Separabilization of Optical Potentials

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Chapel Hill 89 Global Optical Potential (CH 89) Fourier Transform

- **CH89 short range (central):**

$$U(r) = -V_r f_{ws}(r, a_r, R_r) - iW f_{ws}(r, a_i, R_i) - iW_s(-4a_s)\frac{d}{dr} f_{ws}(r, a_s, R_s)$$

$$f_{ws}(r, a, R) = \frac{1}{1+\exp(\frac{r-R}{a})}.$$

- First term FT: $V(q) = -\frac{V_r}{2\pi^2}\frac{a_r^2}{q}\, Im \int_0^\infty dz \frac{z e^{i\rho z}}{1+e^{z-\alpha}}$:= $-\frac{V_r}{2\pi^2}\frac{a_r^2}{q}\, Im\ I_1(\rho, \alpha)$

$q = |\mathbf{k}' - \mathbf{k}|$, $\rho = qa$, $\alpha = R/a$ and $\gamma = e^{-\alpha}$

- Residues: $-[\alpha + i\pi(2n+1)]e^{i\rho\alpha}e^{-(2n+1)\pi\rho}$ where $n \in \mathbf{Z}$

- Path: (1) straight: 0 to ∞, (2) circular: ∞ to $i\infty$, and (3) straight: $i\infty$ to 0

- $Im\ I_1(\rho, \alpha) = \frac{2\pi e^{-\pi\rho}}{(1-e^{-2\pi\rho})^2}\left[\pi(1+e^{-2\pi\rho})\sin(\rho\alpha) - \alpha(1-e^{-2\pi\rho})\cos(\rho\alpha)\right]$

$$-2\sum_{n=0}^{\infty}(-1)^n\gamma^n\frac{n\rho}{(\rho^2+n^2)^2}.$$

- sum converges fast: $\gamma \sim 3\times 10^{-3}$ for ^{40}Ca.

Separabilization: EST Scheme for Non-Hermitian Potentials

- EST separable potential [1]:

$$\mathbf{V} = \sum_{ij} v|\Psi_i^{(+)}\rangle\langle\Psi_i^{(+)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|v$$

$$\delta_{ik} = \sum_j \langle\Psi_i^{(+)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|v|\Psi_k^{(+)}\rangle = \sum_j \langle\Psi_i^{(+)}|v|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|M|\Psi_k^{(+)}\rangle$$

- Not invariant under time reversal for $V^\dagger \neq V$. (i.e. $KVK \neq V$).

- Modified EST obeys reciprocity:

$$\mathbf{V} = \sum_{ij} v|\Psi_i^{(+)}\rangle\langle\Psi_i^{(-)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(-)}|v \;:= \sum_{ij} v|\Psi_i^{(+)}\rangle\lambda_{ij}\langle\Psi_j^{(+)}|v$$

$$\delta_{ik} = \sum_j \langle\Psi_i^{(-)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(-)}|v|\Psi_k^{(+)}\rangle = \sum_j \langle\Psi_i^{(-)}|v|\Psi_j^{(+)}\rangle\langle\Psi_j^{(-)}|M|\Psi_k^{(+)}\rangle$$

- $v|\Psi_i^{(+)}\rangle$, $\langle\Psi_i^{(-)}|v$:= half-shell t-matrix at E_i:= EST support points

- $V|\Psi_i^{(+)}\rangle = v|\Psi_i^{(+)}\rangle \Leftrightarrow \langle\Psi_i^{(-)}|V = \langle\Psi_i^{(-)}|v \Rightarrow$ exact
  $\&$ separable half-shell t-matrices match at support points

