

# Theory for (d,p) reactions connecting new and old

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TORUS collaboration

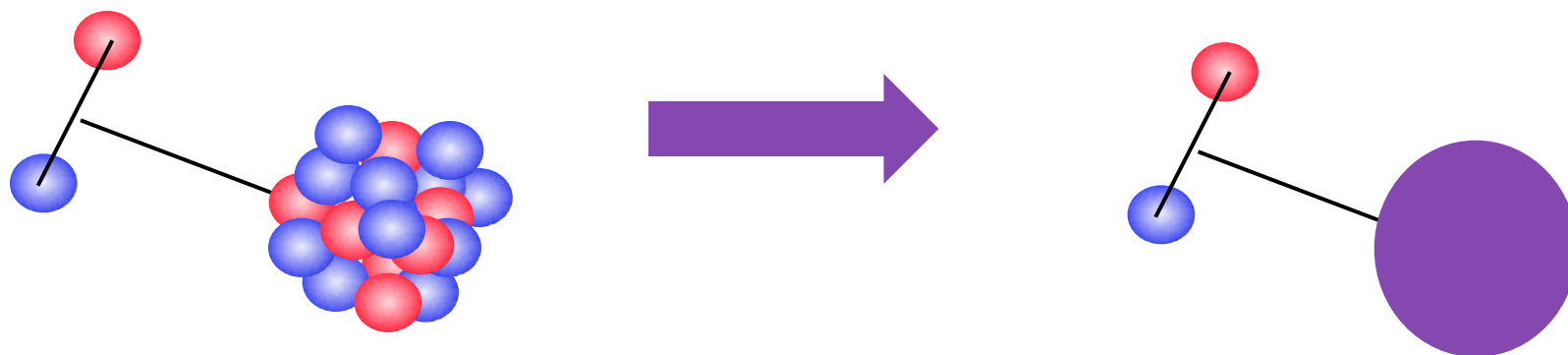
in collaboration with A. Deltuva (Lisbon)

what sort of reaction are we interested in?



?

# reducing the many body to a few body problem



- ❑ isolating the important degrees of freedom in a reaction  
(keeping track of all relevant channels)
- ❑ connecting back to the many-body problem

- ❑ many-body to few-body
  - ❑ overlap function
  - ❑ effective interactions (optical potentials)
- ❑ solving the few-body problem

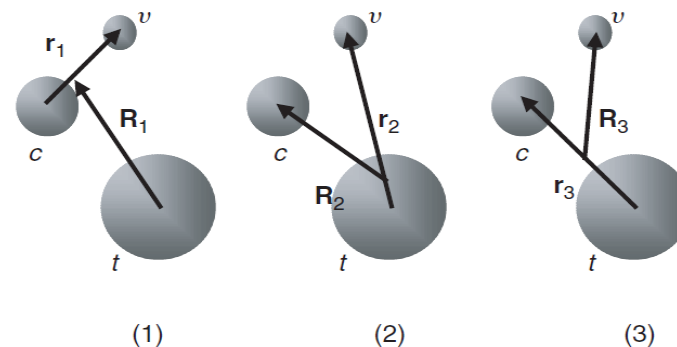
# differences between three-body methods



## Faddeev AGS:

- all three Jacobi components are included
- elastic, breakup and rearrangement channels are fully coupled
- computationally expensive

Deltuva and Fonseca, Phys. Rev. C **79**, 014606 (2009).



3 jacobi coordinate sets

## CDCC:

- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- computationally expensive

## ADWA:

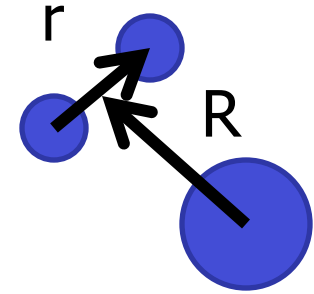
- only one Jacobi component
- elastic and breakup fully coupled (no rearrangement)
- adiabatic approximation for breakup
- runs on desktop – practical for experimentalists

# what do we learn from these comparisons?



- importance of fully coupling to rearrangement channels
- potential problems with optical potentials
- quantifying accuracy of approximations

# (d,p) reactions: three body model



Start from a 3B Hamiltonian

$$\mathcal{H}_{3B} = T_{\mathbf{r}} + T_{\mathbf{R}} + U_{nA} + U_{pA} + V_{np}$$

Solve for 3B wfn and use in exact T-matrix

$$T = \langle \phi_{nA} \chi_{pB}^{(-)} | V_{np} + \Delta_{rem} | \Psi^{(+)} \rangle$$

# (d,p) reactions: Johnson and Tandy theory



Expand 3-body wfn in deuteron Weinberg states

$$(T_r + \alpha_i V_{np})\phi_i(\vec{r}) = -\varepsilon_i \phi_i(\vec{r}) \quad \Psi^{(+)}(\vec{r}, \vec{R}) = \sum_{i=1}^{\infty} \phi_i(\vec{r}) \chi_i(\vec{R})$$

➤ set of scattering coupled channel equations

Johnson and Tandy potential

$$U_{ij}(\vec{R}) = -\langle \phi_i | V_{np} (U_{nA} + U_{pA}) | \phi_j \rangle$$

If only first term of the expansion is included:  
coupled equations reduce to single channel!



# Systematic comparison: FR-ADWA vs Faddeev

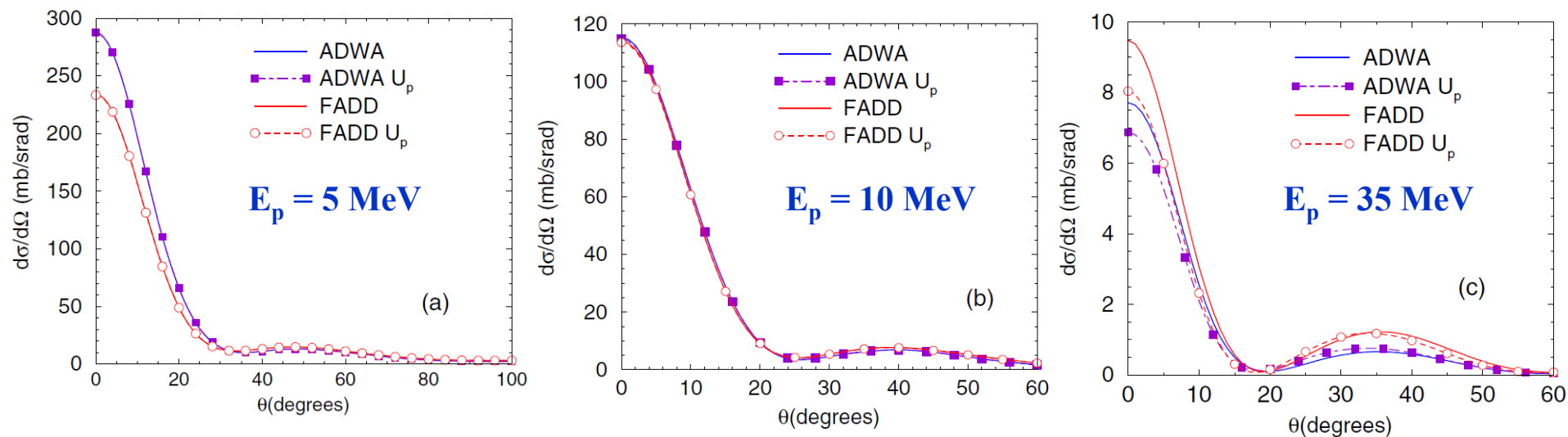


FIG. 1: Angular distributions for  $^{11}\text{Be}(p,d)^{10}\text{Be}$ : (a)  $E_p = 5$  MeV, (b)  $E_p = 10$  MeV, and (c)  $E_p = 35$  MeV.

