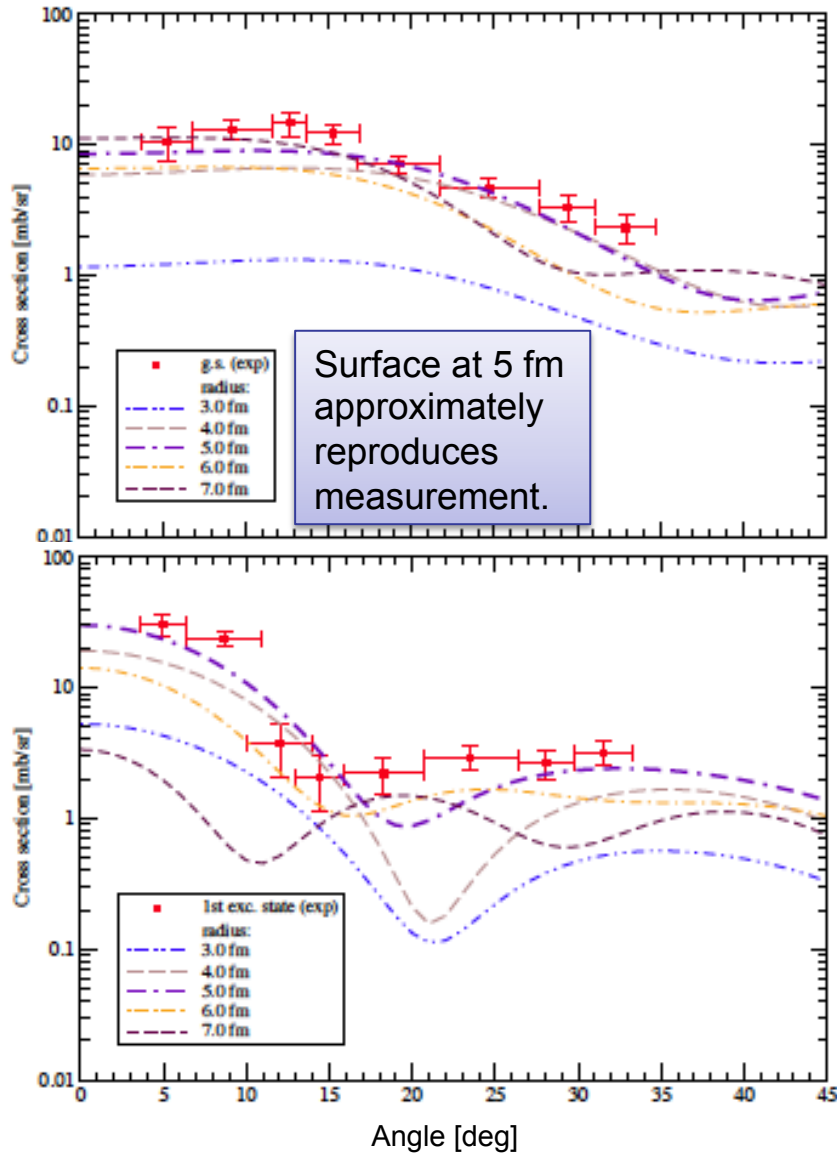


↑
Surface term approximation improves with decreasing energy