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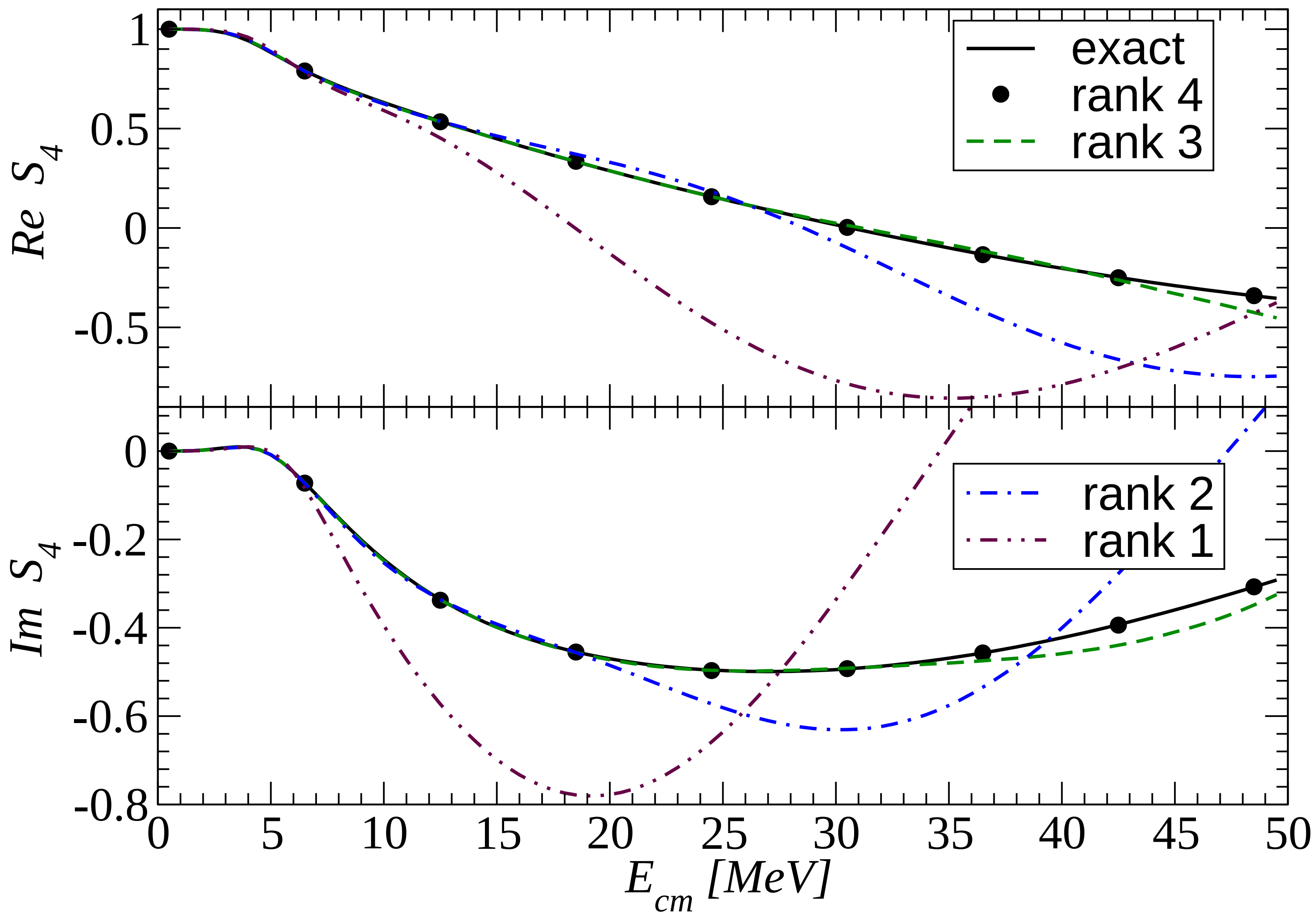
[1] Ernst, Shakin and Thaler (EST), PRC **8**, 48 (1973)

EST Scheme in Momentum Space

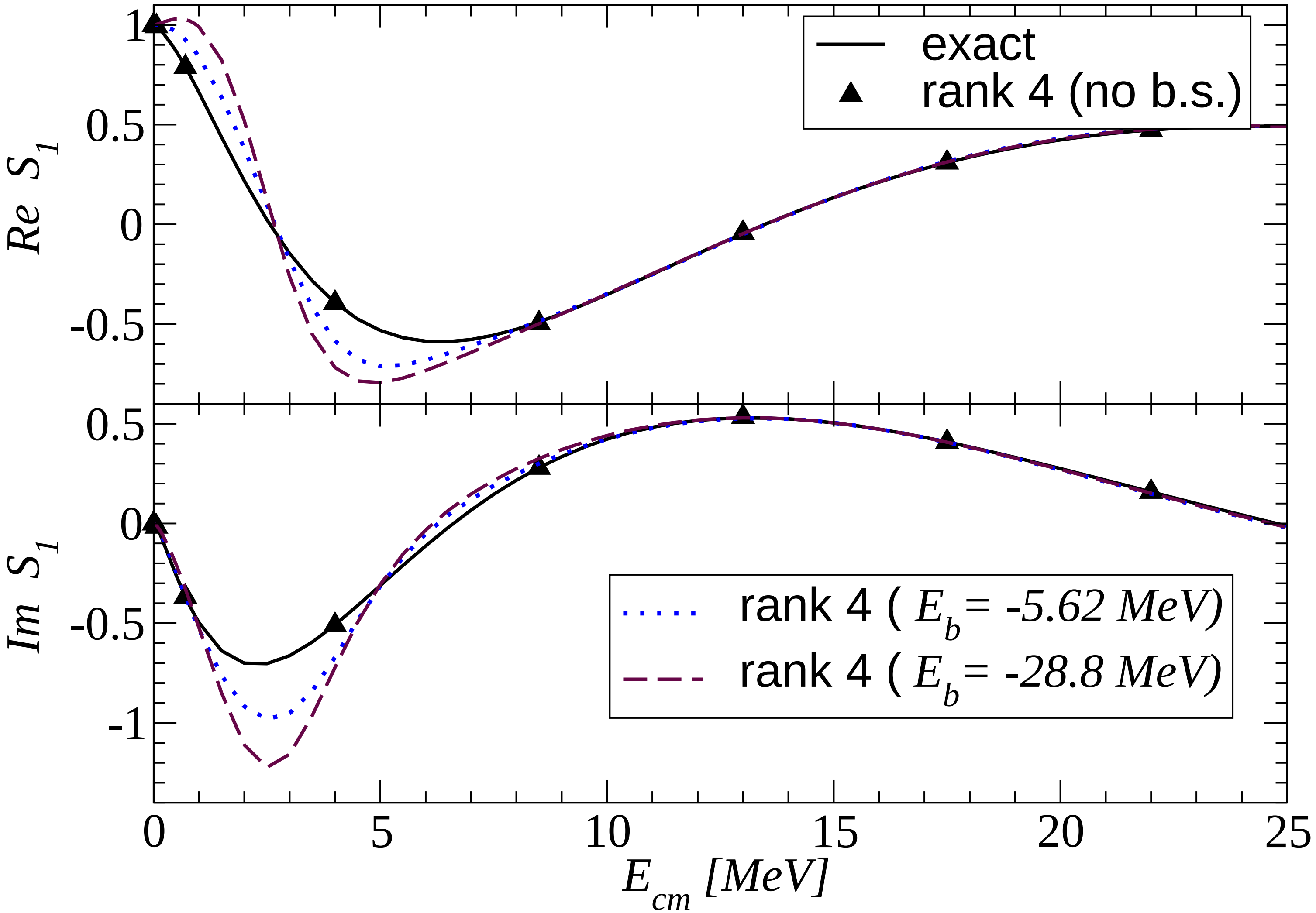
- Lippmann-Schwinger equation: $\hat{t}(E) = \mathbf{V} + \mathbf{V} g_0(E)\hat{t}(E)$
- Separable t-matrix: $\hat{t}(E) = \sum_{ij} v|\Psi_i^{(+)}\rangle \tau_{ij}(E) \langle \Psi_j^{(-)}|v.$
- we thus have: $\sum_j \tau_{ij}(E)\ \langle \Psi_j^{(-)}|v - v g_0(E) v|\Psi_k^{(+)}\rangle = \delta_{ik}$
- momentum space potential and t-matrix:

