Advancing deuteron stripping reactions

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TORUS collaboration

- **Why nuclear reactions are measured?**
- **What important information for nuclear structure, nuclear astrophysics and applied physics can be extracted?**

- **Answer to these questions is important part of advancing nuclear reaction theory.**

- **Deuteron stripping reactions since the dawn of nuclear physics served as one of the main tools to extract spectroscopic information, in particular spectroscopic factors (SFs).**

- **"Unitary correlation in nuclear reaction theory: Separation of nuclear reactions and spectroscopic factors", A. M. Mukhamedzhanov and A. S. Kadyrov, Phys. Rev. C 82, 051601(R) (2010). First published paper related to TORUS.**

- **Nuclear reactions and SFs - long standing misguided connection .**

- **The exact amplitudes for the (d,p), (d,pn), and (e,e'p) reactions determining the asymptotic behavior of the scattering wave functions in the corresponding channels, in contrast to SFs, are invariant under finite-range unitary transformations.**
- **Moreover, the exact reaction amplitudes are not parameterized in terms of the SFs and nuclear reactions cannot provide a tool to determine SFs, which are not observable!**

- **The only spectroscopic quantity, invariant under finite-range unitary transformations, which can be extracted from reaction analysis is the asymptotic normalization coefficient (ANC) .**

- **Extension of existing and construction of new facilities are underway-exotic nuclei.**
- **The important part of the new programs, as it has been for previous fifty years, is determination of SFs from nuclear reactions to compare with theoretical calculations.**

- **This paper calls for revision of the existing and future programs.**

My part in TORUS

Generalized Faddeev equations (AGS), target degrees of freedom are included

Equations are from
E. O. Alt, A. M. M. et al,
Phys. Rev. C 75, 054003 (2007).
Starting code is available

Collaboration with
C. Elster and N. Upadhyay

2-nd year: beginning code

Theory of deuteron stripping into bound and resonance states

Equations -completed

2-nd year: code and test of the theory, application for nuclear astrophysics

Collaboration with I. Thompson, F. Nunes and J. Escher

Important: both theories provide ANCs-residues in the S-matrix poles (bound states and resonances).
No SFs!

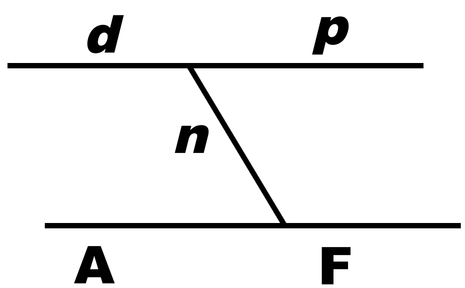
- **Goal: deliver the theory of (d,p) stripping reactions populating resonance states.**
- **Problems: spectroscopic information, convergence, multi-channel, multi-level(interfering) broad and narrow resonances, connection to direct resonant binary reactions, nuclear astrophysics.**

- **New theory of (d,p) reactions- Generalized R-matrix method.**

Based on "Surface-integral formulation of scattering theory " A.S. Kadyrov , A.M. M. et al., Ann. Phys. 324, Issue 7, 1516-1546 , July 2009 Special Issue

Binary resonant reactions:
R –matrix parameters: observable partial widths, resonance energies, boundary conditions and channel radii.

A(d,p)F



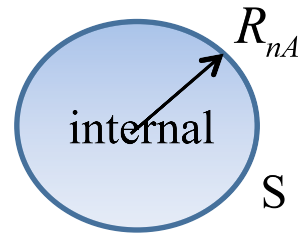
➢**New theory: the stripping amplitude is parameterized, as the R-matrix amplitude for the binary resonance reactions , in terms of the same quantities.**

➢**The theory has been developed both for stripping to bound and resonance states.**
Many-level and many-channel cases are included, both narrow and broad resonances.
Convergence problem is resolved in the CDCC approach (coupling of the initial elastic and deuteron breakup channels).

➢**Provides experimentalists with a consistent tool to analyze both resonance binary reactions and deuteron stripping reactions in terms of the same quantities.**

Stripping to bound states

CDCC $M^{CDCC(post)}(\mathbf{k}_{pF},\mathbf{k}_{dA}) = < \chi^{(-)}_{pF} I^F_A |\Delta V^P_{pF}|\Psi_i^{CDCC(+)} > .$



Internal region, $r_{nA} \le R_{nA}$,

External region, $r_{nA} > R_{nA}$.

