Applying the surface formalism for (d,p) reactions to bound and resonance states

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TORUS Collaboration

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Present theories provide valuable information on angular momenta …but have serious limitations in resonance cases

- Problem: present theories rely heavily on one-body overlap function of A and A+1 systems

$$I_A^F(r) = < \varphi_A \mid \varphi_F >$$

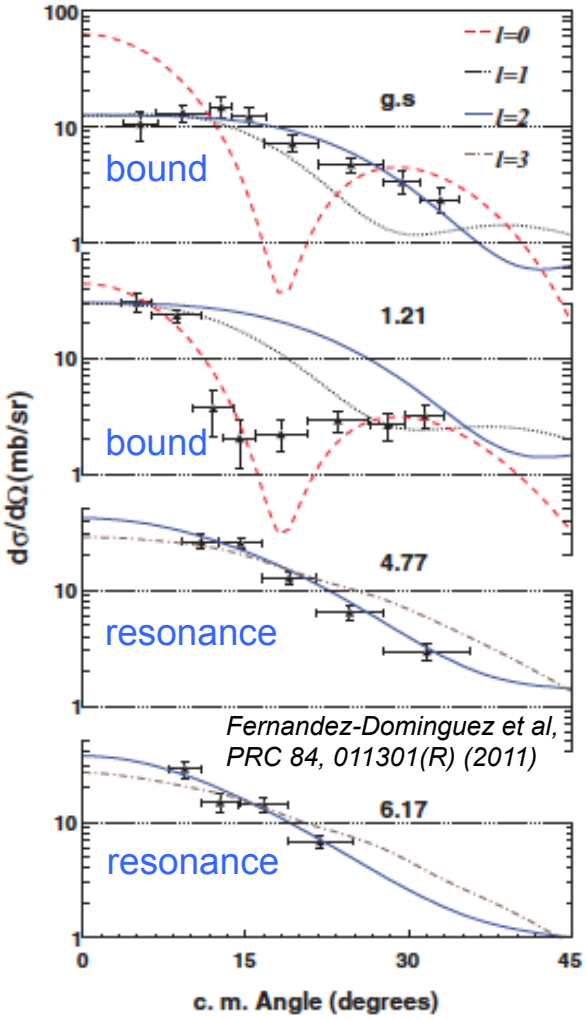
  - carries structure information
  - not well-known in nuclear interior
  - typically approximated by single-particle function

- Calculations converge very slowly
- Not appropriate for describing reactions involving wide resonances
- Desired resonance properties (energies and widths) cannot be reliably obtained

Mukhamedzhanov's suggestion: Extend R-matrix description to transfer reactions

➔ Surface Formalism

Example: $^{20}O(d,p)^{21}O$ inverse-kinematics experiment - Intepreted the traditional way



The surface formalism – DWBA and CDCC approximations

Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(post)} = \langle \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} \rangle$$

DWBA

$$\langle \varphi_F \chi_{pF}^{(-)} \mid \Delta V_{pF} \mid \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_d \chi_{dA}^{(+)} \rangle$$

3-body

$$\langle \varphi_F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_A \Psi_i^{3B(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{3B(+)} \rangle$$

CDCC

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{CDCC(+)} \rangle$$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = \langle \varphi_A \mid \varphi_F \rangle$ one-body overlap

$$M^{(prior)} = \langle \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} \rangle$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Some details of the surface formalism

$$M^{(post)} = \langle \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} \rangle$$

Transition matrix element M:

- Cross section $\sigma \sim M^2$
- Is split into interior and exterior parts

DWBA

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_d \chi_{dA}^{(+)} \rangle$$

$$I_A^F = \langle \varphi_A \mid \varphi_F \rangle = I_A^F(r_{nA})$$

One-body overlap function of A and A+1 systems

Mukhamed-zhanov

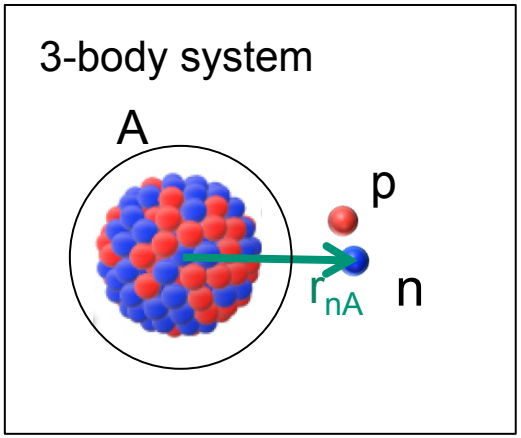
Interior + exterior

$$M^{(post)} = M^{(post)}(0,a) + M^{(post)}(a,\infty)$$

$$M_{surf}(a) + M^{(prior)}(a,\infty)$$

Surface formulation

$$\mathbf{M = M^{(post)}(0,a) + M_{surf}(a) + M^{(prior)}(a,\infty)}$$



Post: $\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

Prior: $\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$

The surface formalism: a closer look

Features:

- Model dependence contained in interior term
- Surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.
- Exterior term vanishes in CDCC implementation

Questions:

- **Is the surface term dominant? Where?**
- **Do we get reduced dependence on model for interior?**
  Under which circumstances?
- **Can we extract useful spectroscopic quantities from comparison to experiment?**

Surface formulation

$$M = \underbrace{M^{(post)}(0,a)}_{\text{model dependence}} + \underbrace{M_{surf}(a) + M^{(prior)}(a,\infty)}_{\text{asymptotic quantities}}$$