↓

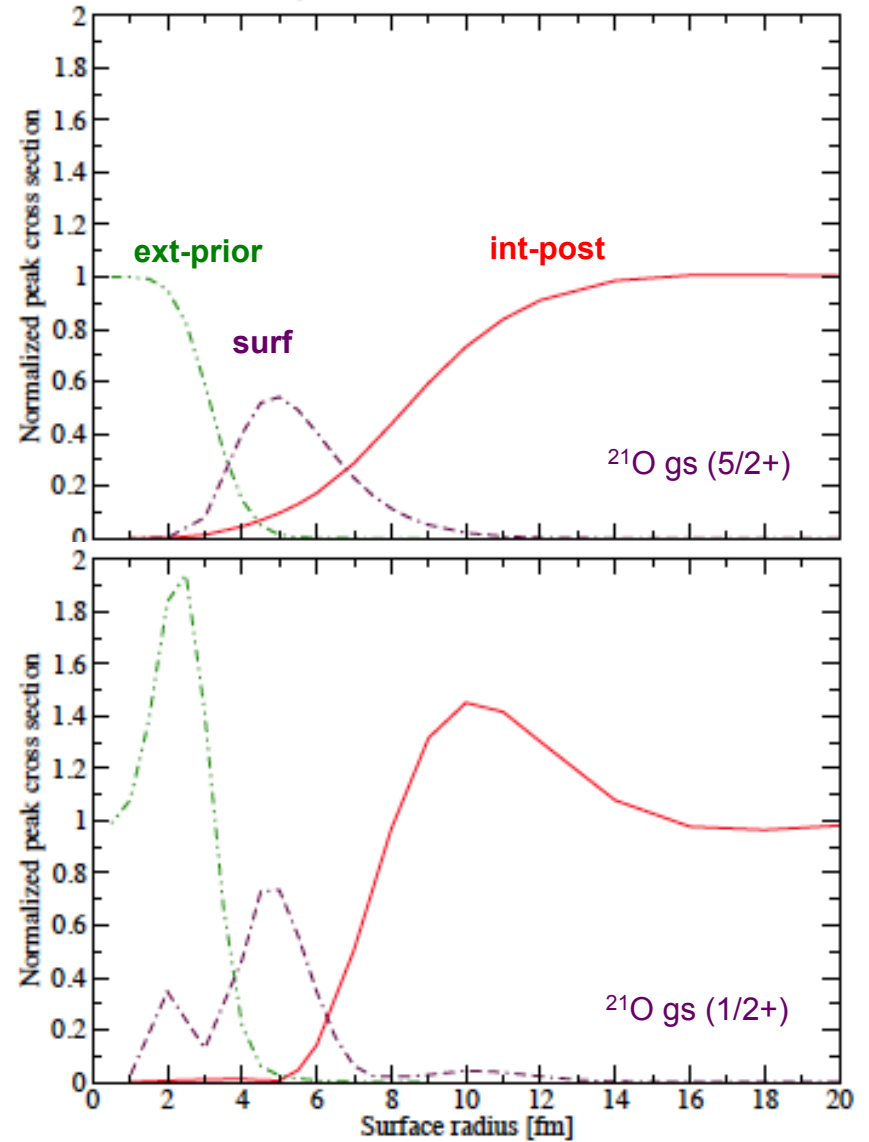
Calculations for ^{49}Ca 1st (1/2-) give similar results