Second Jacobian variable, r_{pF}. Integration over the full space.

$$M^{CDCC(post)}(\mathbf{k}_{pF},\mathbf{k}_{dA}) = \overset{\textit{small}}{M_{int}^{CDCC(post)}(\mathbf{k}_{pF},\mathbf{k}_{dA})} + \overset{C_{An}}{M_S^{CDCC}(\mathbf{k}_{pF},\mathbf{k}_{dA})}.$$

No external term, only small internal and dominant surface term

$$M_S^{CDCC}(\mathbf{k}_{pF},\mathbf{k}_{dA}) = < \chi^{(-)}_{pF} I^F_A |\overleftarrow{T} - \overrightarrow{T}|\Psi_i^{CDCC(+)} >$$

$$= \boxed{\gamma_{nA}}\; i^{-l_{nA}} \sqrt{R_{nA}} \sqrt{\frac{\hbar^2}{2\mu_{nA}}} \int d\mathbf{r}_{pF}\, \chi^{(+)}_{-\mathbf{k}_{pF}}(\mathbf{r}_{pF}) \int d\Omega_{nA}$$

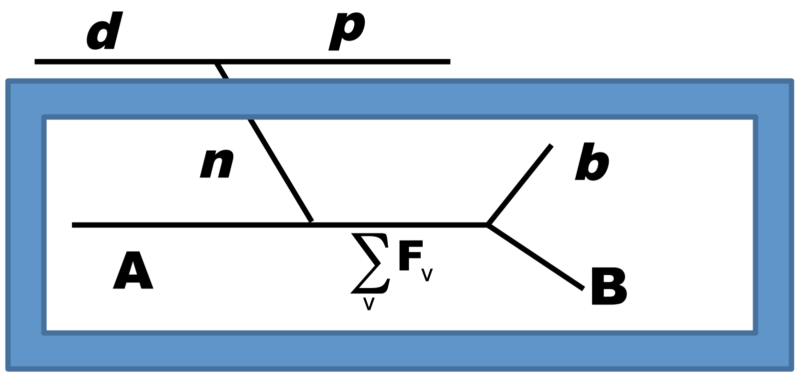
$$\left[\Psi_i^{CDCC(+)} \boxed{(B_{nA} - 1)} - R_{nA} \frac{\partial \Psi_i^{CDCC(+)}}{\partial r_{nA}} \right]\Big|_{r_{nA}=R_{nA}}$$

-parameterized in terms of the reduced width amplitude and boundary condition

Stripping to resonance states

$$M^{CDCC(post)}(\mathbf{k}_{pF},\mathbf{k}_{dA}) = M^{CDCC(post)}_{int}(\mathbf{k}_{pF},\mathbf{k}_{dA}) \;+\; M^{CDCC}_{S}(\mathbf{k}_{pF},\mathbf{k}_{dA}).$$

Two-channel, many-level case



$$M^{DW}_{S_{R_{nA}}}(P,\mathbf{k}_{dA}) = i^{-l_{nA}+1}\sqrt{R_{nA}}\sqrt{\frac{\hbar^2}{2\mu_{nA}}}\sqrt{\frac{k_{bB}}{\mu_{bB}}}\,e^{-i\delta^{ss}_{bB}}\,e^{-i\delta^{ss}_{nA}}\sum_{\nu\tau}^{N}\boxed{\Gamma^{1/2}_{\nu(bB)}(E_{bB})}\,[\mathbf{A}^{-1}]_{\nu\tau}\,\boxed{\gamma_{\tau(nA)}}$$

$$\times\int d\mathbf{r}_{pF}\,\chi^{(+)}_{-\mathbf{k}_{pF}}(\mathbf{r}_{pF})\int d\Omega_{nA}\Big[\Psi^{CDCC(+)}_{i}\,\boxed{(B_{nA}-1)}\,-R_{nA}\frac{\partial\Psi^{CDCC(+)}_{i}}{\partial r_{nA}}\Big]\Big|_{r_{nA}=R_{nA}}$$