$$
\begin{aligned}
V(p', p) &= \sum_{ij} \langle p'|v_l|\psi_i^{(+)}\rangle \lambda_{ij} \langle \psi_j^{(-)}|v_l|p\rangle \\
&= \sum_{ij} t(p', k_{E_i}; E_i)\lambda_{ij} t(p, k_{E_j}; E_j) \\
&:= \sum_{ij} h_i(p')\lambda_{ij} h_j(p) \\
t(p', p; E) &= \sum_{ij} \langle p'|v_l|\psi_i^{(+)}\rangle \tau_{ij}(E) \langle \psi_j^{(-)}|v_l|p\rangle \\
&= \sum_{ij} t(p', k_{E_i}; E_i)\tau_{ij}(E) t(p, k_{E_j}; E_j) \\
&:= \sum_{ij} h_i(p')\tau_{ij}(E) h_j(p)
\end{aligned}
$$



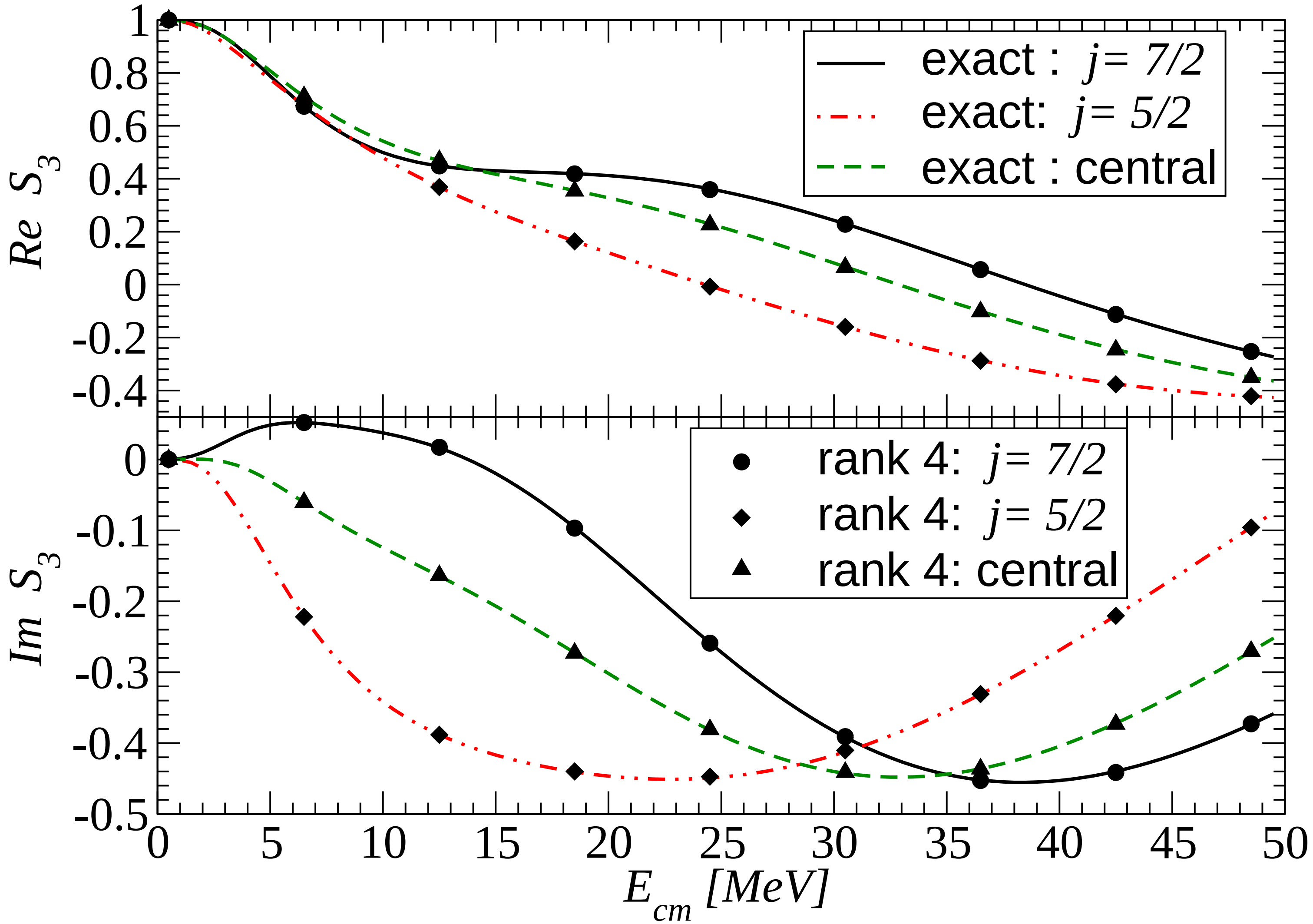
EST pts (MeV): **rank 4:** 6, 15, 36, and 47; **rank 3:** 6, 15, and 25; **rank 2:** 6 and 12; **rank 1:** 6.

- a longer energy range requires an increasingly higher rank



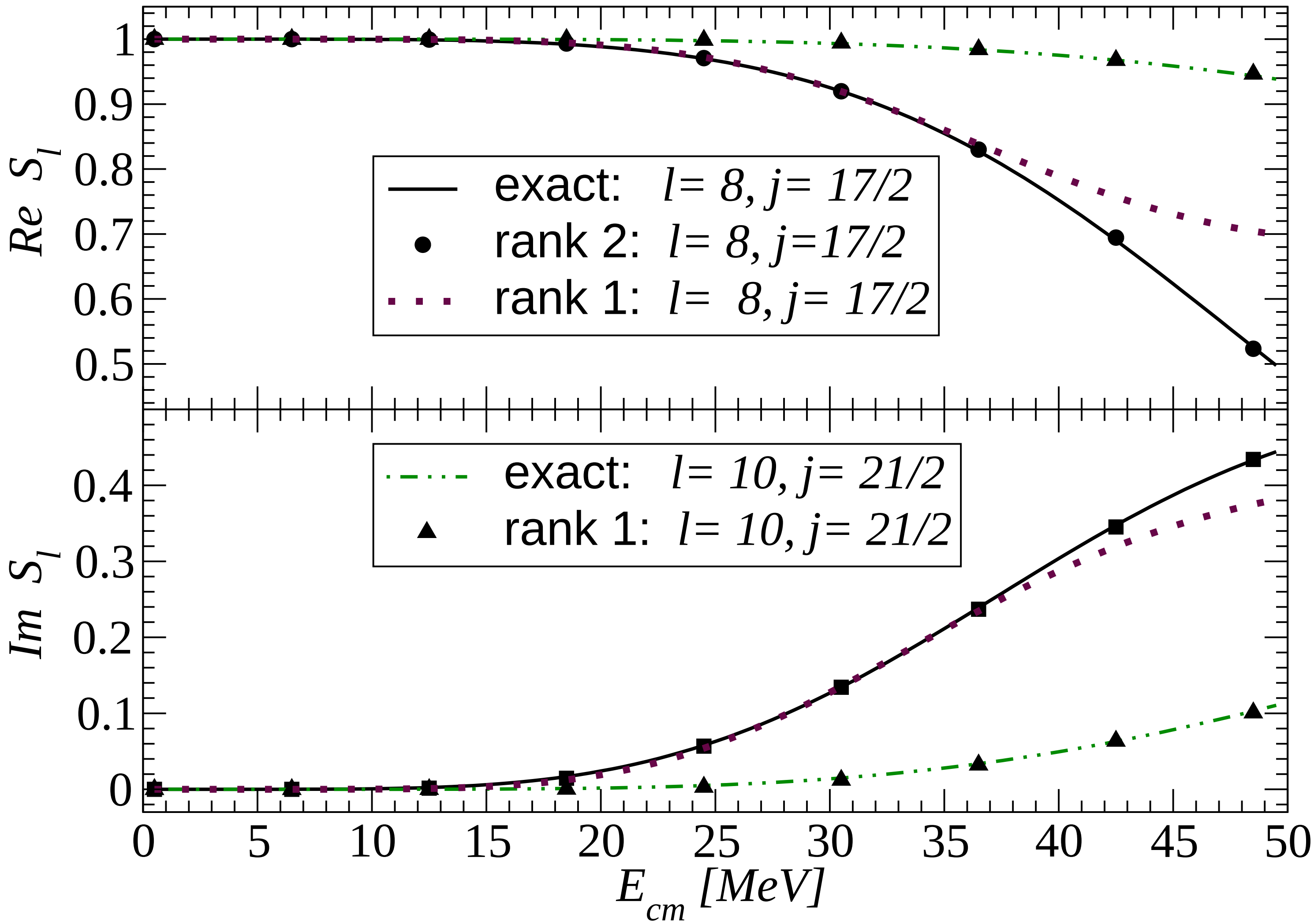
EST pts (MeV): 2, 18, 30, and 45.