# Systematic comparison: FR-ADWA vs Faddeev

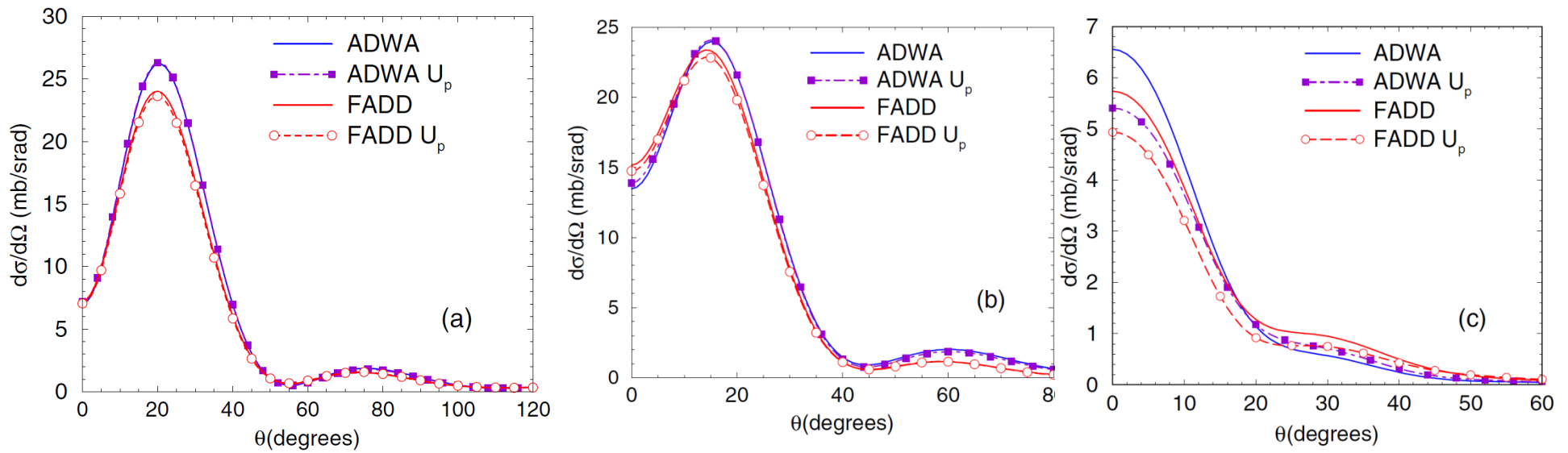


FIG. 2: Angular distributions for  $^{12}\text{C}(d,p)^{13}\text{C}$ : (a)  $E_d = 7.15$  MeV, (b)  $E_d = 12$  MeV and (c)  $E_d = 56$  MeV

# Systematic comparison: FR-ADWA vs Faddeev

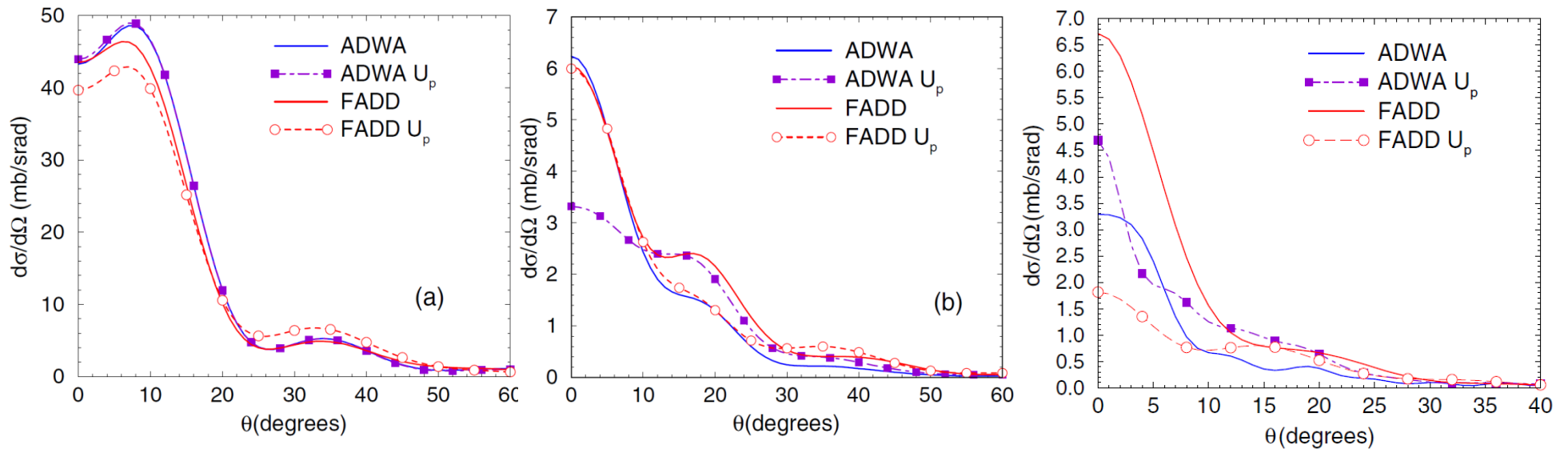
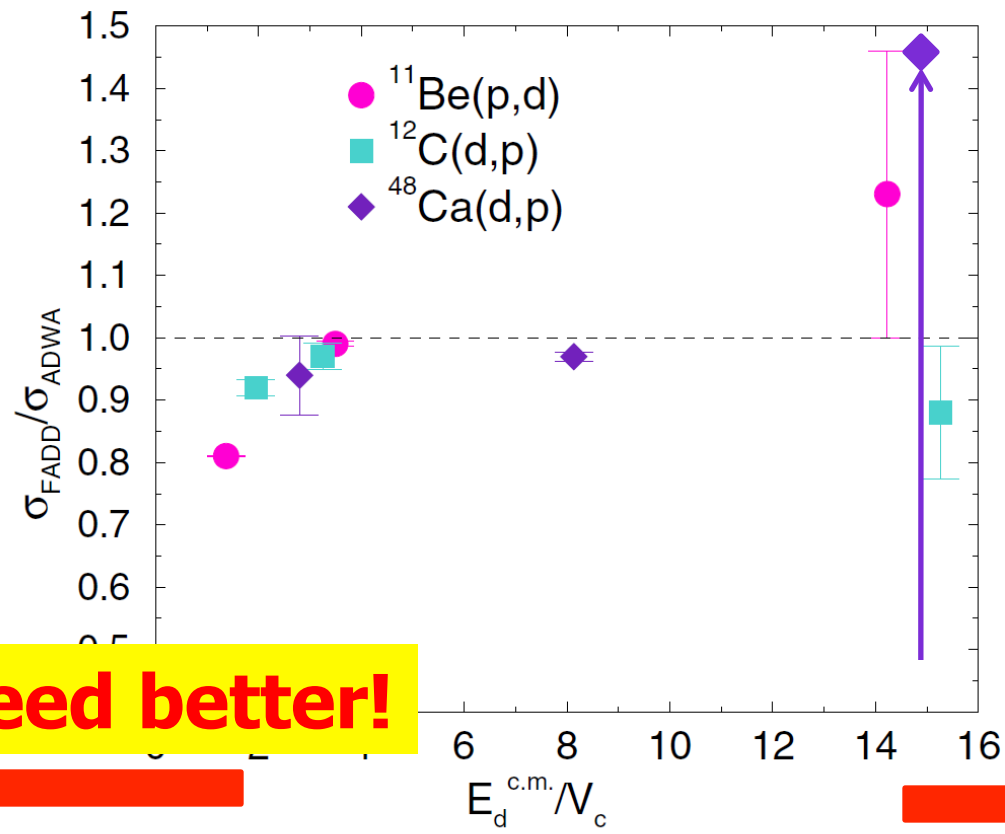


FIG. 3: Angular distributions for  $^{48}\text{Ca}(d,p)^{49}\text{Ca}$ : (a)  $E_d = 19.3$  MeV, (b)  $E_d = 56$  MeV and (c)  $E_d = 100$  MeV.

# Systematic comparison: FR-ADWA vs Faddeev



**need better!**

FIG. 4: Ratio of Faddeev prediction for the cross section at the peak of the angular distribution versus the Adiabatic counterpart plotted in term of the deuteron energy in the c.m. over the Coulomb Barrier.