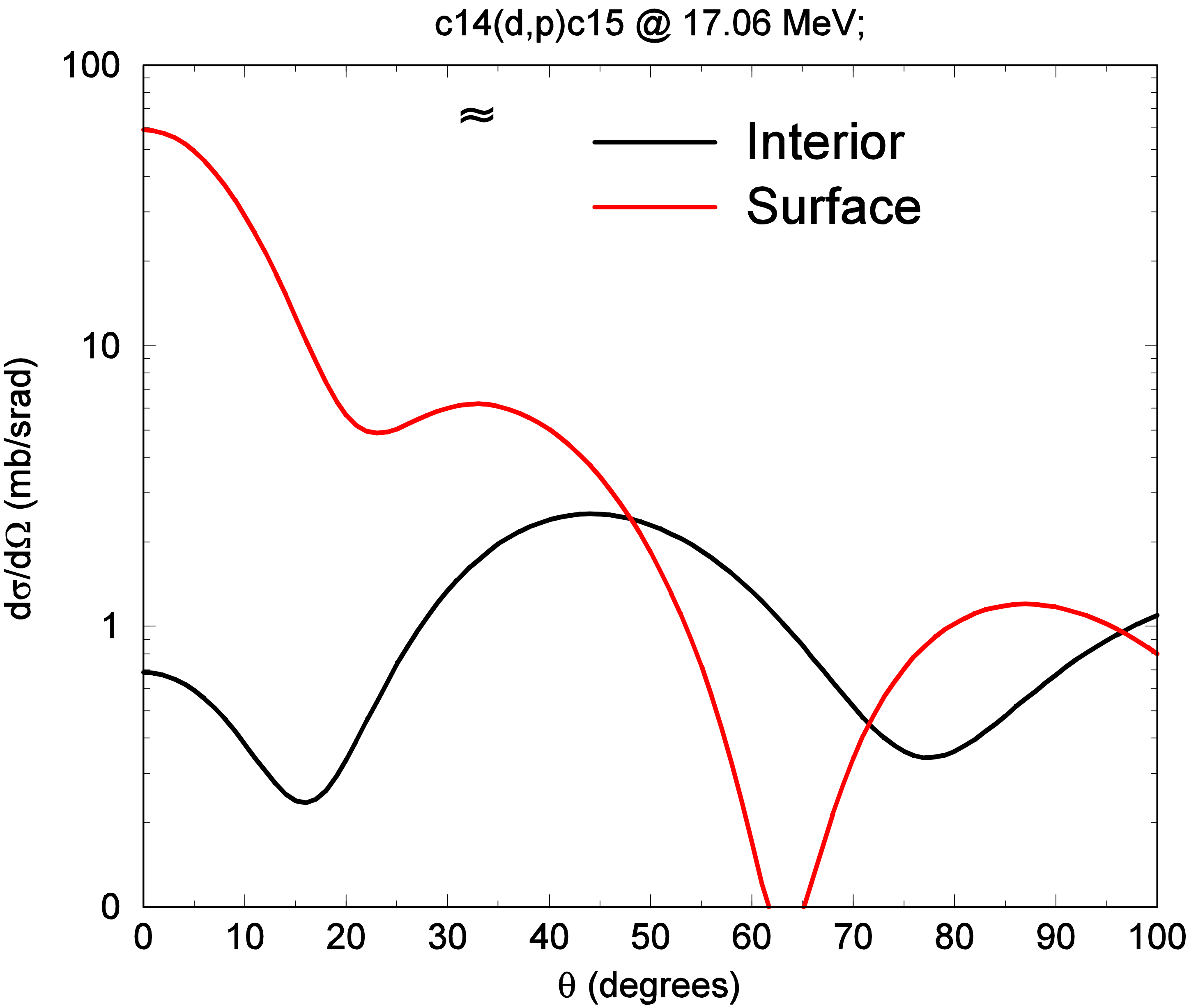
Stripping to resonance, single-level, single channel case

$$M_S^{CDCC}(P,\mathbf{k}_{dA}) = -i\,e^{-i2\delta_{nA}^{ss}} \boxed{\frac{[\Gamma_{(nA)}(E_{nA})]^{1/2}\gamma_{nA}}{E_R - E_{nA} - i\frac{\Gamma_{(nA)}(E_{nA})}{2}}} \frac{R_{nA}}{2\mu_{nA}}\sqrt{2k_{nA}R_{nA}}$$