- bound state closer to zero: keeps S-matrix values physical at low positive energies



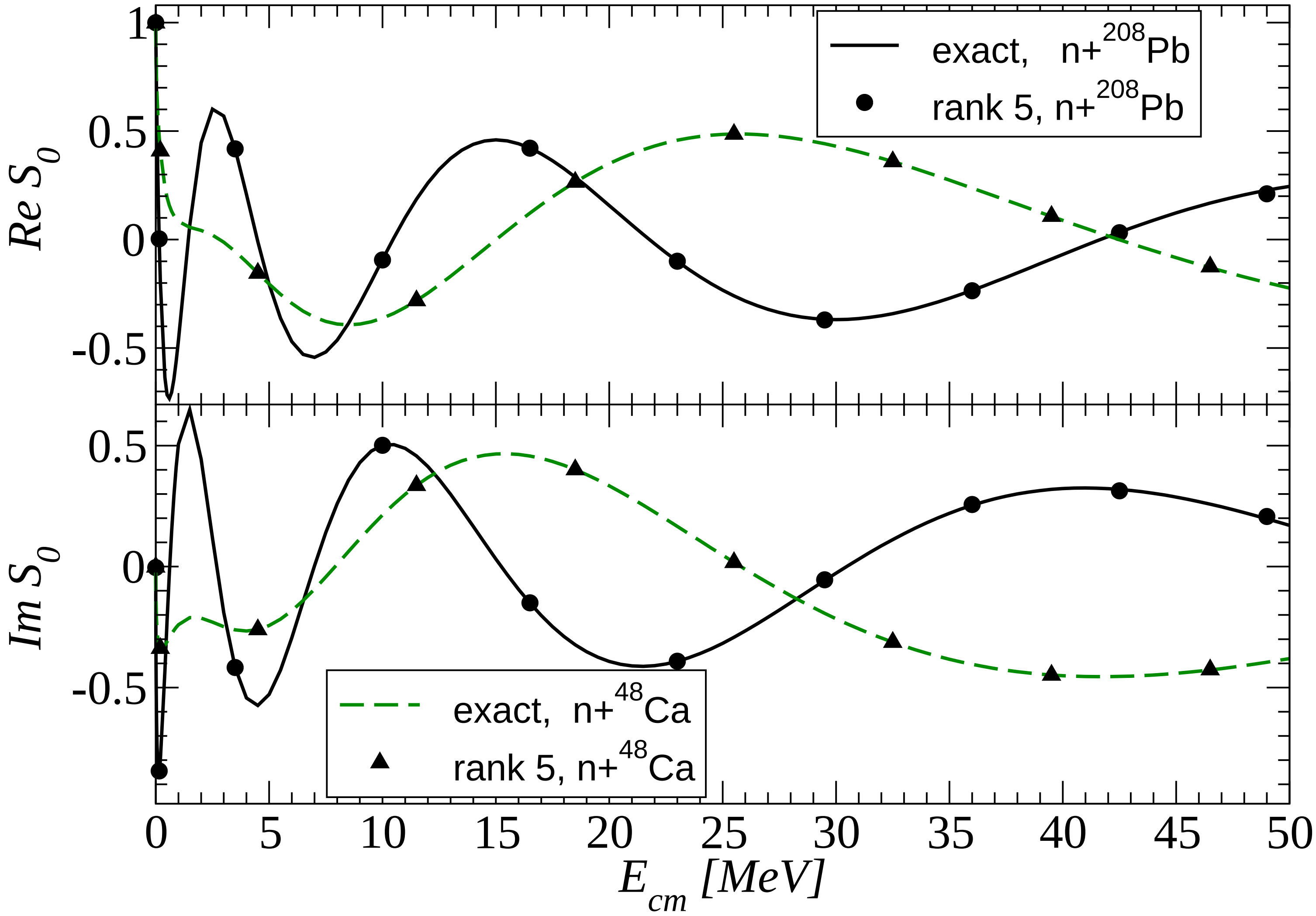
EST pts (MeV): 2, 8, 23, and 45.

