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Towards proton-nucleus formfactors

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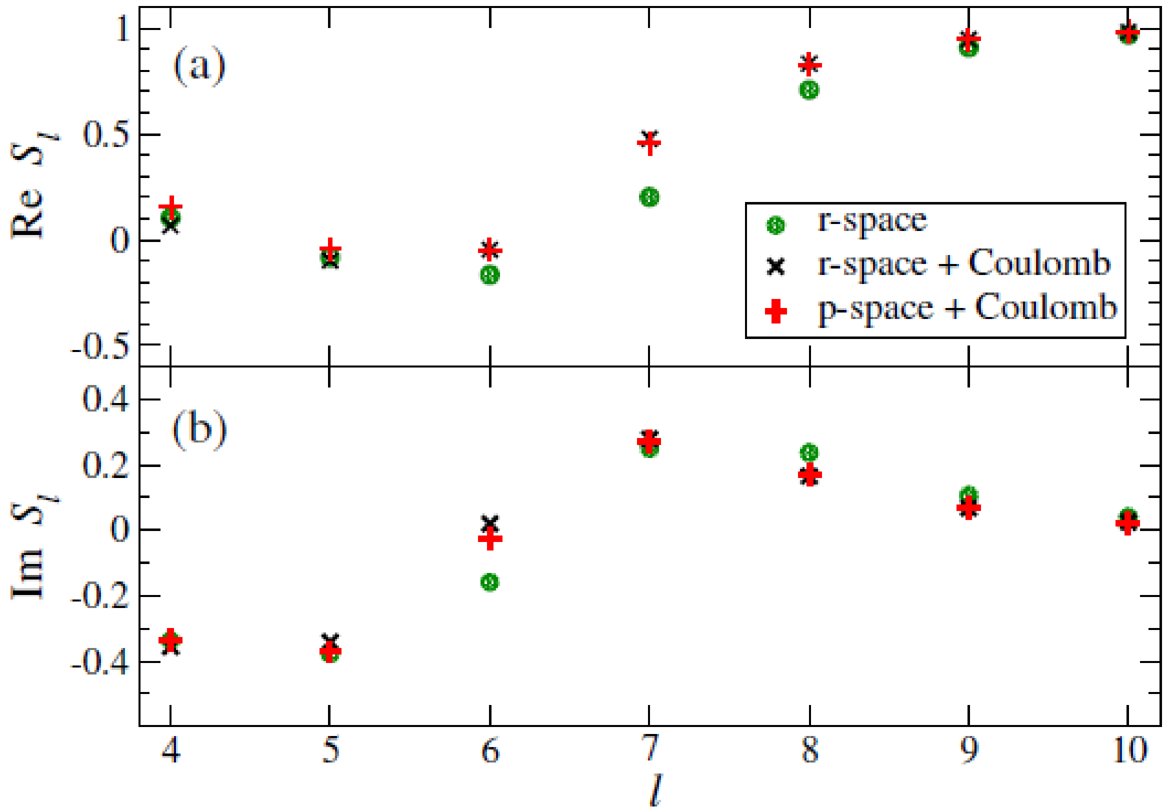

partial wave S-matrix $S_{L+1/2}$, for the p+^{48}Ca @ 38 MeV

For higher partial waves r-space and p-space agree
For lower they do not.

First: understand this calculation

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Reminder: Ernst-Shakin-Thaler (EST)

partial wave t-matrix :

$$\langle p'|t(E)|p\rangle = \frac{\langle p'|V|\Psi_{k_E}^{(+)}\rangle\langle\Psi_{k_E}^{(+)}|V|p\rangle}{\langle\Psi_{k_E}^{(+)}|V - V g_0(E) V|\Psi_{k_E}^{(+)}\rangle}$$

Reminder:

$$V|\Psi_{k_E}^{(+)}\rangle := t|k_E\rangle$$

The EST construction guarantees:

At a given scattering energy E_{kE} the scattering wave functions obtained with the original potential V and the separable potential V are identical . ➔ the half-shell t-matrices are identical

EST construction carried out in plane wave basis

The derivation uses that one has a complete set of states.