# (d,p) reactions: beyond FR-ADWA



FR-ADWA: deuteron breakup plus finite-range

[Nguyen, Nunes, Johnson, Phys. Rev. C **82**, 014611(2010)]

If only first term of the expansion is included:  
coupled equations reduce to single channel!

Continuum discretized coupled channel does not make  
this approximation



Milestone for yr 1: comparison CDCC vs Faddeev

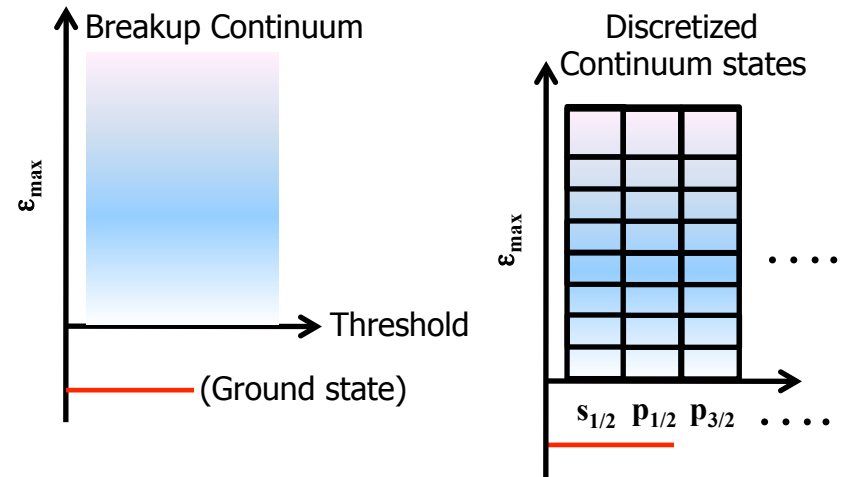
# (d,p) reactions: CDCC

Expand 3-body wfn in deuteron eigenstates

$$H_{\text{int}}(\mathbf{r}) = T_r + V_{\text{pn}}(\mathbf{r})$$

$$\Psi^{(+)}(\vec{r}, \vec{R}) = \sum_{i=1}^{\infty} \phi_i(\vec{r}) \chi_i(\vec{R})$$

➤ Discretize the continuum



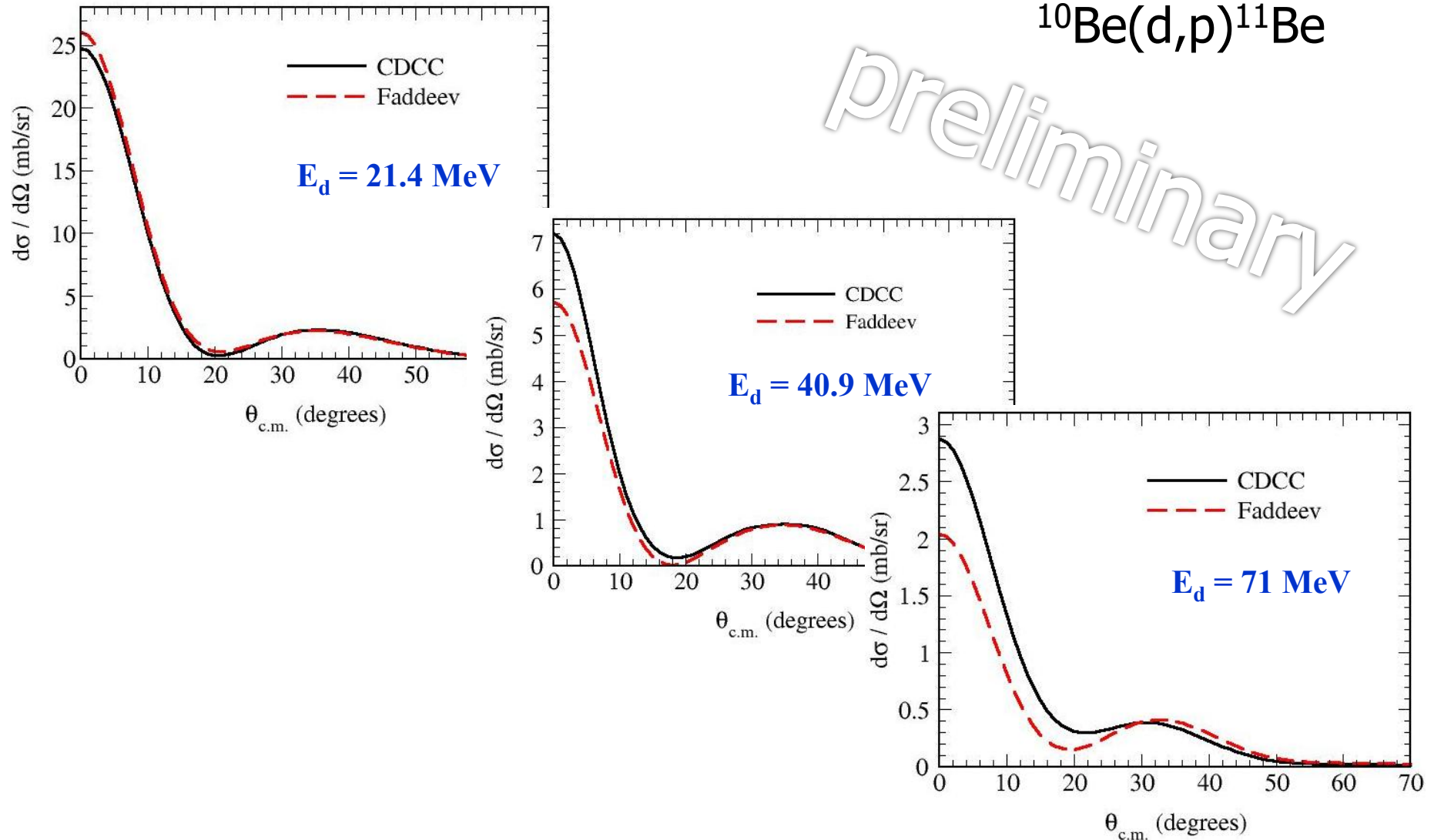
➤ continuum discretized coupled channel (CDCC) equations