- EST support points are determined by the central term



EST pts (MeV): **rank 2:** 29 and 47; **rank 1**, $l = 8$: 29; **rank 1**, $l = 10$: 40.

- higher partial waves require fewer EST support point



EST pts (MeV): 2, 8, 17, 30, 40 and 50.

- this rank 5 potential fits all partial waves for ^{208}Pb and lighter nuclei $\Rightarrow$ universal separable potential

EST support points summary (0 to 50 MeV energy range)

system	partial wave(s)	rank	EST support point(s) [MeV]
n+^{48}Ca	$l \geq 10$	1	40
	$l \geq 8$	2	29, 47
	$l \geq 6$	3	16, 36, 47
	$l \geq 0$	4	6, 15, 36, 47
n+^{208}Pb	$l \geq 16$	1	40
	$l \geq 13$	2	35, 48
	$l \geq 11$	3	24, 39, 48
	$l \geq 6$	4	11, 21, 36, 45
	$\mathbf{l \geq 0}$	**5**	$\mathbf{5, 11, 21, 36, 47}$

⋆ universal set (last row) works for ^{208}Pb and lighter nuclei, for all partial waves

Summary: On-Shell

- Separabilization of phenomenological optical potentials of Woods-Saxon type with the EST scheme works well for light and heavy nuclei

- The number of support points depend on:
    - desired accuracy for S-matrix elements
    - nucleus size: heavier nuclei require more points
    - desired energy range

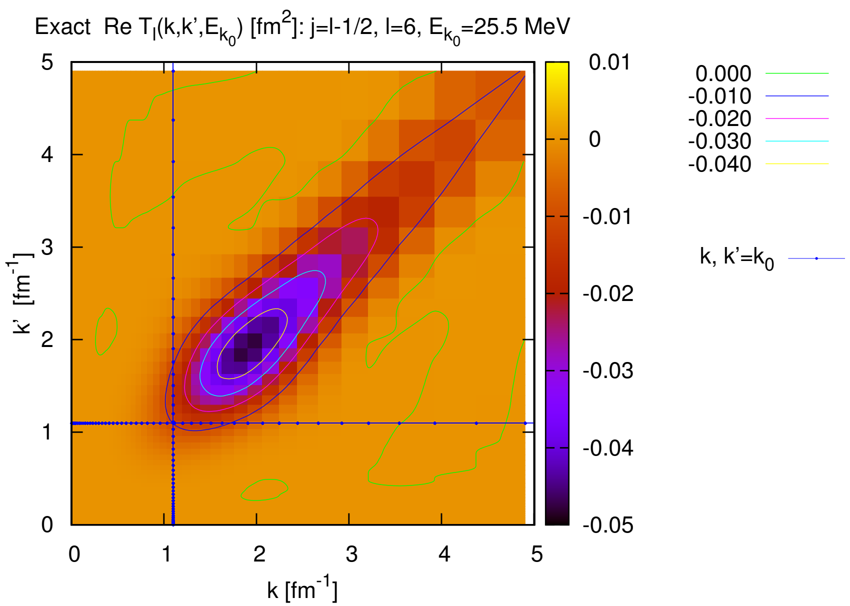
- The central term determines the EST support points

- Higher partial waves tend to require fewer EST support points

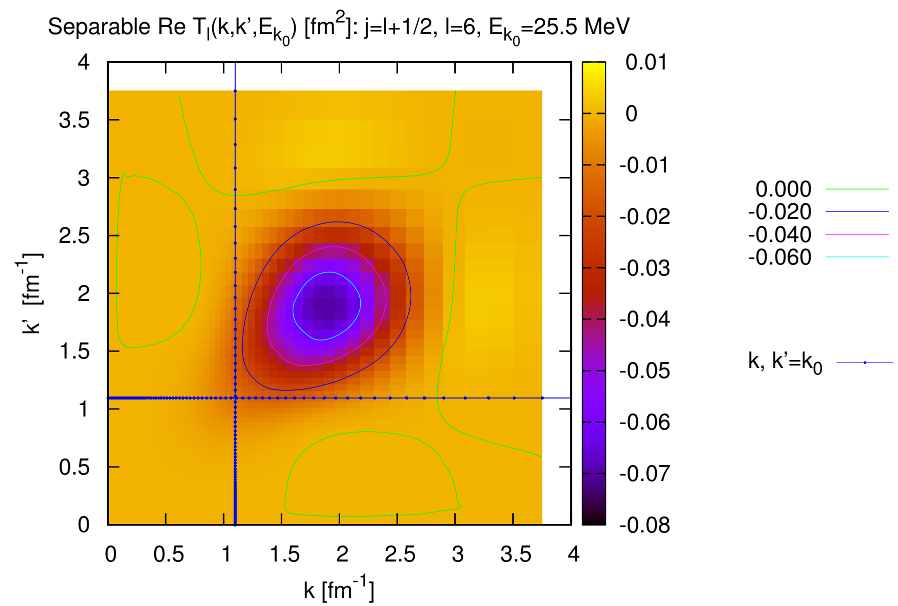
- Including a bound state as an EST support point has an effect if it is close to zero