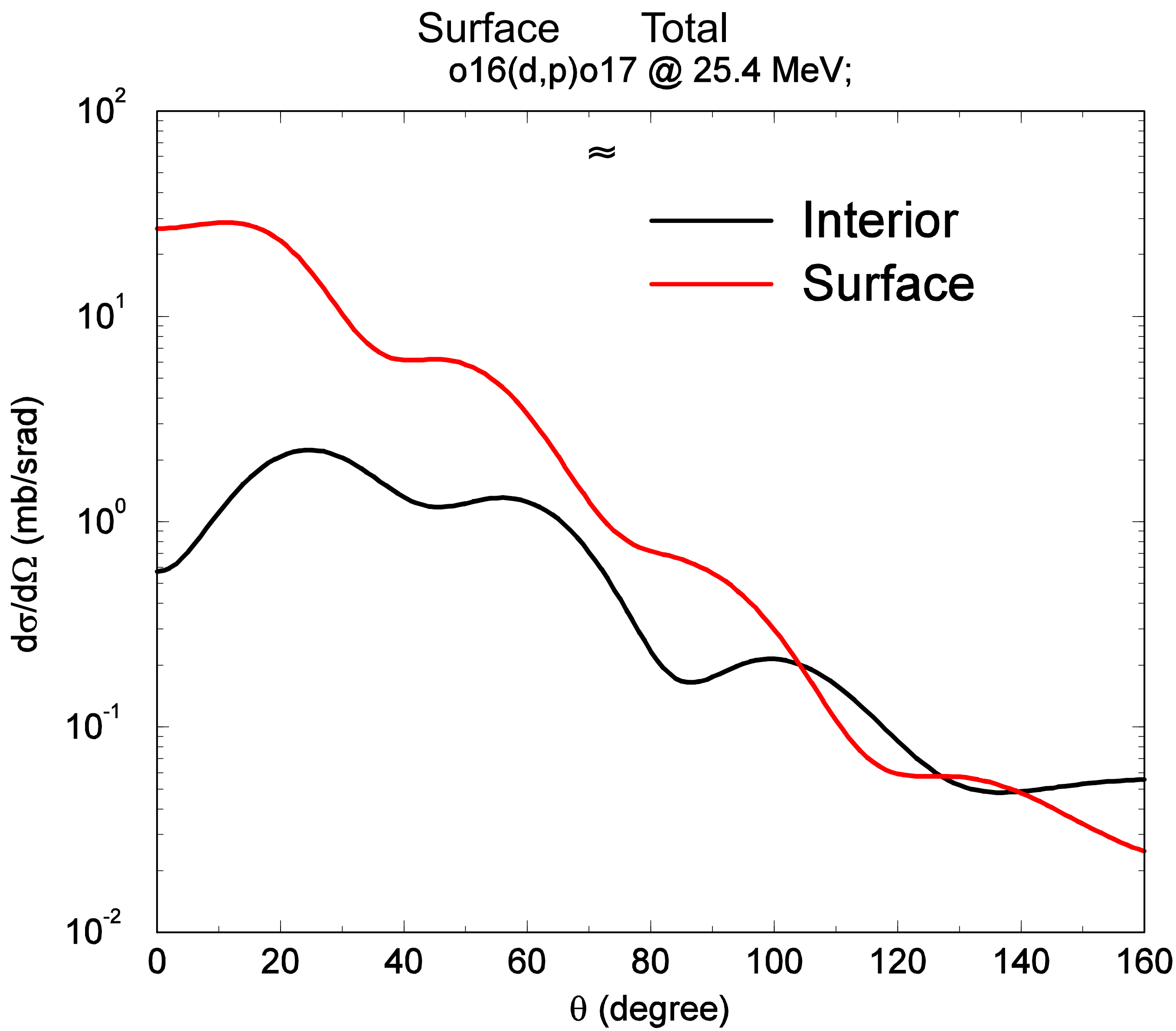
$$\int d\mathbf{r}_{pF}\,\chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF})\int d\Omega_{nA}[\Psi_i^{CDCC(+)}\boxed{(B_{nA}-1)} - \frac{\partial\Psi_i^{CDCC(+)}}{\partial r_{nA}}]\,|_{r_{nA}=R_{nA}}$$