$M_{surf}(a) = f(a, C_A^F, B_{nA})$

B_{nA} = log derivative of I_A^F at surface radius a

ANC: C_A^F defined through: $I_A^F(r_{nA}) \rightarrow C_A^F W(kr_{nA})$

related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

Tests completed:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs, 1st excited state, **$2f_{7/2}$ resonance**
- ^{48}Ca(d,p) for E_d=13, 19.3, 56 MeV
  - ^{49}Ca gs, 1st excited state
- ^{20}O(d,p) for E_d=21 MeV
  - ^{21}O gs, 1st excited state, **$1d_{3/2}$ and $1f_{7/2}$ resonances**
- Akram:
  - ^{12}C(d,p) for E_d=30 MeV
  - ^{40}Ca(d,p) for E_d=34.4 MeV
  - ^{209}Pb(d,p) for E_d=52 MeV

Internal, surface, external contributions – ^{90}Zr(d,p) at E_d=11 MeV

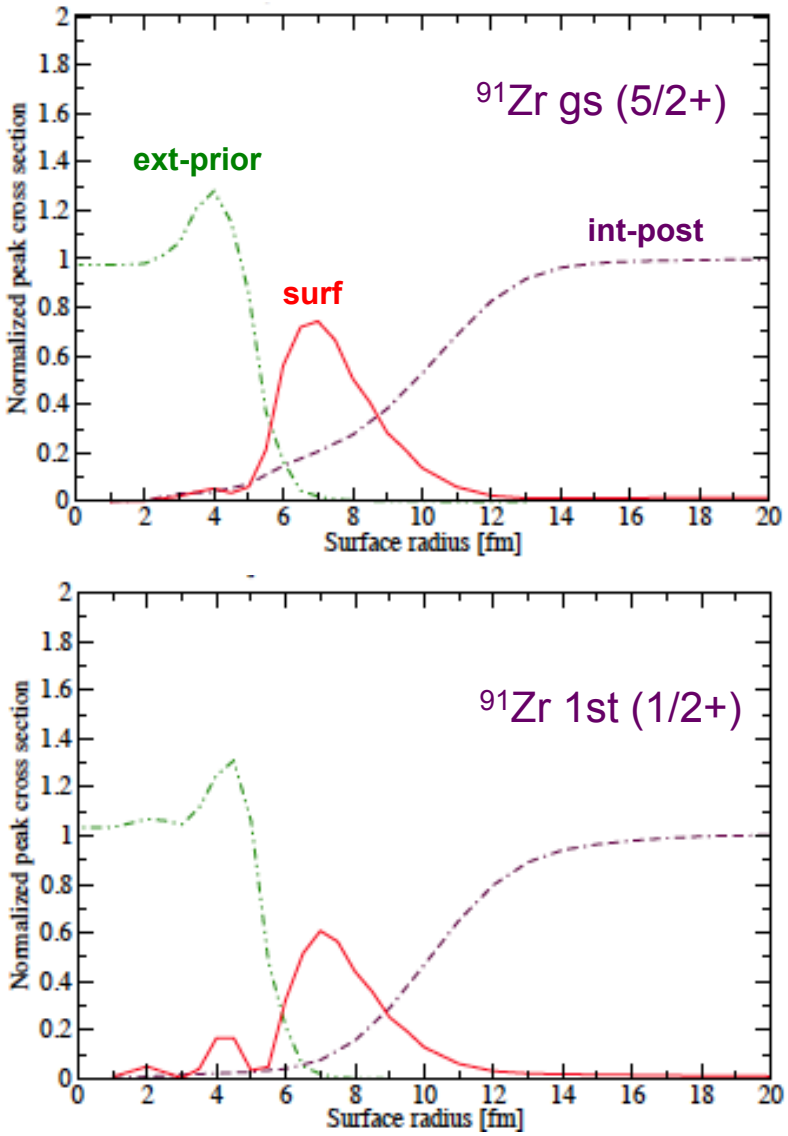
$$M = \underbrace{M^{(post)}(0,a)}_{\text{model dependence}} + \underbrace{M_{(surf)}(a) + M^{(prior)}(a,\infty)}_{\text{asymptotic quantities}}$$

Observations

- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

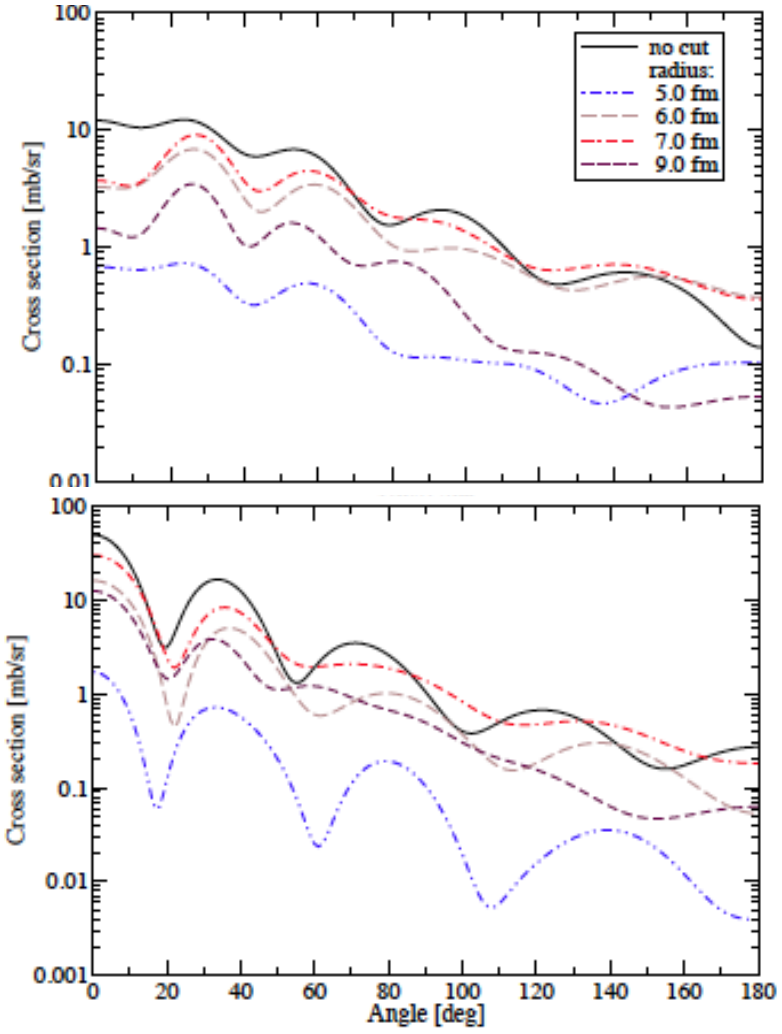
Peak cross section relative to full calculation