Reminder: Technical details of generalized EST

Let $|f_{l,kE}>$ be a radial wave function and $K_0|f_{l,kE}> = |f^*_{l,kE}>$

Rank-1 separable t-matrix: $\langle p'|t(E)|p\rangle = \dfrac{\langle p'|u|f_{l,k_E}\rangle\langle f^*_{l,k_E}|u|p\rangle}{\langle f^*_{l,k_E}|u - ug_0(E)u|f_{l,k_E}\rangle}$

With $t(p',k_E,E_{k_E}) = \langle f^*_{l,k_E}|u|p'\rangle$ and $t(p,k_E,E_{k_E}) = \langle p|u|f_{l,k_E}\rangle$

$$\boxed{\langle p'|t(E)|p\rangle = \frac{t(p',k_E,E_{k_E})\,t(p,k_E,E_{k_E})}{\langle f^*_{l,k_E}|u(1-g_0(E)u)|f_{l,k_E}\rangle} \equiv t(p',k_E,E)\;\tau(E)\;t(p,k_E,E)}$$

and

$$\begin{aligned}
\tau(E)^{-1} &= t(k_E,k_E,E_{k_E}) \\
&+2\mu\left[\mathcal{P}\int dpp^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_E^2-p^2} - \mathcal{P}\int dpp^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_0^2-p^2}\right] \\
&+i\pi\mu\Big[k_0 t(k_0,k_E,E_{k_E})t(k_0,k_E,E_{k_E}) - k_E\; t(k_E,k_E,E_{k_E})t(k_E,k_E,E_{k_E})\Big]\;.
\end{aligned}$$

EST construction based on:

- solve the scattering problem in complete basis
- require that for a set energies E $_i$ the wave functions (half-shell t-matrices) obtained with the original and separable potential coincide.

➔ EST construction can be performed in the Coulomb basis

$$t_l^{CN}(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}^C(E) \langle f_{l,k_{E_j}}^*|u$$

$$\sum_j \tau_{ij}^C(E) \langle f_{l,k_{E_j}}^*|u - u g_C(E) u|f_{l,k_{E_k}}\rangle = \delta_{ik}$$

$$\hat{g}_C(E_{p_0}) = (E - \hat{H}^C + i\varepsilon)^{-1} \qquad \hat{H}^C = H_0 + \hat{V}^C$$

Coulomb Green's function

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$$t_l^{CN}(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}^C(E) \langle f^*_{l,k_{E_j}}|u$$

Multiply from left and right with a Coulomb state:

$$\langle \psi^C_{l,k_E}|u|f_{l,k_E}\rangle = \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, u_l(p)\, (\psi^C_{l,k_E})^*(p) \equiv u_l^C(k_E)$$