The oxygen case - ^{20}O at $E_d=21$ MeV

Angular cross section – Surface term only

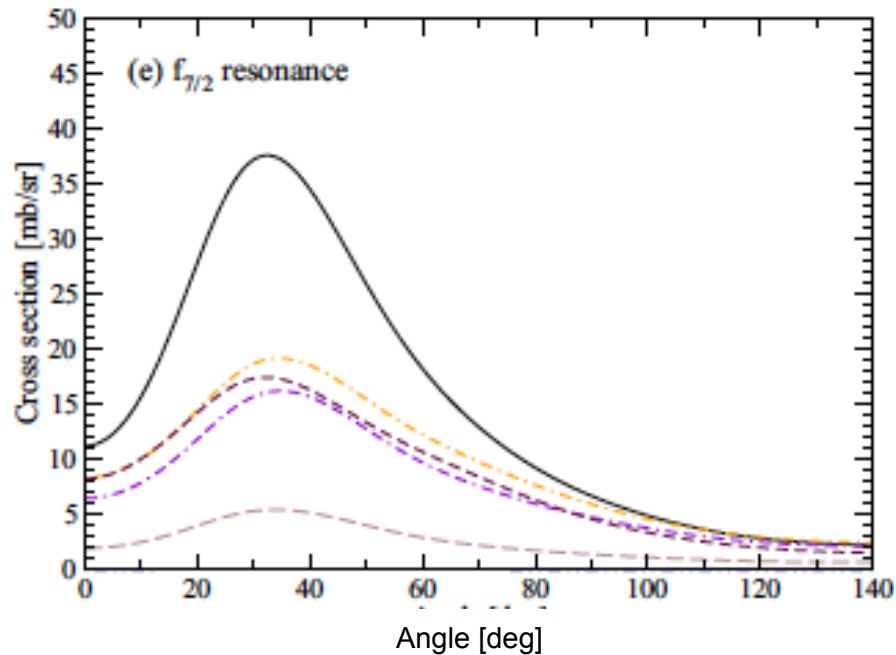


Peak cross section relative to full calculation

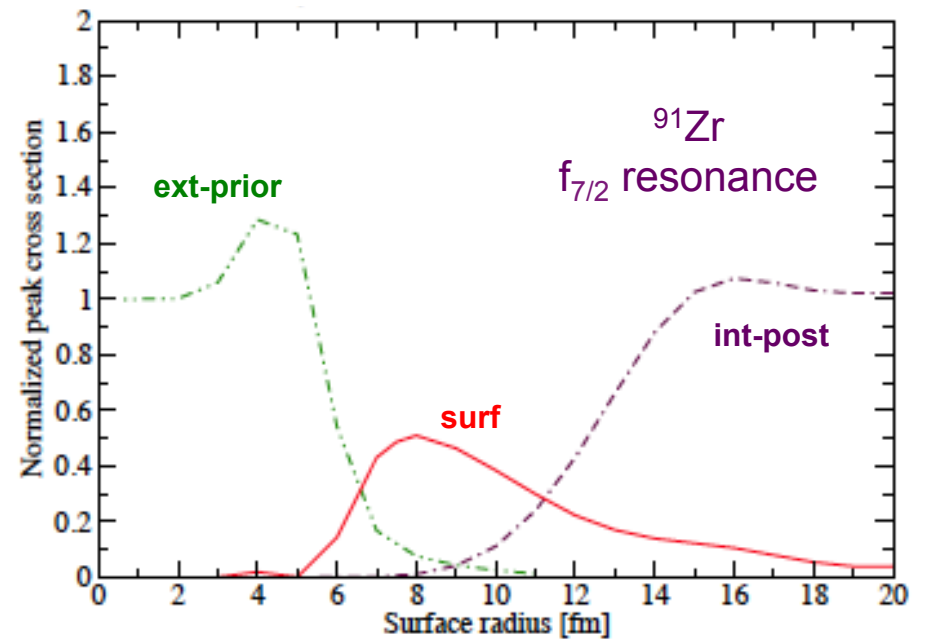


Resonances – ^{90}Zr at $E_d=11$ MeV

Angular cross section – Surface term only



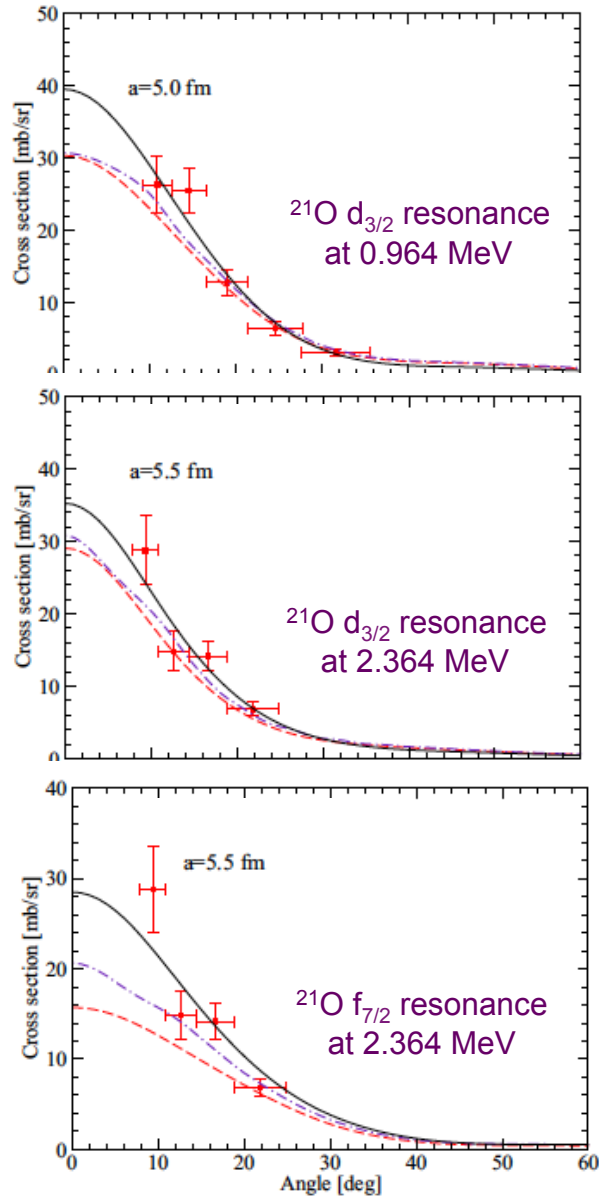
Peak cross section relative to full calculation