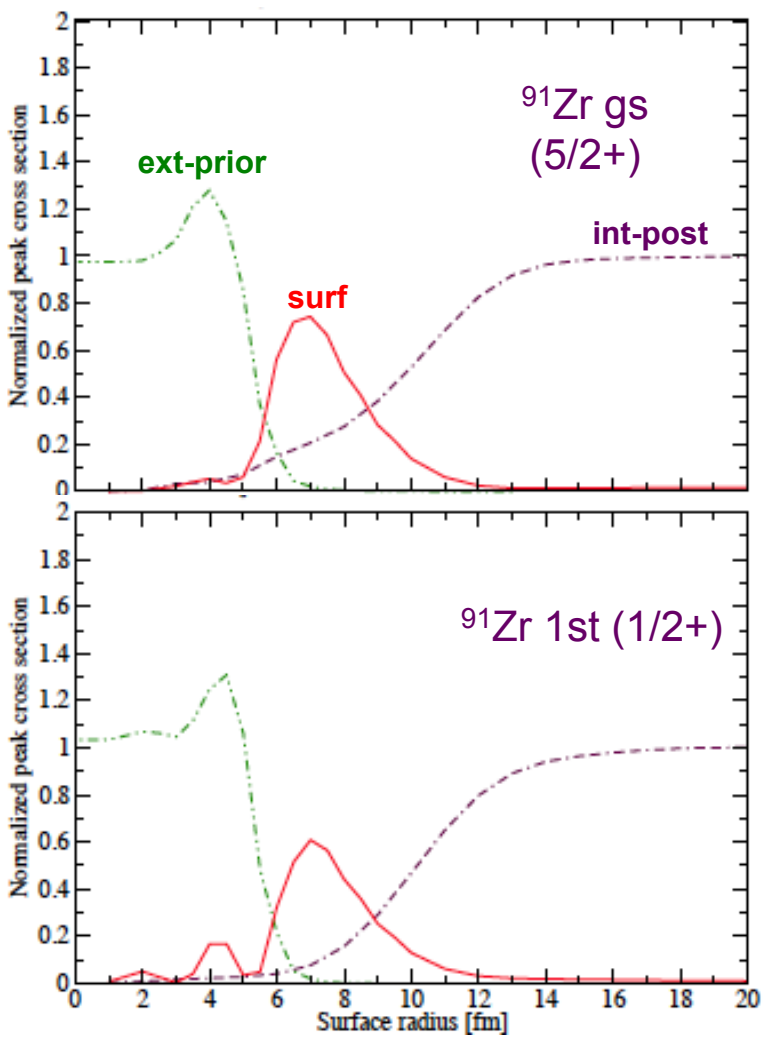
Escher, Thompson, Mukhamedzhanov, JPCS (2012).

The surface contribution – ^{90}Zr(d,p) at E_d=11 MeV

Angular cross section – Surface term only



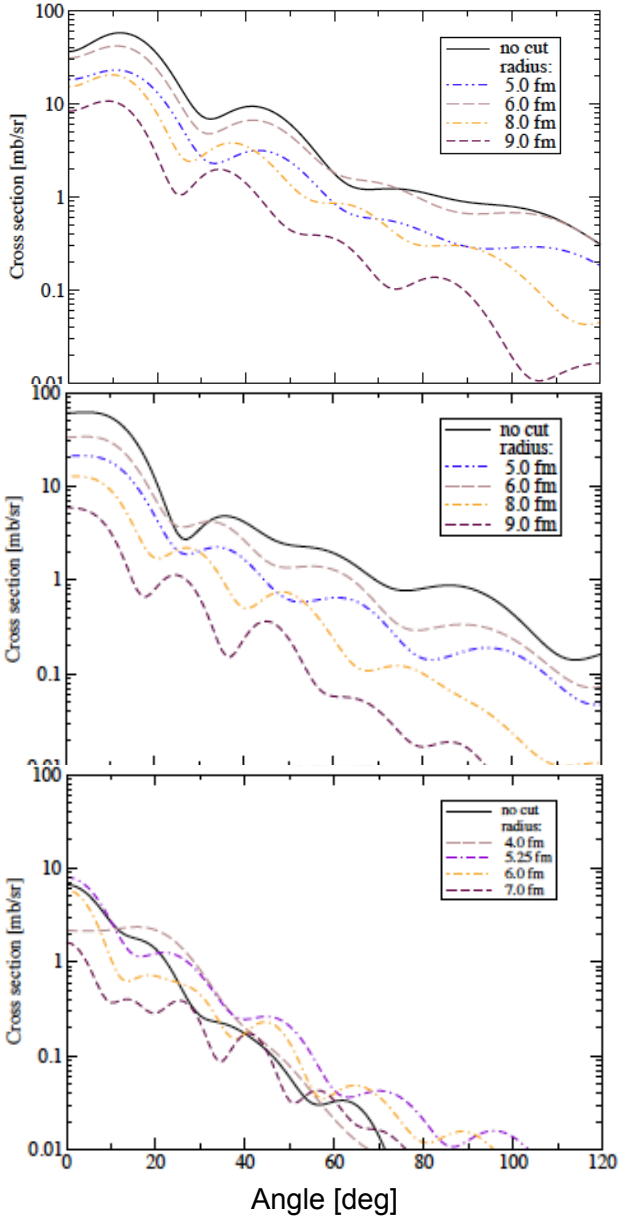
Peak cross section relative to full calculation



- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain

Effect of varying the beam energy – ^{48}Ca(d,p) at E_d=13, 19.3, 56 MeV

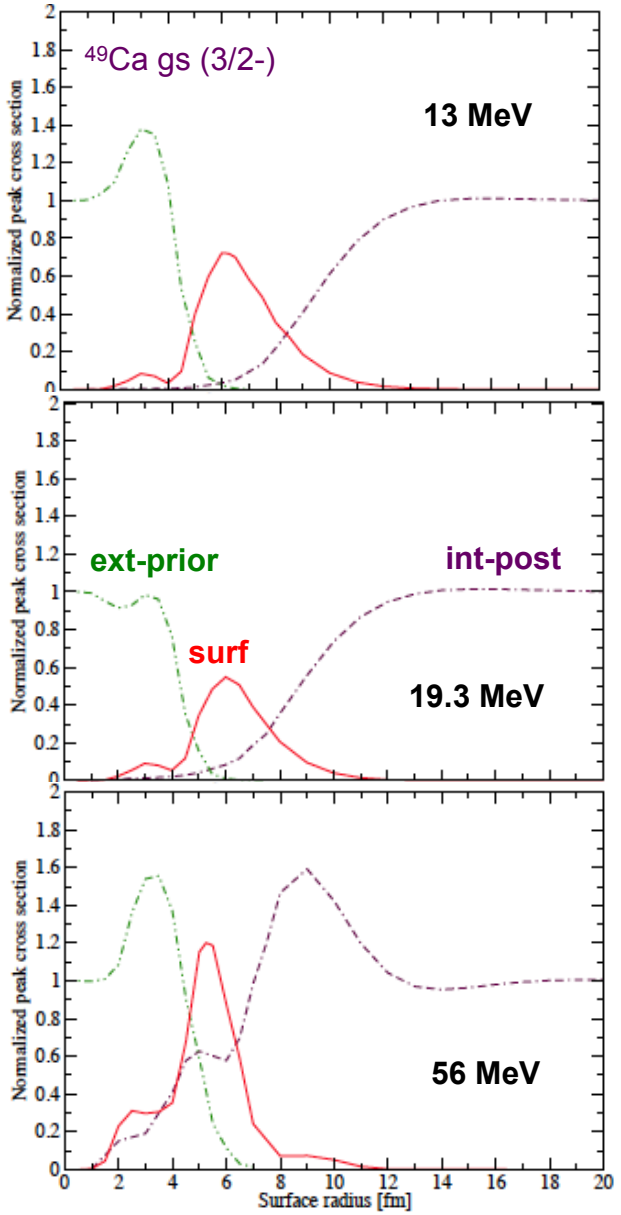
Angular cross section – Surface term only



Surface term approximation improves with decreasing energy

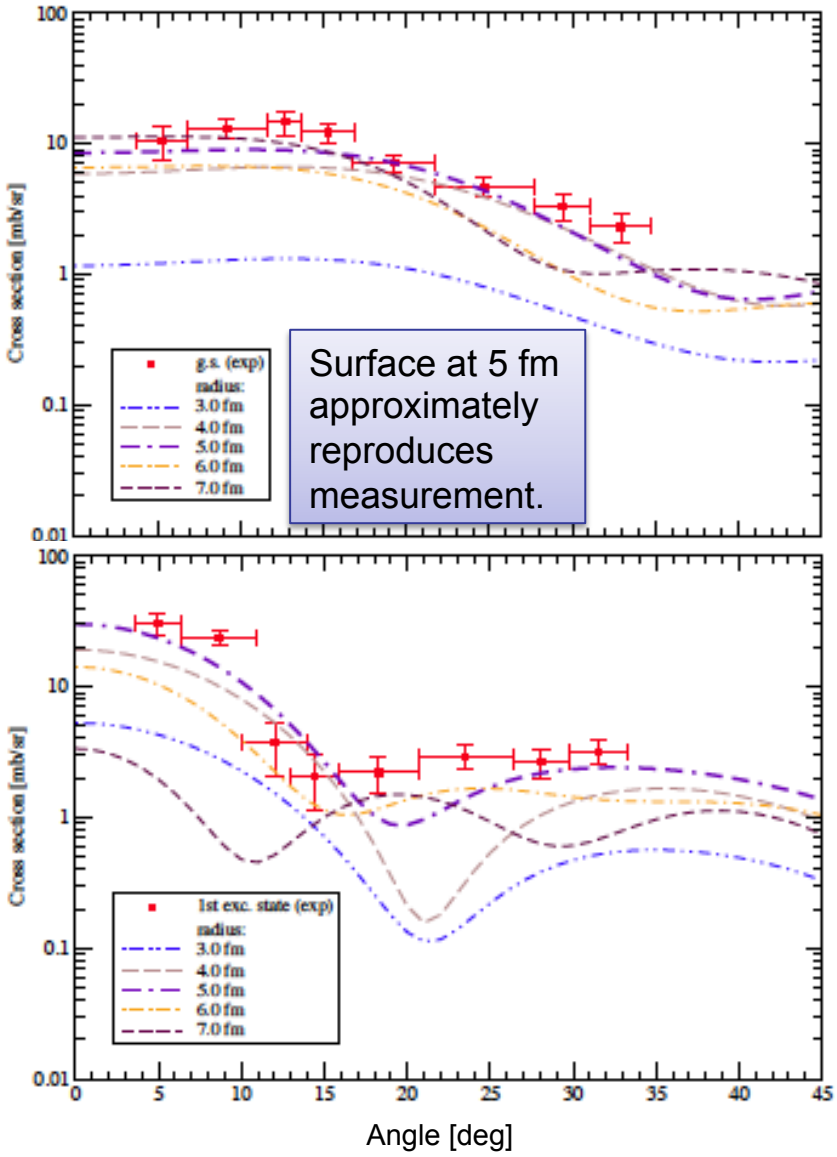
Calculations for ^{49}Ca 1st (1/2-) give similar results