# systematic comparison: CDCC vs Faddeev

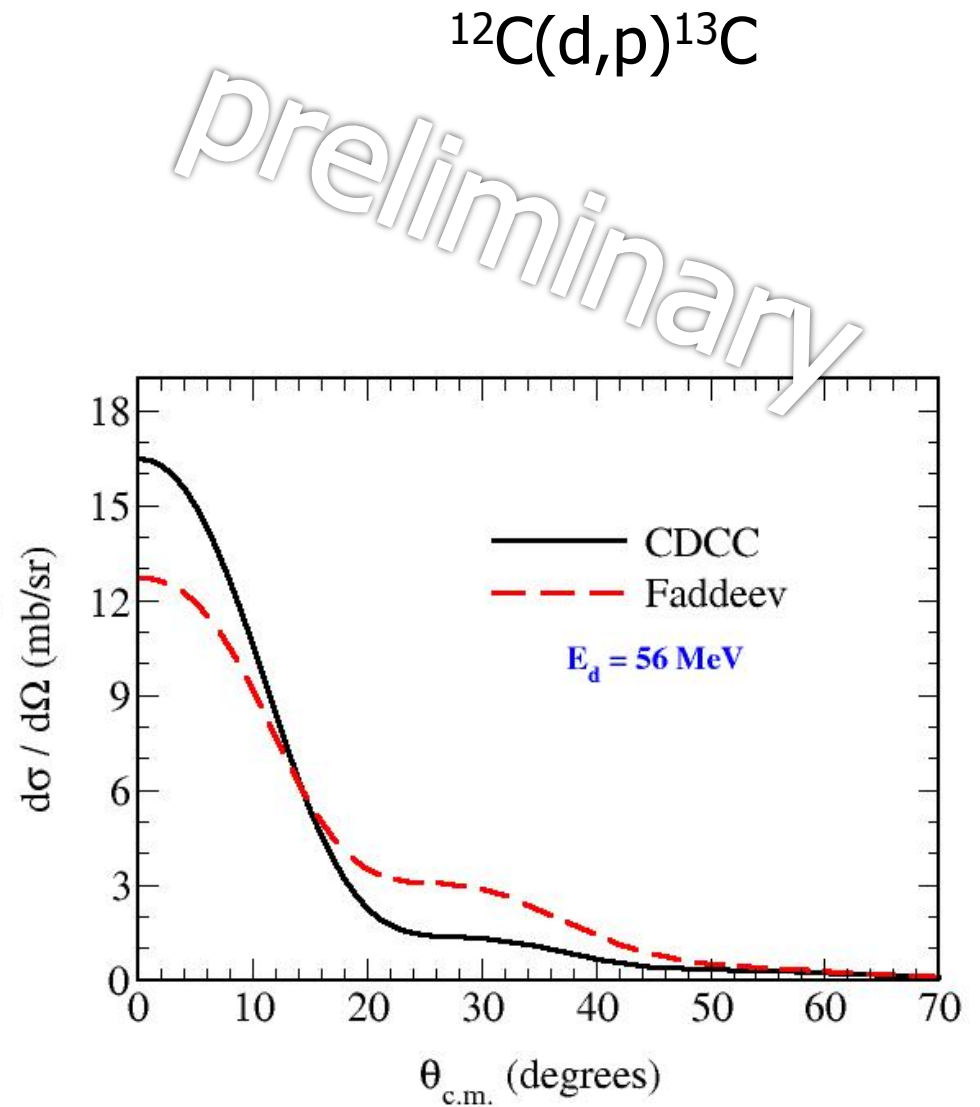
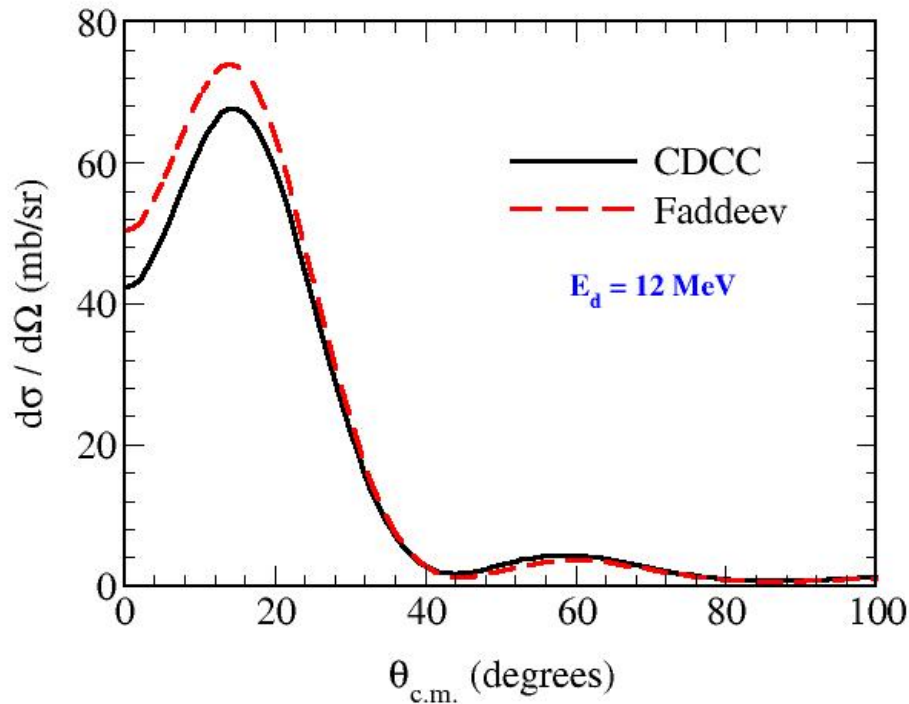


$^{10}\text{Be}(d,p)^{11}\text{Be}$

preliminary

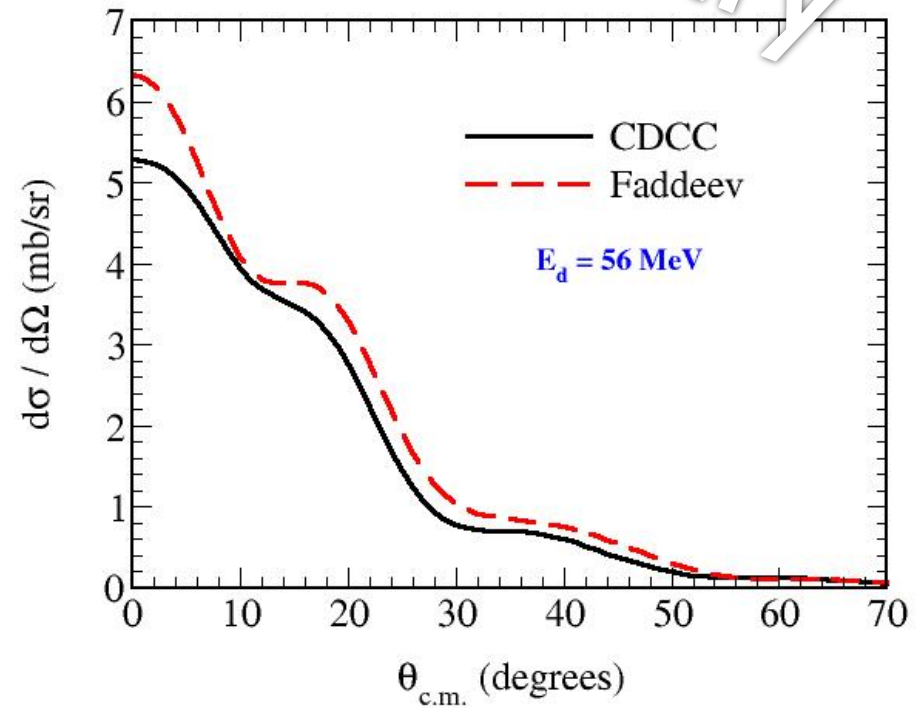
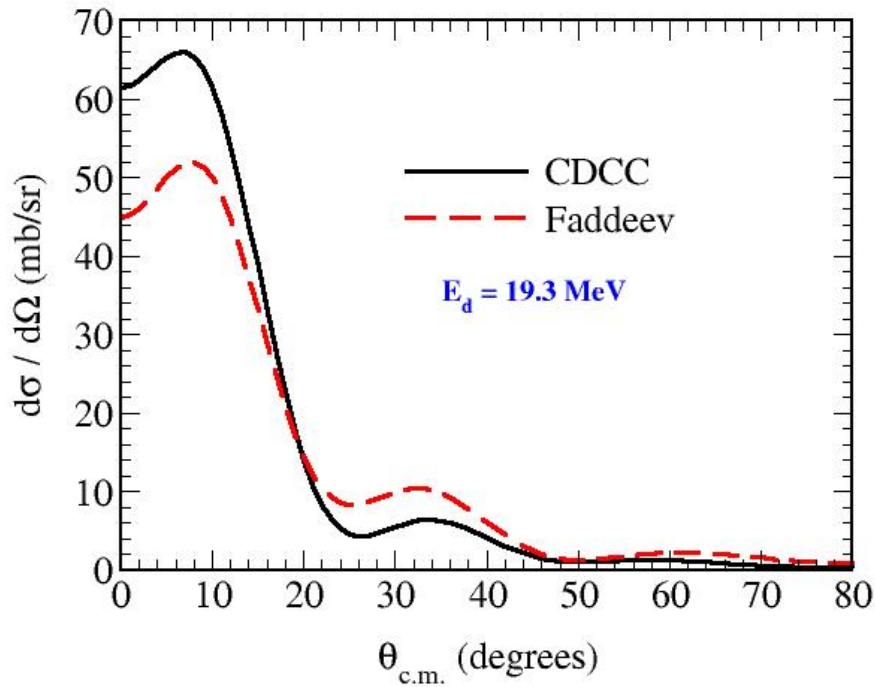