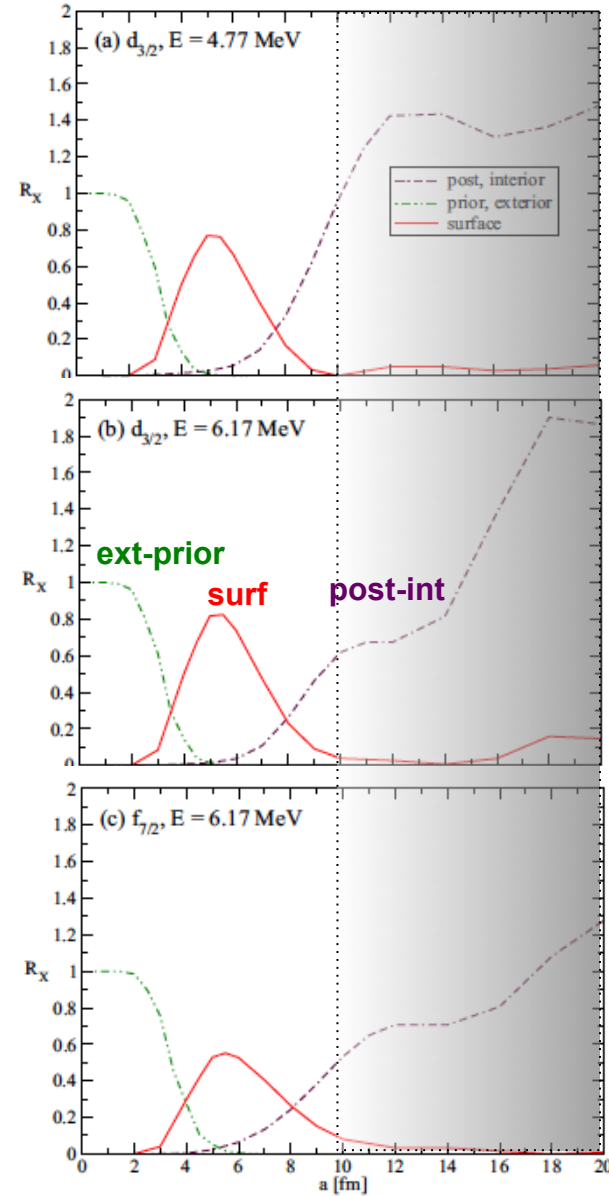
- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute

Resonances - ^{20}O at $E_d=21$ MeV

Angular cross section – Surface term only

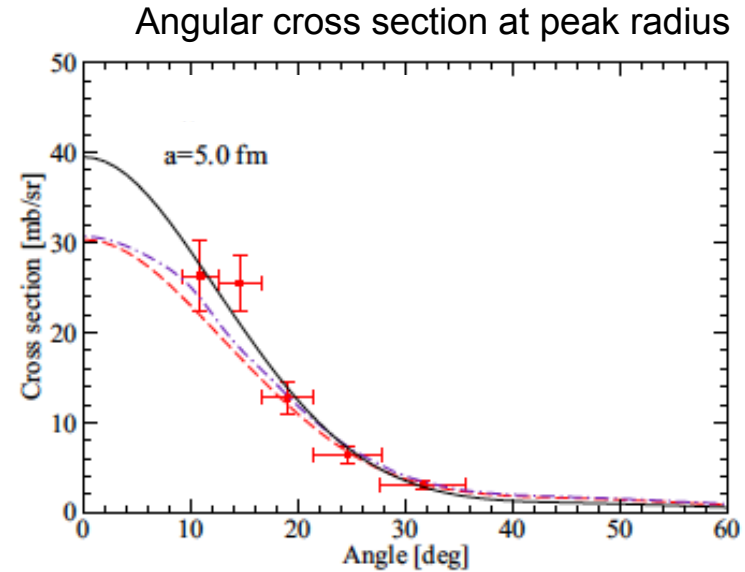
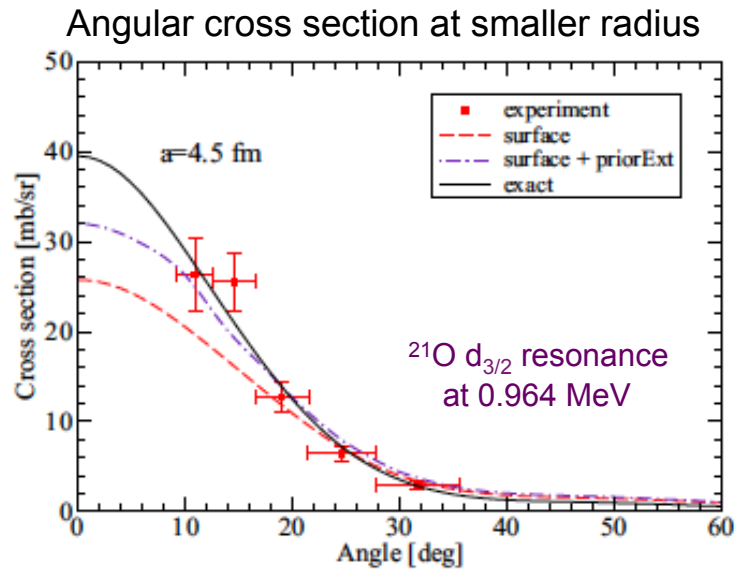