$$M_{int}^{DW(post)}(P,\mathbf{k}_{dA}) = -i\sqrt{\frac{1}{k_{nA}\mu_{nA}}}\,e^{-i\delta_{nA}^{ss}}[\Gamma_{\nu(nA)}(E_{nA})]^{1/2}[\mathbf{A}^{-1}]_{\nu\tau} < \chi_{pF}^{(-)} X_\tau|\Delta\bar{V}_{pF}|\Psi_i^{CDCC(+)} >$$

Surface Total



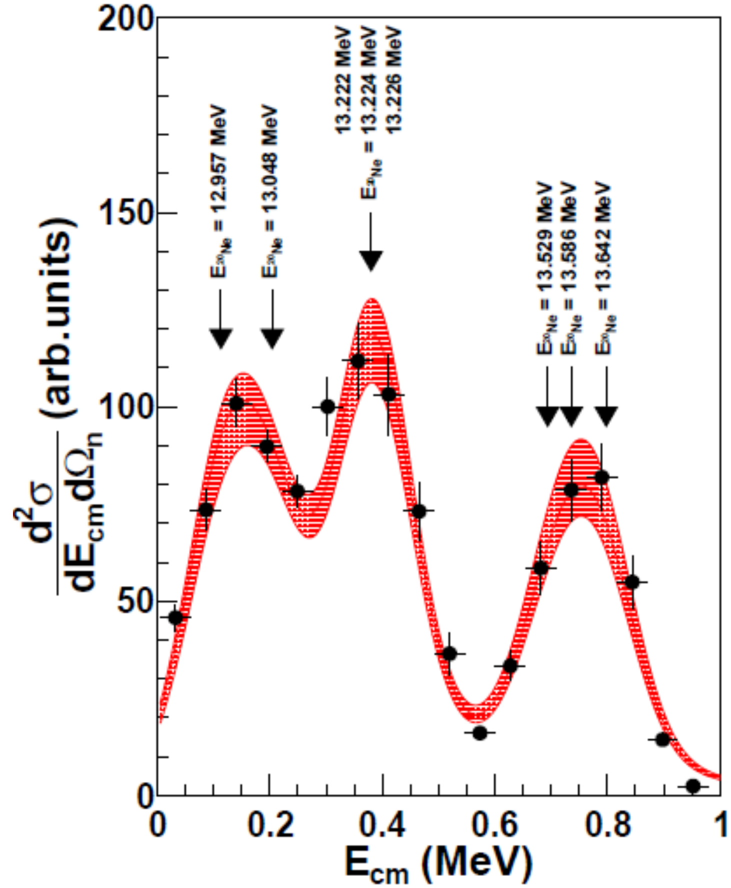
Calculations by Filomena Nunes, MSU/NSCL



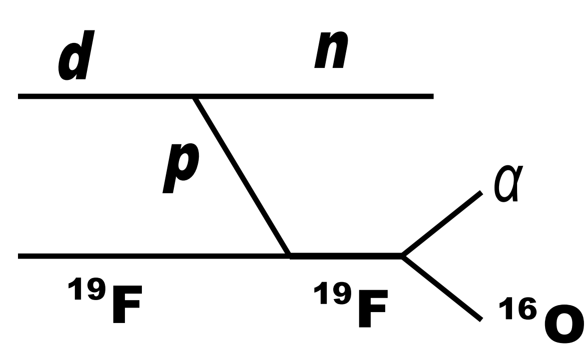
Calculations by Filomena Nunes, MSU/NSCL

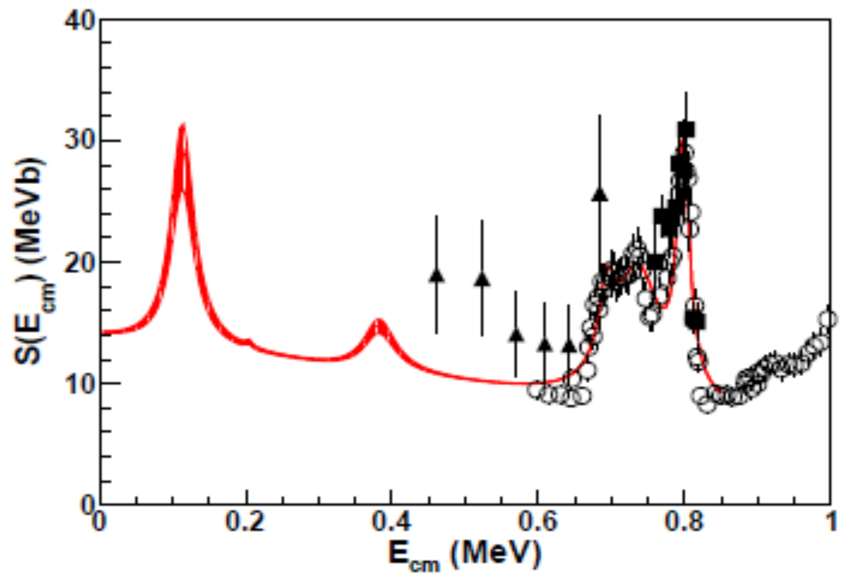
Trojan Horse reaction $^{2}H(^{19}F, \alpha\,^{16}O)n$

M. La Cognata, A. M. M. et al., submitted to APJ 2011



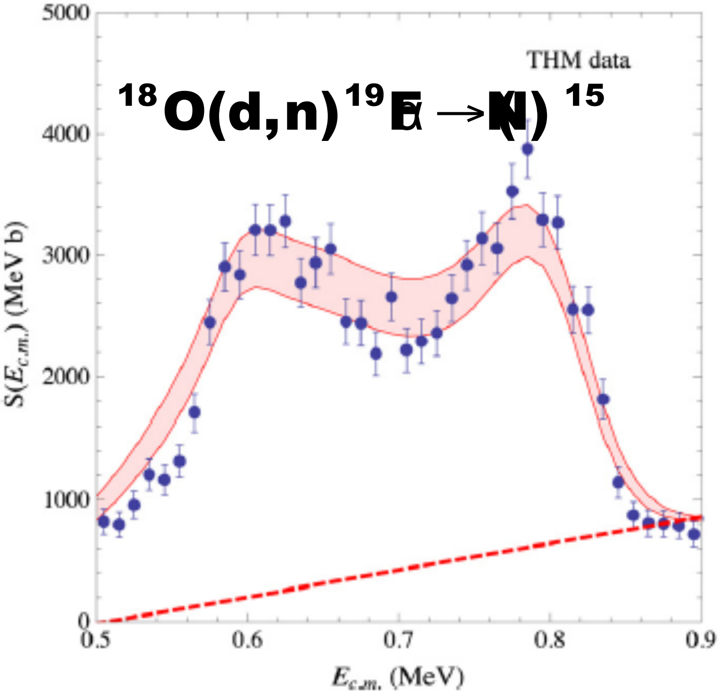
QF cross section for $^{2}H(^{19}F, \alpha\,^{16}O)n$

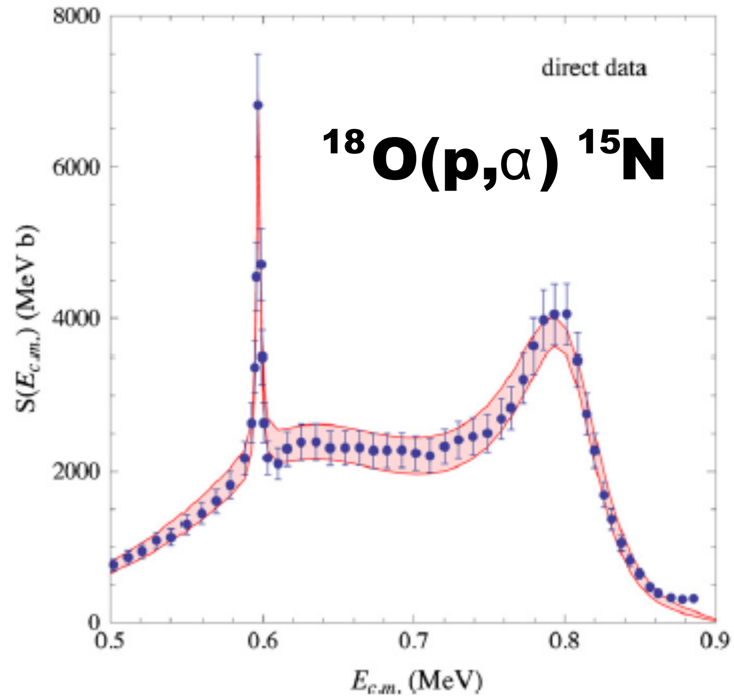




R-matrix parameterization of the $^{19}F(p,\alpha)\,^{16}O$ S fractor. Open circles – Isoya et al. (1959), black squares – Caracciolo et al. (1974), black triangles – Breuer (1959). Red solid line – THM, which is normalized to direct data at energies > 0.6 MeV.

M. La Cognata, C. Spitaleri and A.M.M., APJ 722, 1 (2010)





H. Lorentz-Wirzba et al., NPA 313, 346 (1979)

Two-level, two-channel R-matrix fit

The fitting parameters are determined by simultaneous fitting of direct and THM data

2 papers have been published, one under review, 1 submitted, 2 to be submitted

Future work in TORUS

2-nd year: code for generalized AGS equations including target degrees of freedom

Collaboration with C. Elster, F. Nunes and N. Upadhyay

2-nd year: code and test of the theory, application for nuclear astrophysics

Collaboration with I. Thompson, F. Nunes and J. Escher

$$H_{\mathrm{int}}\,\varphi\,|\;\rangle^{\rho} = \square\;|\square^{\rho}\;\rangle^{\rho}$$

Projection of A+2 problems on three-body space

$$\underline{H} = [H^{\rho\sigma}]\;\varphi = \langle\,|H^{\rho}|\square\;\rangle^{\sigma}$$

Projected coupled AGS equations

$$X^{\sigma\rho}_{\beta n,\square m}(q'_{\square}, q_{\square}; z) = \square\square(q'_{\square}, q_{\square}; z)^{\rho\rho}_{n,\square m} + \square \sum_{\gamma,r,s} \sum_{\square=1} \int dq''\, X^{\sigma\tau}_{\square n,\square r}(q'_{\square}, q''_{\square}; z)$$

$$\hat{T}^{\tau\rho}_{\gamma;rs}(z - q''^{2}_{\gamma}/2M_{\gamma})\, Z^{\rho\rho}_{\gamma r,\square m}(q''_{\square}, q_{\square}; z)$$

$$Z^{\rho\rho}_{\beta n,\alpha m}(\mathbf{q}'_{\beta}, \mathbf{q}_{\alpha}; z) = \bar{\delta}_{\alpha\beta} c^{\rho *}_{n} c^{\rho}_{m} \langle \mathbf{q}'_{\beta} | \langle \chi^{\rho}_{\beta n} | \mathcal{G}^{\rho\rho}_{0}(z) | \chi^{\rho}_{\alpha m} \rangle | \mathbf{q}_{\alpha} \rangle.$$

$$\mathcal{V}^{\rho\sigma}_{\alpha} = \sum_{mn} |\chi^{\rho}_{\alpha m}\rangle \lambda^{\rho\sigma}_{\alpha;mn} \langle \chi^{\sigma}_{\alpha n}|$$

$$\hat{\tau}^{\rho\sigma}_{\alpha;mn}(\hat{z}) = \lambda^{\rho\sigma}_{\alpha;mn} + \sum_{\tau=1}^{N} \sum_{r} \lambda^{\rho\tau}_{\alpha;mr} \times \langle \chi^{\tau}_{\alpha r} | \frac{1}{\hat{z} - \epsilon^{\tau} - P^{2}_{\alpha}/2\mu_{\alpha}} | \chi^{\tau}_{\alpha r} \rangle \hat{\tau}^{\tau\sigma}_{\alpha;rn}(\hat{z})$$