# systematic comparison: CDCC vs Faddeev





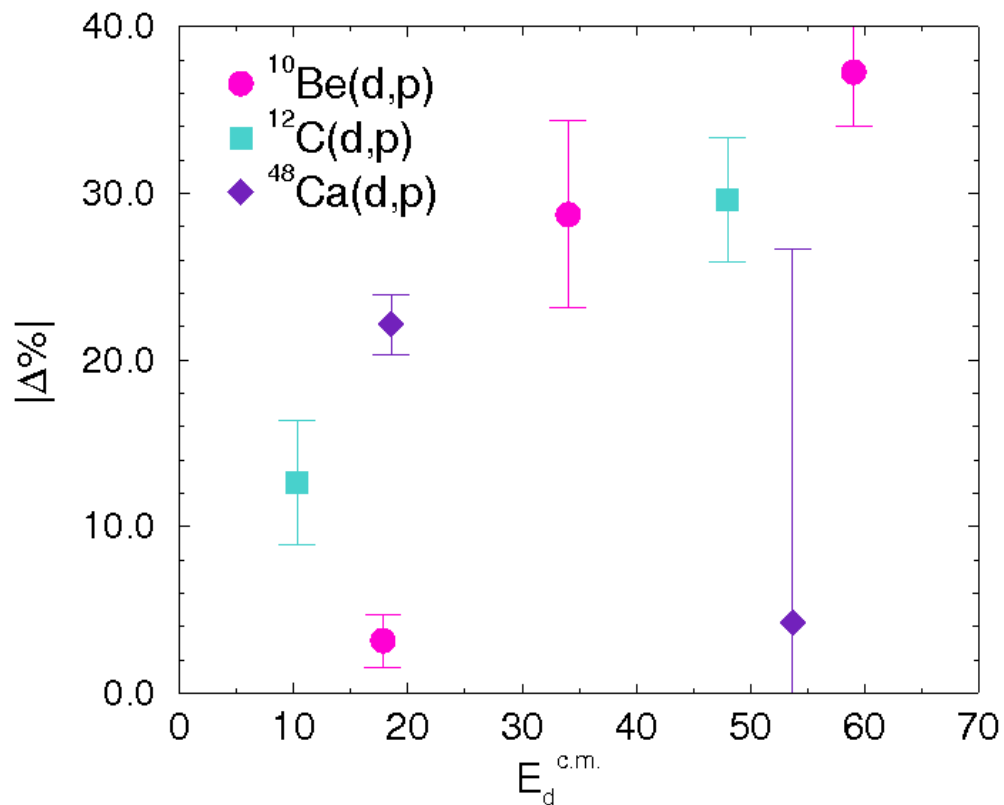
# systematic comparison: CDCC vs Faddeev



# Systematic comparison: CDCC vs Faddeev



## Comparative differences CDCC/FADD



(errors based on remnant)

# summary and conclusions



- preliminary project
  - comparisons Faddeev and Adiabatic **(completed)**
- 1<sup>st</sup> yr milestone
  - comparisons Faddeev and CDCC **(nearly completed)**

## Conclusions:

- agreement around 10 MeV/u
- agreement deteriorates with increasing beam energy
- ambiguities in optical potentials have higher impact at

higher E

## next steps



- extending new AGS code for nuclear reactions
  - starting code development
- capability of including target excitation
- separable optical potentials
  - examining advantages/disadvantages

# acknowledgements



TORUS collaboration  
Ngoc Bich Nguyen (grad student at MSU)  
Muslema Pervin (post doc at MSU)  
Anissa Bey (post doc at UT)

Arnas Deltuva (Lisbon)



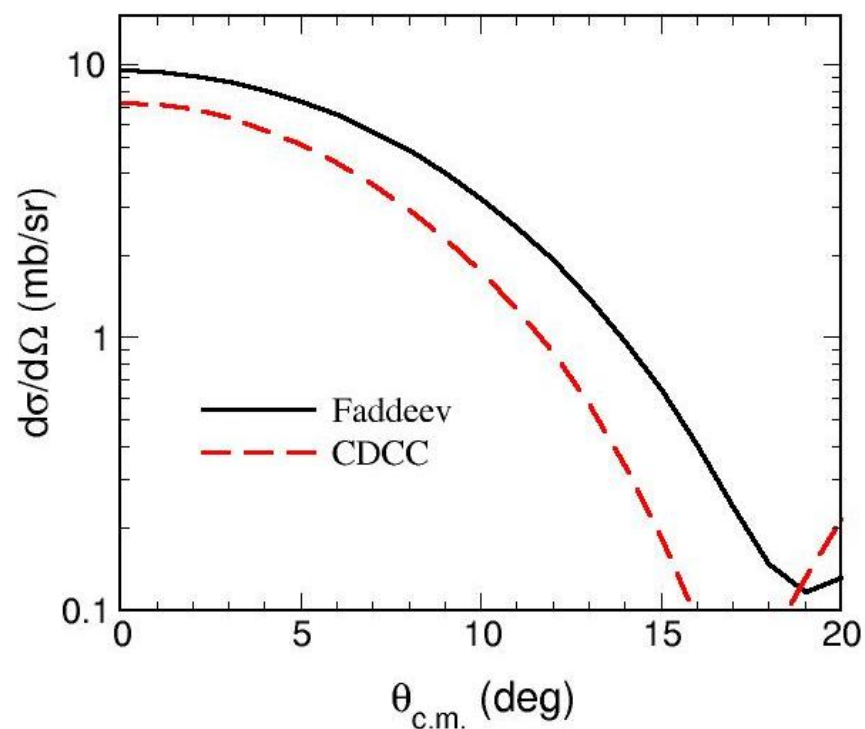
extra slides



# Comparing CDCC vs Faddeev (old)



$^{11}\text{Be}(p,d)^{10}\text{Be}$  @35 MeV



# Percentage effects in the comparison



TABLE I: Percentage differences between the differential cross section at the peak of the distribution for the various formulations:  $\Delta_{F-AD}$  comparing Faddeev with Adiabatic,  $\Delta_{F-Up}(\Delta_{AD-Up})$  the effect of changing the energy at which the proton-target interaction is calculated in the Faddeev (Adiabatic), and finally  $\Delta_{AD-rem}$  the effect of the remnant term in the adiabatic.

reaction	$E(\text{MeV})$	$\theta(\text{deg})$	$\Delta_{F-AD}$	$\Delta_{F-Up}$	$\Delta_{AD-Up}$	$\Delta_{AD-rem}$
$^{11}\text{Be}(p,d)$	5	1	-22.90	0.17	-0.10	-1.36
	10	1	-1.12	0.42	0.44	-1.13
	35	1	18.50	15.25	10.91	26.82
$^{12}\text{C}(d,p)$	7	20	-9.08	1.61	-0.42	-3.74
	12	15	-2.74	2.18	-0.50	-3.92
	56	1	-14.26	13.95	17.50	3.34
$^{48}\text{Ca}(d,p)$	19	8	-6.12	7.16	-0.64	-0.74
	56	1	-3.05	0.76	46.42	0.57
	100	0	51.0	72.9	-42.71	9.73