Peak cross section relative to full calculation

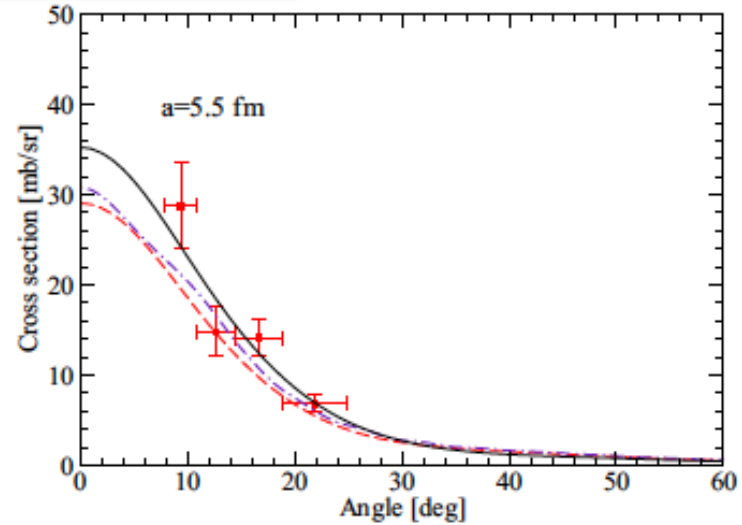
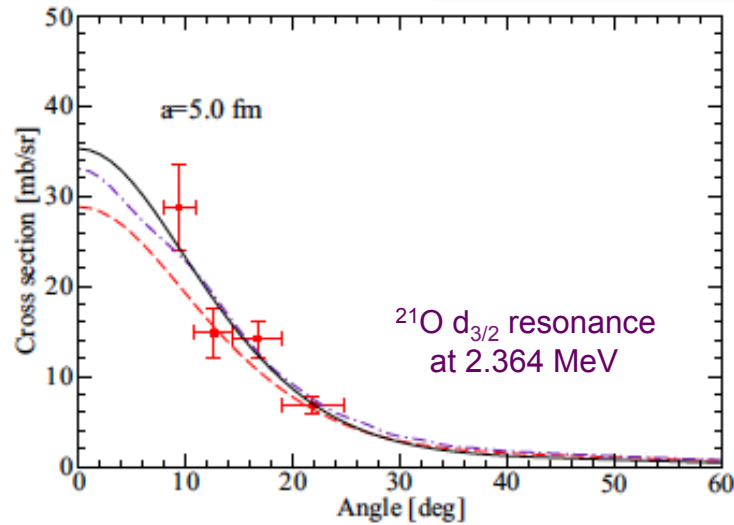


- Convergence difficult for resonance cases
- Surface term seems able to reproduce data
- Additional contribution play a role

Moving forward - ^{20}O at $E_d=21$ MeV



- ←
- reducing the surface radius
 - adding prior-exterior contribution
-



TORUS Collaboration

ReactionTheory.org

TORUS: Theory of Reactions for Unstable iSotopes
A Topical Collaboration for Nuclear Theory

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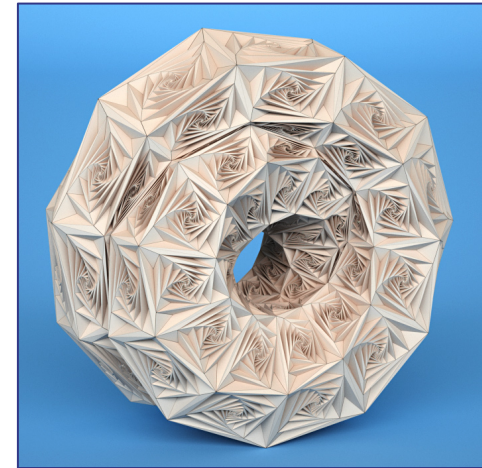
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Theory of Reactions for Unstable iSotopes

A Topical Collaboration to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations and by developing a new partial-fusion theory to integrate descriptions of direct and compound-nucleus reactions. This multi-institution collaborative effort is directly relevant to three areas of interest identified in the solicitation: (b) properties of nuclei far from stability; (c) microscopic studies of nuclear input parameters for astrophysics and (e) microscopic nuclear reaction theory.



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