n+^{48}Ca off-shell t-matrix: l= 6, E= 25.5 MeV

Exact

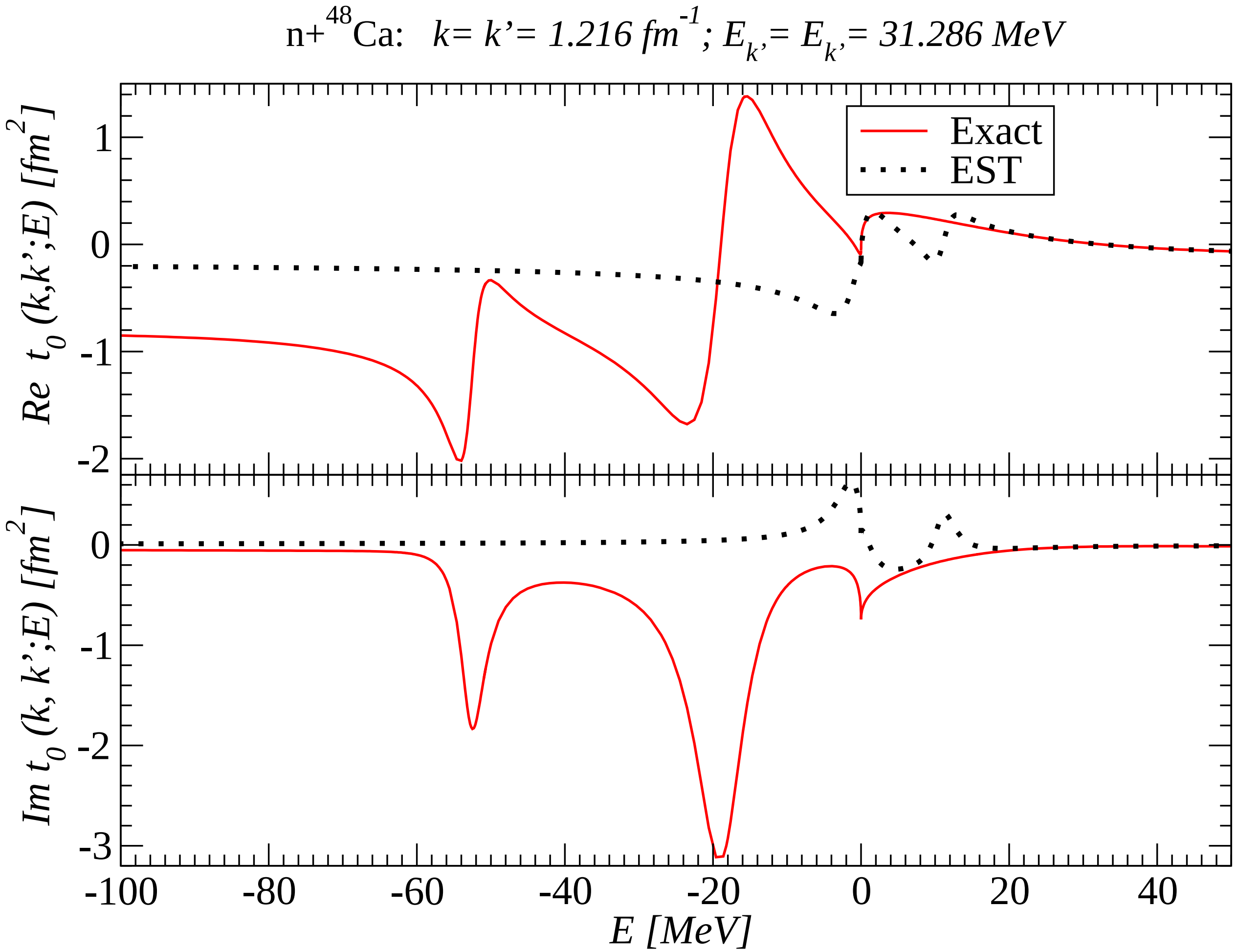


Separable



EST pts (MeV): 5, 11, 28 and 47.