# Systematic comparison: CDCC vs Faddeev



## Comparative differences CDCC(rem)/FADD

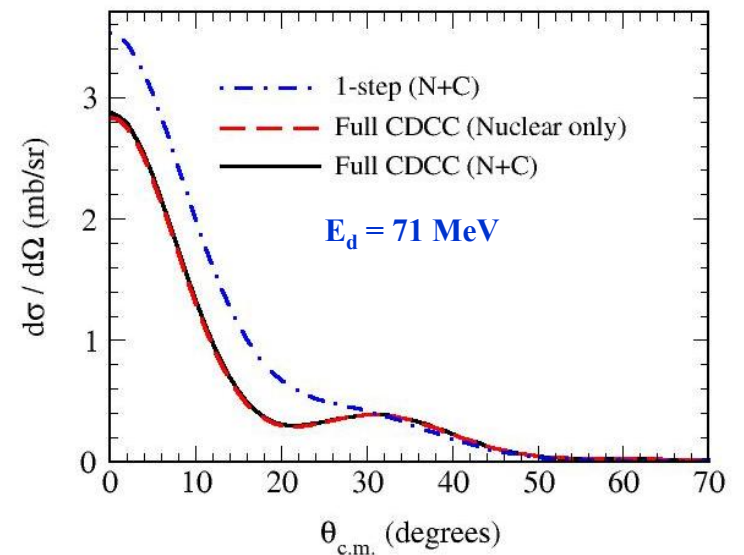
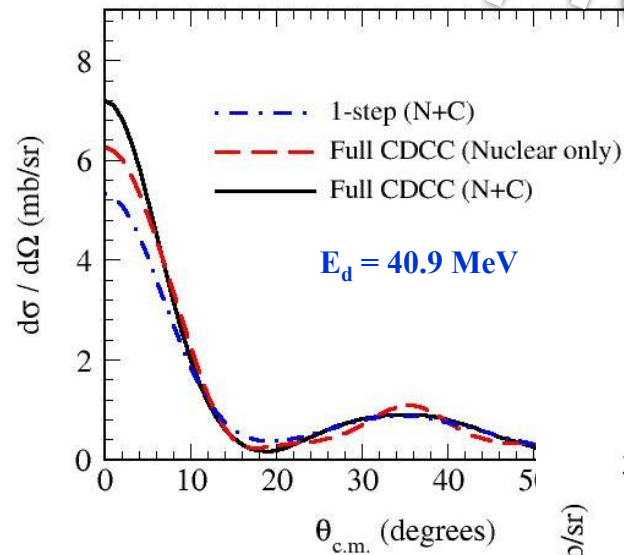
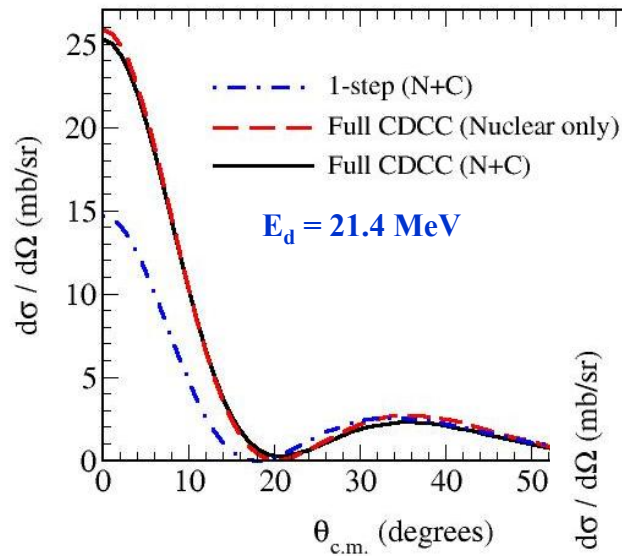
<b><math>^{10}\text{Be}</math></b>		<b><math>^{12}\text{C}</math></b>		<b><math>^{48}\text{Ca}</math></b>	
$E_d$	$\Delta\%$	$E_d$	$\Delta\%$	$E_d$	$\Delta\%$
21 MeV	4.8	12 MeV	9.0	19 MeV	-24.0
41 MeV	-23.1	56 MeV	-25.8	56 MeV	18.1
71 MeV	-34.0				

# systematic comparison: CDCC vs 1-step



$^{10}\text{Be}(d,p)^{11}\text{Be}$

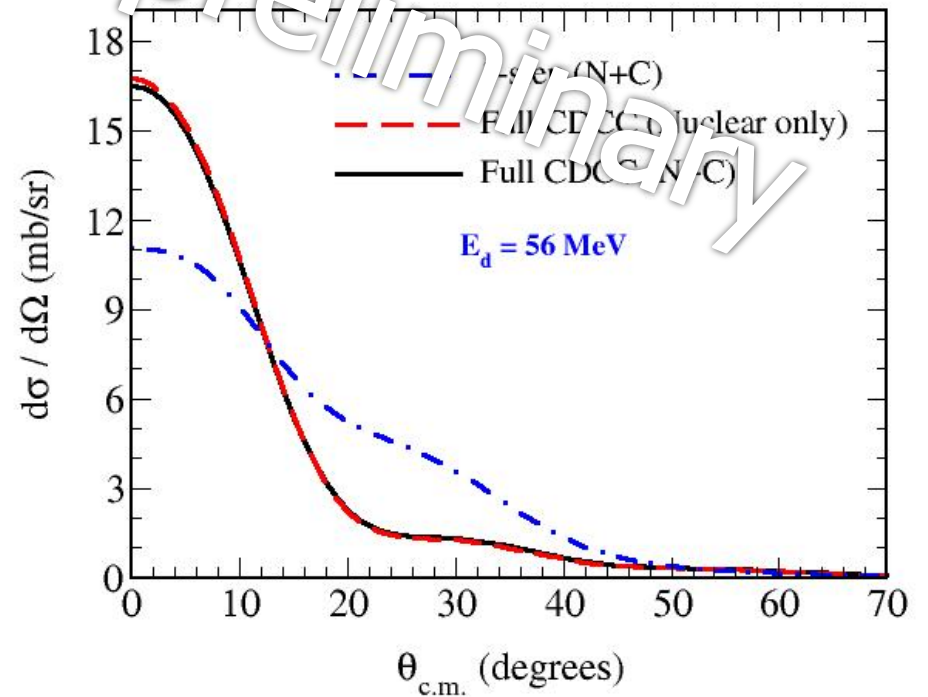
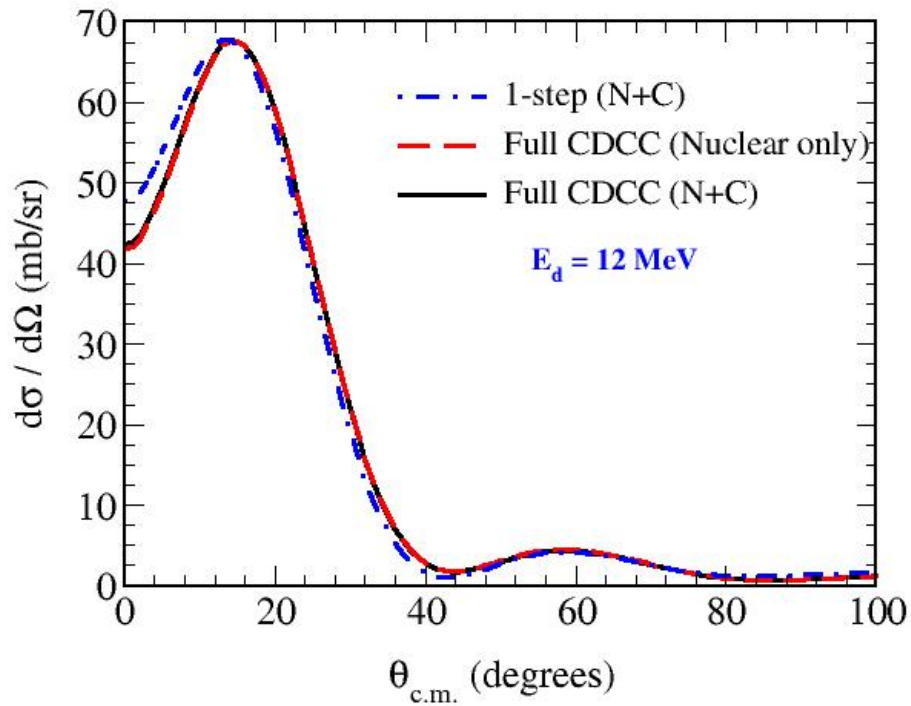
preliminary