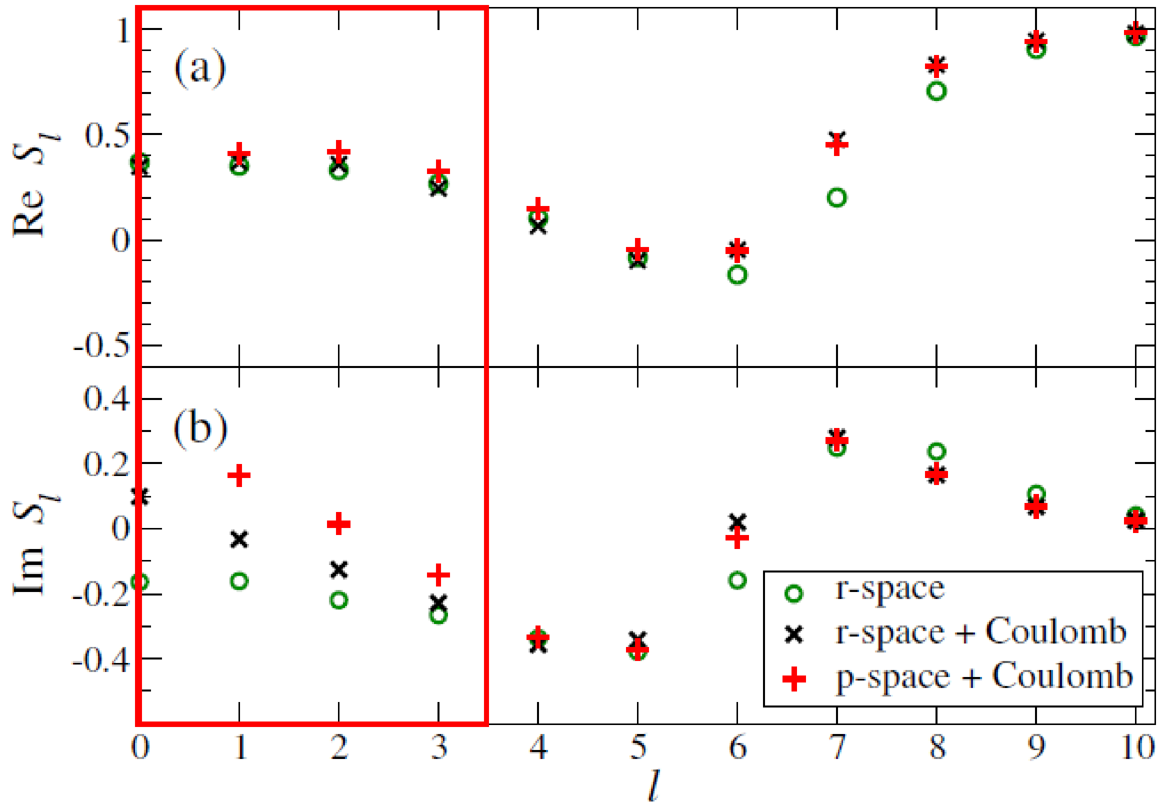
$$\langle f^*_{l,k_E}|u|\psi^C_{l,k_E}(k_E)\rangle = \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, u_l(p)\, \psi^C_{l,k_E}(p) \equiv (u_l^C)^\dagger(k_E)$$

Graphs in PRC paper

However: Not a consistent EST construction.

We used the n-A t-matrix calculated in a plane wave basis as input.

➔ Wave functions obtained with original and separable potential are not the same at the support points if calculated in different bases.



Why do the higher partial waves agree?

General: the higher l and the higher E for given charge Z:
the closer Coulomb functions resemble plane waves

For the higher l's the n+A t-matrices probably are already very close to the p+A t-matrices.

We need the p+A half-shell t-matrices
for a charged particle EST generalization

Non-trivial (due to "pinch singularity")

Use approach by Elster, Liu, Thaler, JPG 19, 2123 (1993)

Solve Lippmann-Schwinger equation in Coulomb distorted basis:

$$\langle k'|\tau_l(E)|k\rangle = \langle k'|U_l|k\rangle + \int \langle k'|U_l|k''\rangle \frac{4\pi k''^2\,\mathrm{d}k''}{E - E'' + \mathrm{i}\varepsilon} \langle k''|\tau_l(E)|k\rangle$$

Looks like a regular LS equation if potential element

$$\langle k'|U_l|k\rangle = \langle (\phi_l^{\mathrm{C}})^{(+)}(k')|V^{\mathrm{S}}|(\phi_l^{\mathrm{C}})^{(+)}(k)\rangle$$

This is the hard part!

$$\langle k'|U_l|k\rangle = \langle(\phi_l^{C})^{(+)}(k')|V^{S}|(\phi_l^{C})^{(+)}(k)\rangle$$

Matrix elements exist and are well defined if V^S is finite-ranged ✔

We solved this as:

$$\langle k'|U_l|k\rangle = \int \langle\phi_l^{C}(k')|r'\rangle r'^2\,\mathrm{d}r'\,\underline{\langle r'|V_l^{S}|r''\rangle}\,r''^2\,\mathrm{d}r''\,\langle r''|\phi_l^{C}(k)\rangle$$

Linda has this already

$$\langle r'|V_l^{S}|r\rangle = \frac{2}{\pi}\int j_l(k''r')k''^2\,\mathrm{d}k''\,\langle k''|V_l^{S}|k'''\rangle k'''^2\,\mathrm{d}k'''\,j_l(k'''r)$$