Peak cross section relative to full calculation



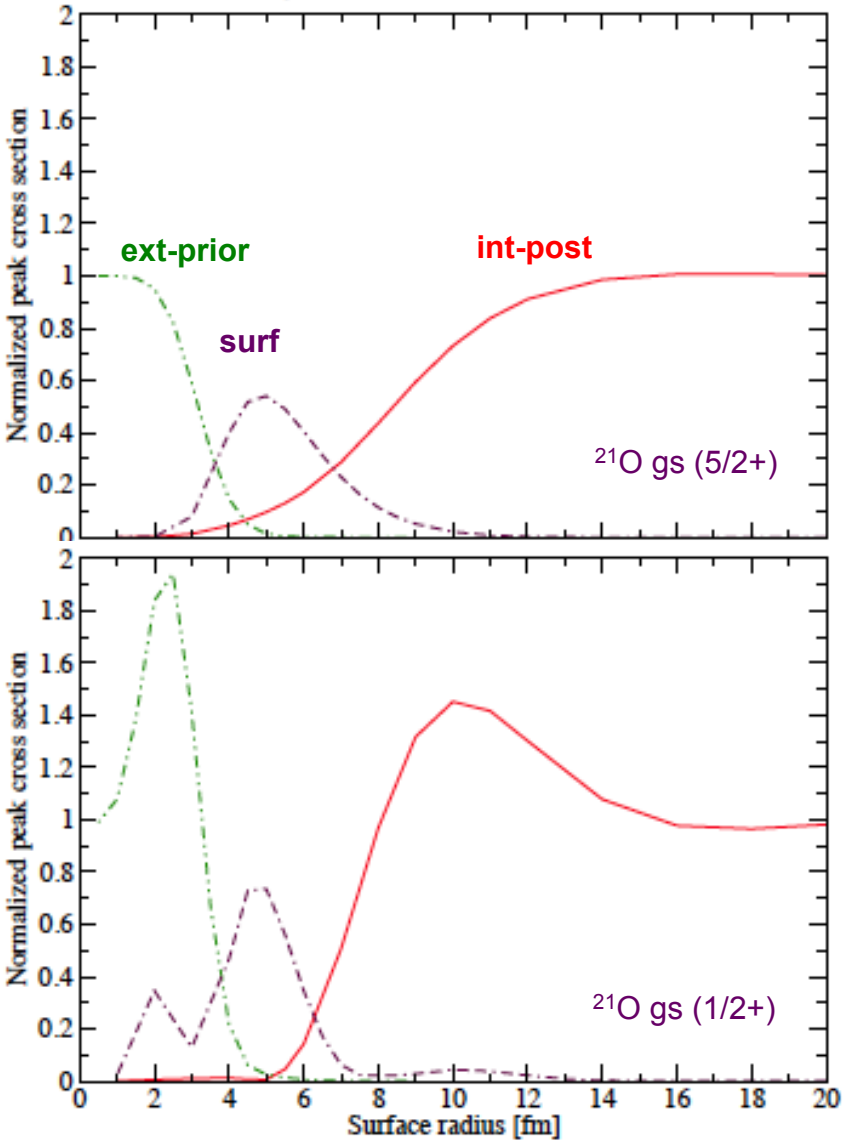
The oxygen case - ^{20}O at E_d=21 MeV

Angular cross section – Surface term only



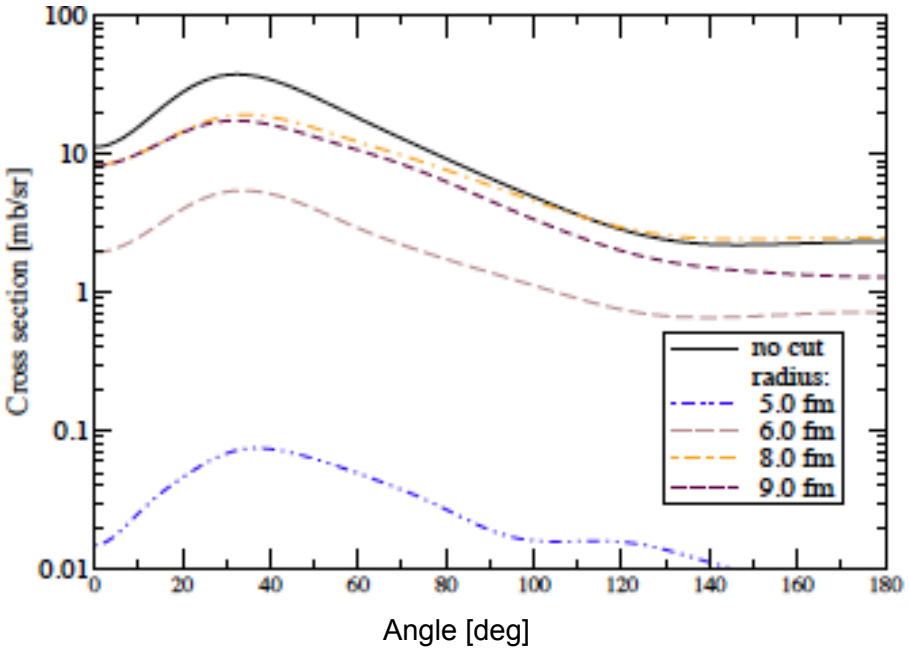
Surface at 5 fm approximately reproduces measurement.