# systematic comparison: CDCC vs 1-step



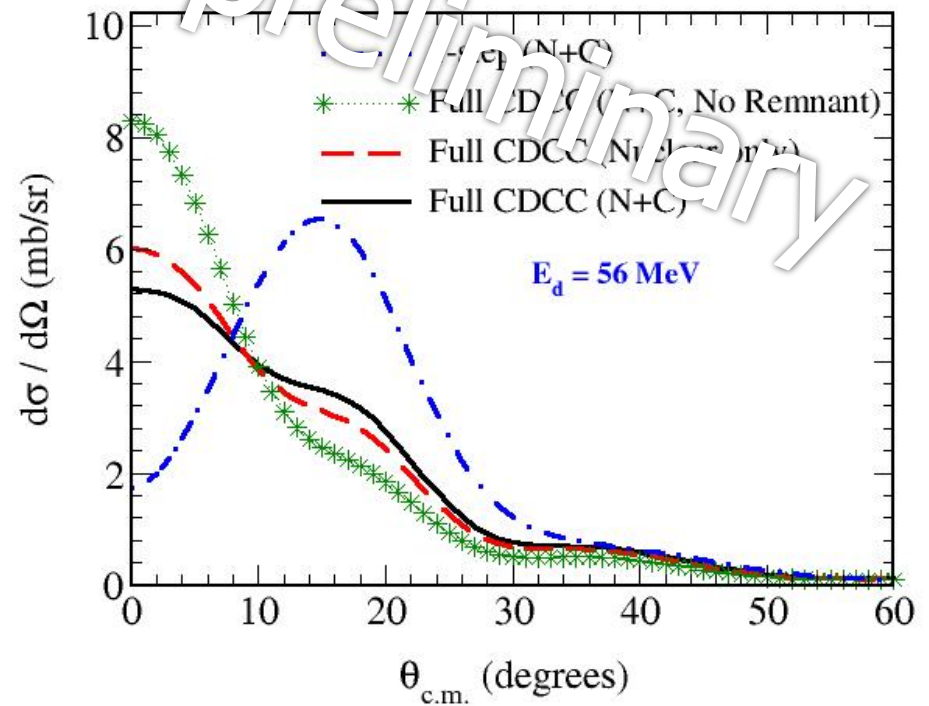
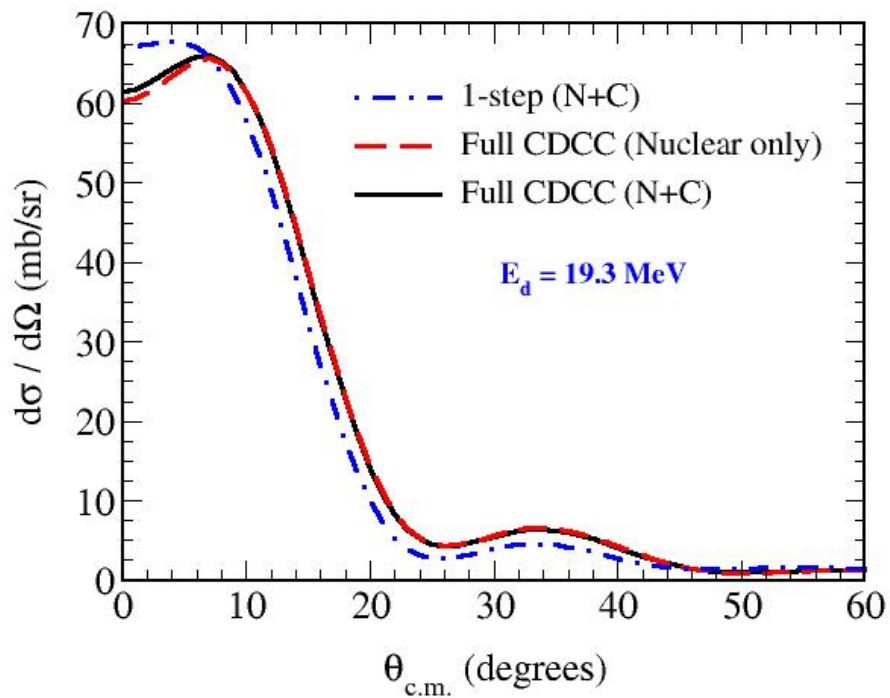
$^{12}\text{C}(d,p)^{13}\text{C}$



# systematic comparison: CDCC vs 1-step



$^{48}\text{Ca}(d,p)^{49}\text{Ca}$

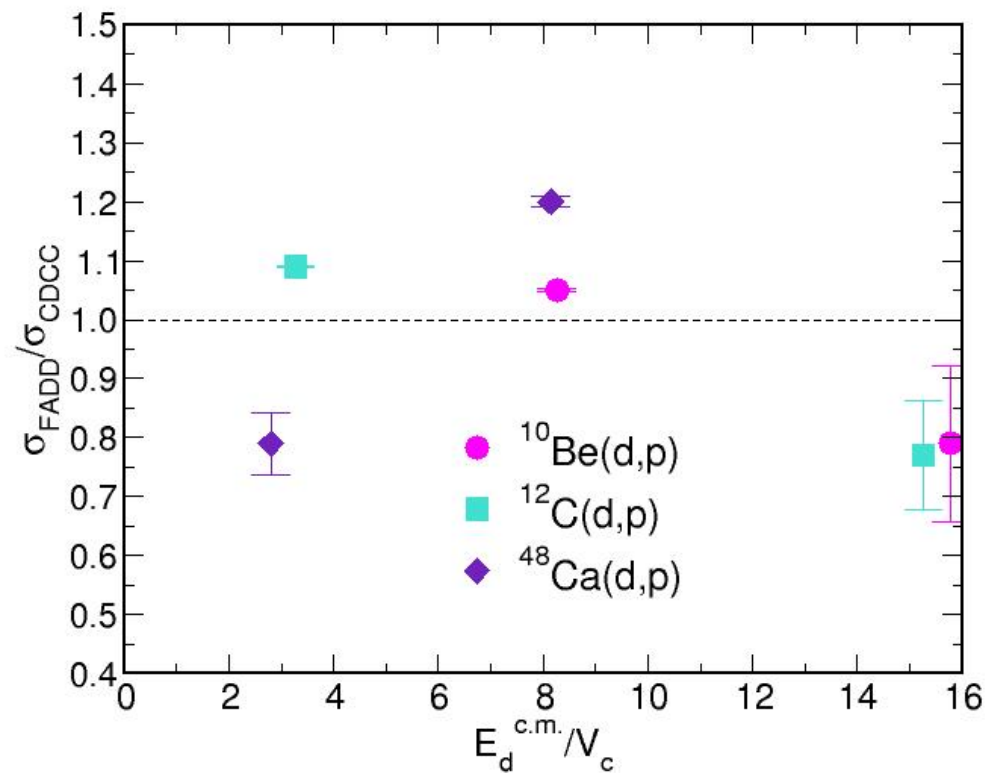
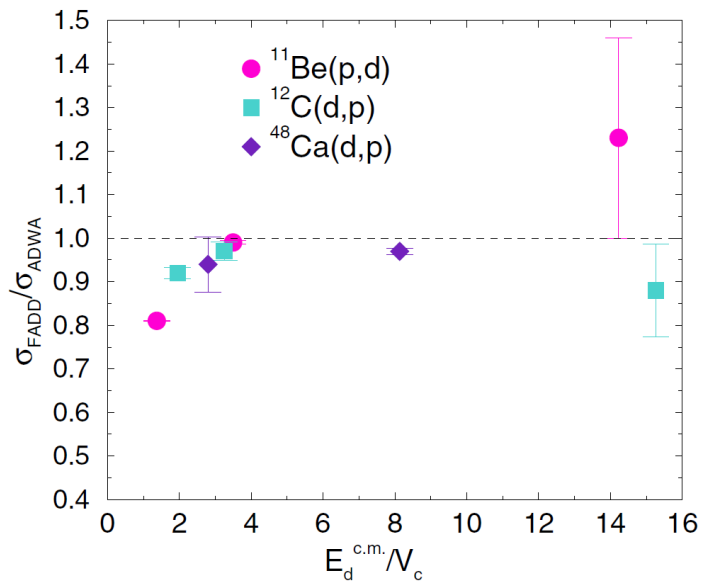


# Systematic comparison: CDCC vs Faddeev



## Comparative differences FADD/CDCC

### FADD/ADWA



F  
a  
counterpart plotted in term of the deuteron energy in the c.m.  
over the Coulomb Barrier.

Upadhyay, Deltuva and Nunes, in preparation



# theory opportunities with FRIB



**DOE Nuclear Physics Mission is to understand the fundamental forces and particles of nature as manifested in nuclear matter, and provide the necessary expertise and tools from nuclear science to meet national needs**

**DOE Nuclear Physics Mission is accomplished by supporting scientists who answer overarching questions in major scientific thrusts of basic nuclear physics research**

## Science Drivers (Thrusts) from NRC RISAC

Nuclear Structure	Nuclear Astrophysics	Tests of Fundamental Symmetries	Applications of Isotopes
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## Overarching Questions from NSAC 2007 LRP

<p>★ What is the nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes?</p> <p>★ What is the origin of simple patterns in complex nuclei?</p>	<p>What is the nature of neutron stars and dense nuclear matter?</p> <p>What is the origin of the elements in the cosmos? ★</p> <p>What are the nuclear reactions that drive stars and stellar explosions? ★</p>	<p>Why is there now more matter than antimatter in the universe?</p>	<p>What are new applications of isotopes to meet the needs of society?</p>
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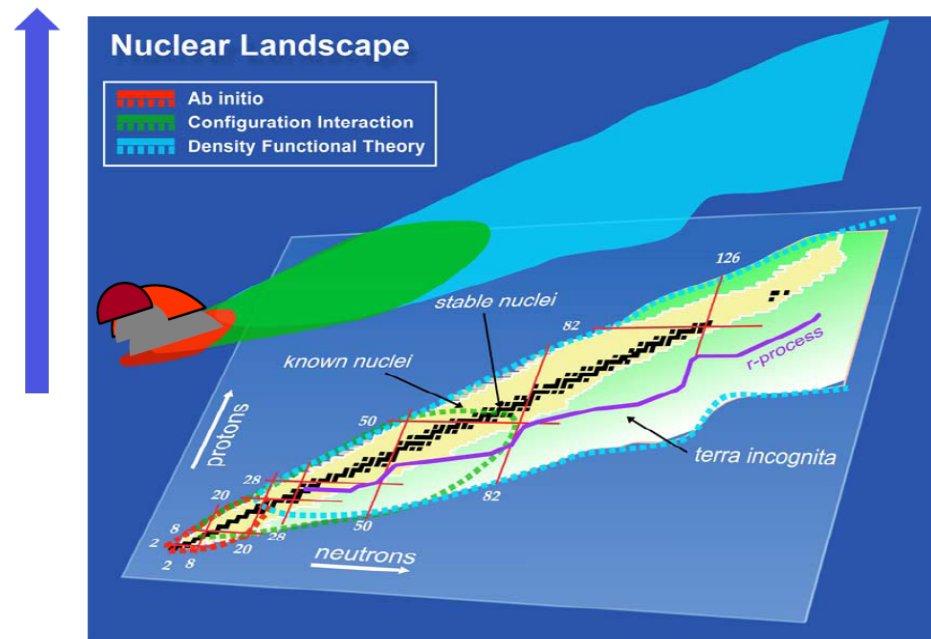
## Overarching questions are answered by rare isotope research

### 17 Benchmarks from NSAC RIB TF measure capability to perform rare isotope research

<p>→ Shell structure</p> <p>2. Superheavies</p> <p>3. Skins</p> <p>→ Pairing</p> <p>5. Symmetries</p> <p>→ Limits of stability</p> <p>→ Weakly bound nuclei</p> <p>15. Mass surface</p>	<p>6. Equation of State (EOS)</p> <p>→ r-Process</p> <p>8. <math>^{15}\text{O}(\alpha, \gamma)</math></p> <p>9. <math>^{59}\text{Fe}</math> supernovae</p> <p>15. Mass surface</p> <p>→ rp-Process</p> <p>17. Weak interactions</p>	<p>12. Atomic electric dipole moment</p>	<p>10. Medical</p> <p>11. Stewardship</p>
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## Theory Of Reactons for Unstable iSotoptes:

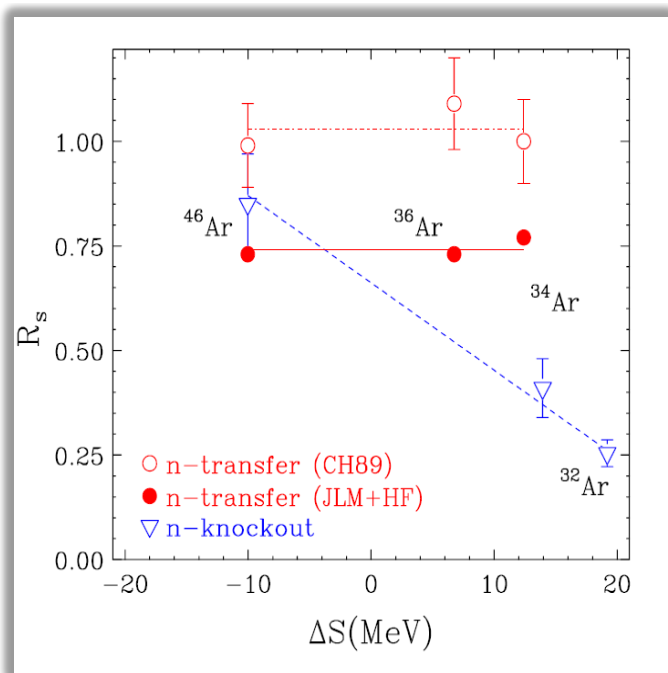
- 1) develop new methods to advance nuclear reaction theory for unstable isotopes, building on Faddeev techniques



# opportunities with FRIB



## transfer versus knockout



[Jenny Lee et al, PRL 2009]

[Gade et al, Phys. Rev. Lett. 93, 042501]

- shell structure
- correlations
- pairing
- weakly bound systems
- role of continuum
- ...

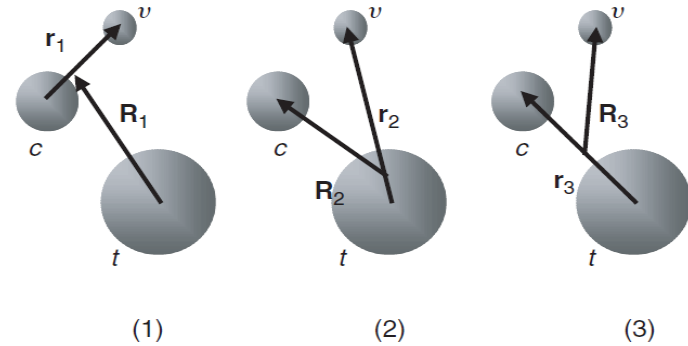
**FRIB needs accurate reaction models!**



# three body problem: exact solution



$$\Psi = \sum_{n=1}^3 \Psi^{(n)}(\mathbf{r}_n, \mathbf{R}_n)$$



3 jacobi coordinate sets

Faddeev Equations:

$$(E - T_1 - V_{vc})\Psi^{(1)} = V_{vc}(\Psi^{(2)} + \Psi^{(3)})$$

$$(E - T_2 - V_{ct})\Psi^{(2)} = V_{ct}(\Psi^{(3)} + \Psi^{(1)})$$

$$(E - T_3 - V_{tv})\Psi^{(3)} = V_{tv}(\Psi^{(1)} + \Psi^{(2)})$$

AGS: T-matrix version and momentum space

# (d,p) reactions: faddeev versus FR-ADWA

FR-ADWA: deuteron breakup plus finite-range

[Nguyen, Nunes, Johnson, Phys. Rev. C **82**, 014611(2010)]

If only first term of the expansion is included:  
coupled equations reduce to single channel!

How good is this approximation?

Faddeev (AGS): solves 3-body problem exactly

[Deltuva, Phys. Rev. C **79**, 021602(2009)]



- breakup and transfer wavefunctions are mixed...
- intensive and expensive computations