- both exact and separable t-matrices are symmetric in $\mathbf{k}$ and $\mathbf{k}'$
- high momentum components are projected out



EST pt. (MeV): 31.286

- t-matrix amplitude much larger for exact than for separable
- t-matrix exhibits cusps at $E = 0$

Summary: Off-Shell

- Off-shell separable t-matrix: symmetric and high momentum components in (d,p) reactions are projected out

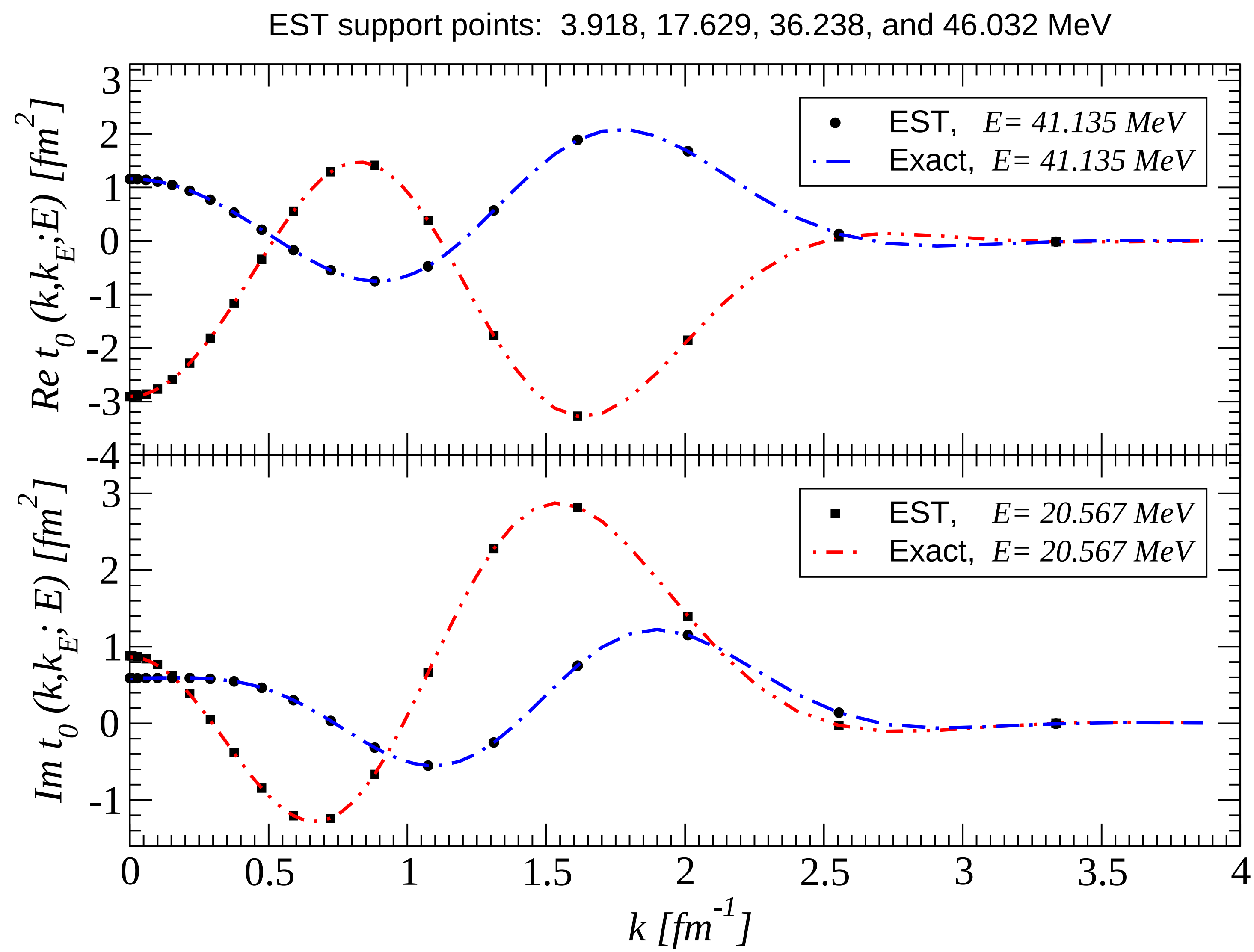
- For corresponding EST approximation (fitted only at positive energies) the off-shell t-matrix amplitude is much smaller

- The Off-shell t-matrix elements exhibit cusps at $E = 0$ and these disappear for higher partial waves

- The cusps gradually disappear as the imaginary is slowly switched off

Separable Potentials

- **Proposed Faddeev formulation of (d,p) reactions in momentum space includes target excitations**
- **Two-body t-matrices (in *separable* form) of sub-systems serve as inputs**
- **Advantages:**
    - reduces the numerical effort in solving momentum space Faddeev integral equations.
    - **Coulomb challenge at high Z:** separable potential simplify integrals with Coulomb wavefunctions
    - **for transfer reactions:** separable representation allows choice of specific states (Effective Pauli projector)



EST support points: 3.918, 17.629, 36.238, and 46.032 MeV
Re t_0 $(k,k_E;E)$ $[fm^2]$
EST, E= 41.135 MeV
Exact, E= 41.135 MeV
Im t_0 $(k,k_E;E)$ $[fm^2]$
EST, E= 20.567 MeV
Exact, E= 20.567 MeV
k $[fm^{-1}]$
3
2
1
0
-1
-2
-3
-4
0
0.5
1
1.5
2
2.5
3
3.5
4