Peak cross section relative to full calculation

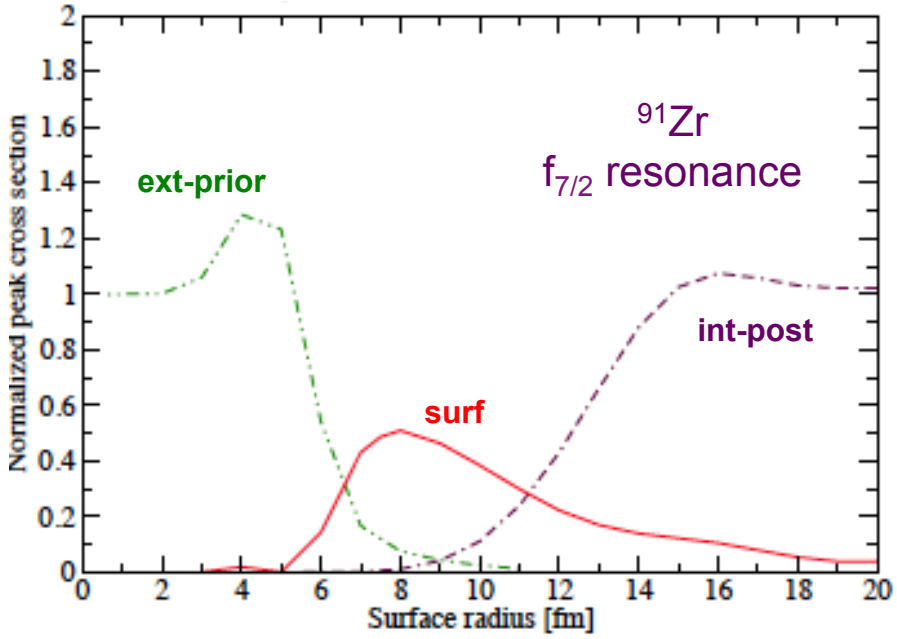


Resonances – ^{90}Zr at E_d=11 MeV

Angular cross section – Surface term only



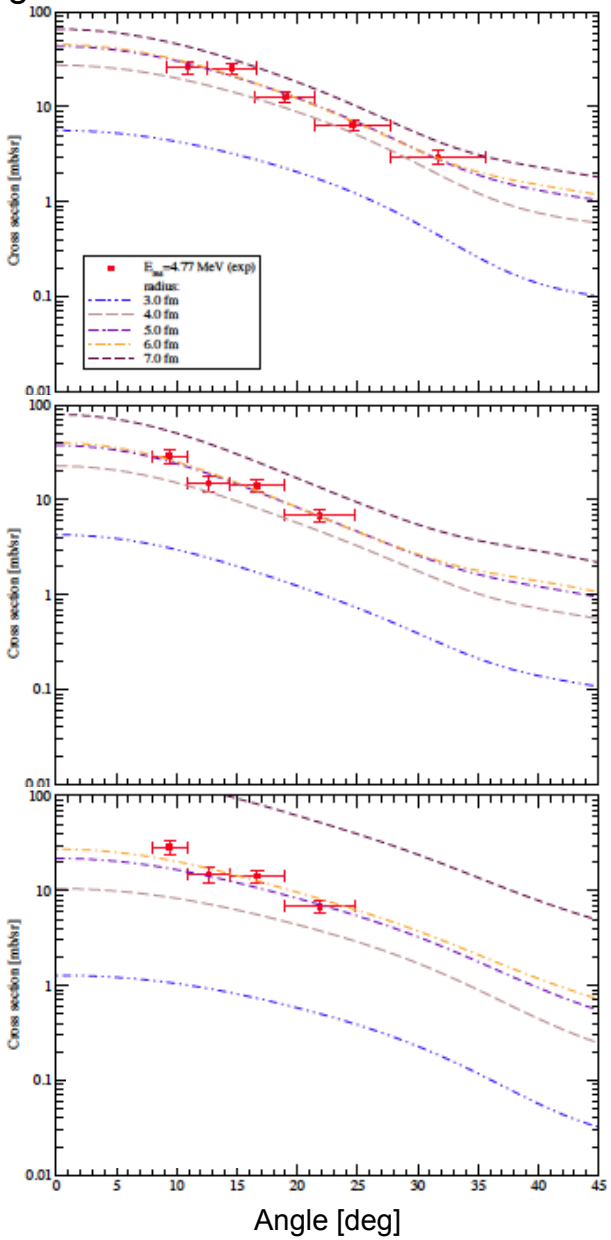
Peak cross section relative to full calculation



- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute

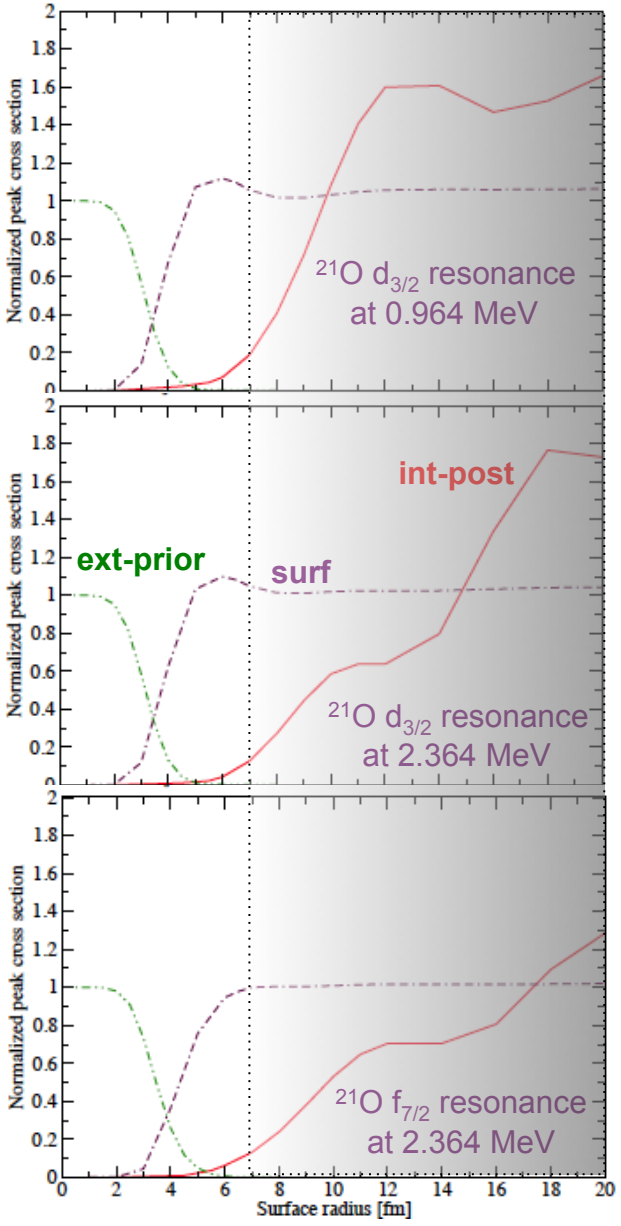
Resonances - ^{20}O at E_d=21 MeV

Angular cross section – Surface term only



- Calculations reasonable only for a<7 fm
- Convergence difficult for resonance cases
- Surface term seems able to reproduce data

Peak cross section relative to full calculation



Lessons so far…

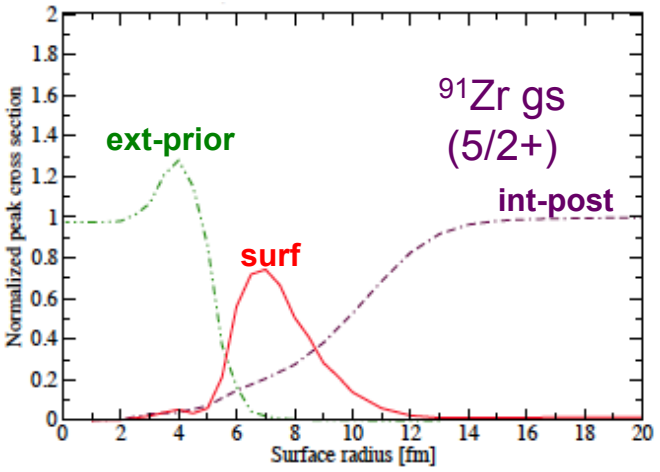
- Varying the surface radius changes the relative contributions from interior, surface, exterior terms.
- The surface term is dominant in the surface region, but contributions from the interior and/or exterior terms are present at all radii.
- The surface term can provide a rough approximation to the (d,p) cross section. The approximation deteriorates for higher beam energies.
- The findings are similar for all mass regions considered.
- Results for resonances are similar to those for bound states.
- Achieving convergence for resonances is difficult, but expected to be simpler in the fully-implemented method.

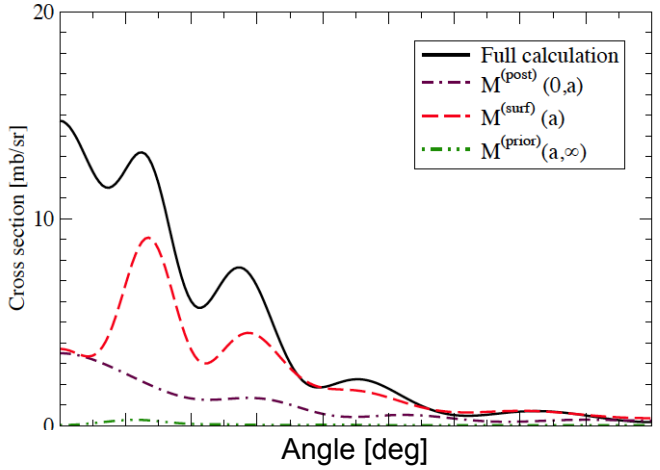
Maximizing the surface term…

- Motivation: The surface term can be written in terms of reduced widths, the surface radius, and derivatives of known functions.
- The calculations suggest: using a slightly smaller radius and the CDCC implementation (which eliminates the ext-prior contribution) ➔ Ian's talk
- AMM: Vary the core-core potential to simultaneously minimize the 2nd-order contributions and the interior-post term ➔ Not a solution (formally or practically)!

$$M^{(post)} = \langle \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} \rangle$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

Peak cross section relative to full calculation





Lessons for moving forward…

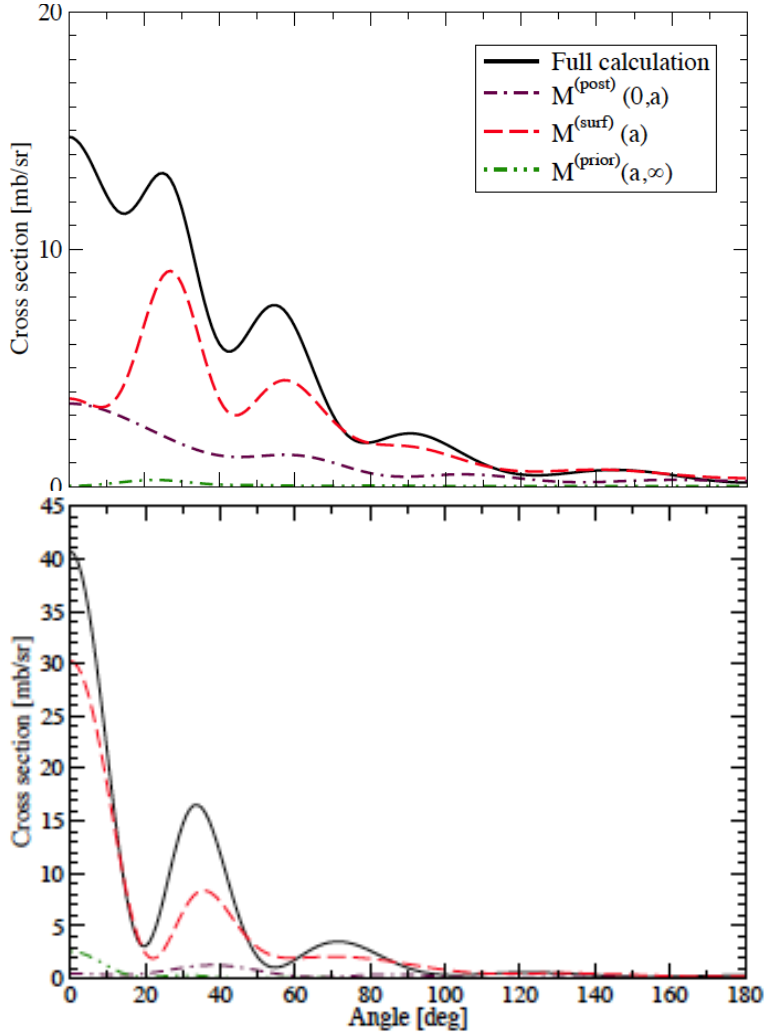
To make the surface approach into a useful tool, we need to:

- Implement the formalism in its CDCC form, to incorporate breakup and eliminate the exterior-prior contributions ➔ Ian's talk
- Minimize the interior-post contributions by finding an optimal radius (corrections may still be necessary)
- Test the approach for bound and resonance states

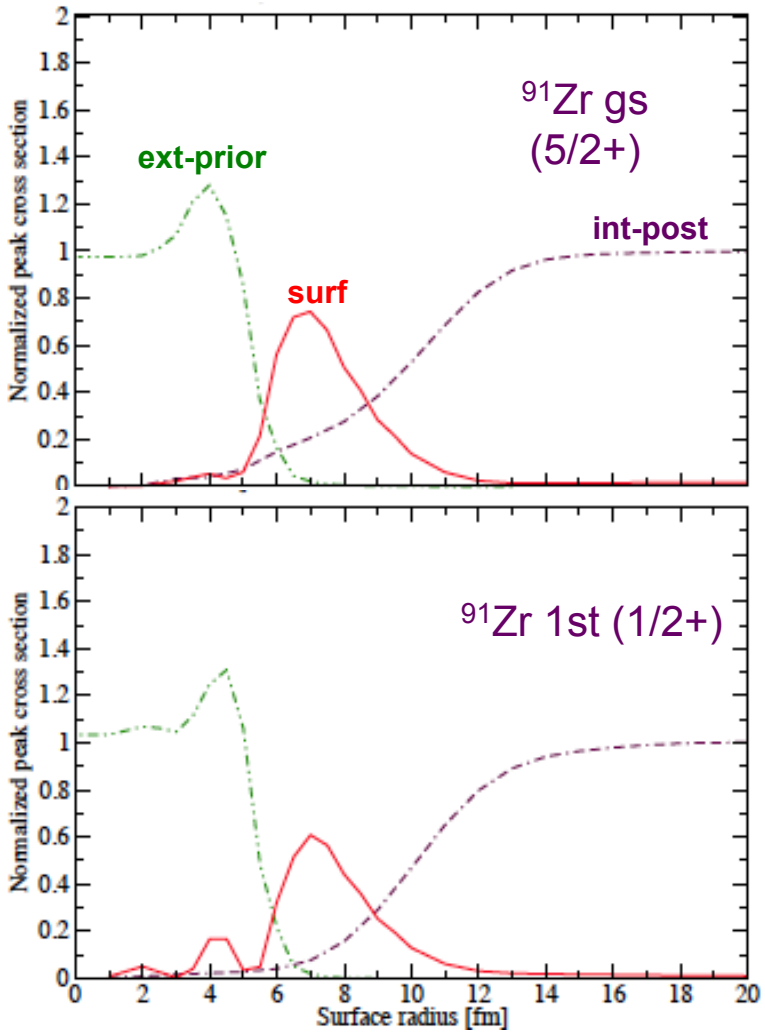
Appendix

The surface contribution – ^{90}Zr(d,p) at E_d=11 MeV

Cross section contributions from the three terms



Peak cross section relative to full calculation



- Interior and exterior terms still contribute to the cross section

The surface formalism: a closer look - II

$$M = \underbrace{M^{(post)}(0,a)}_{\textit{model dependence}} + \underbrace{M_{(surf)}(a) + M^{(prior)}(a,\infty)}_{\textit{asymptotic quantities}}$$

$$M_{(surf)}(a) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{j_{nA} m_{j_{nA}} m_{l_{nA}} M_n} \langle J_A M_A j_{nA} m_{j_{nA}} | J_F M_F \rangle \langle J_n M_n l_{nA} m_{l_{nA}} | j_{nA} m_{j_{nA}} \rangle \langle J_p M_p J_n M_n | J_d M_d \rangle \gamma_{nA j_{nA} l_{nA}}$$

$$\times \int d\mathbf{r}_{pF} \chi^{(+)}_{-\mathbf{k}_{pF}}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{nA}} Y^*_{l_{nA} m_{l_{nA}}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA} = R_{nA}}$$

From: Mukhamedzhanov, PRC 84, 044616 (2011)

Extension of the formalism to include breakup

DWBA matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

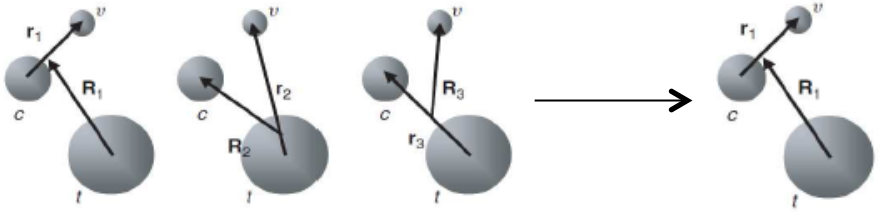
CDCC matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a)$$

$$M^{(prior)}(a,\infty) = 0 \text{ (is included in breakup)}$$

CDCC (Continum-discretized coupled channels)

- Approximate treatment of 3-body problem
- Describes breakup of deuteron



- Successfully used for describing data
- Currently revisited via comparison with Fadeev

CDCC extension of R-matrix formalism

- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant