Faddeev techniques as a theoretical tool to study exotic nuclei

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2015-02-17 · Athens, OH

TORUS Collaboration (`http://reactiontheory.org`)

Scientific questions

Where to look for the elements heavier than iron?

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$$\Downarrow$$

How the distributions of the elements were formed?

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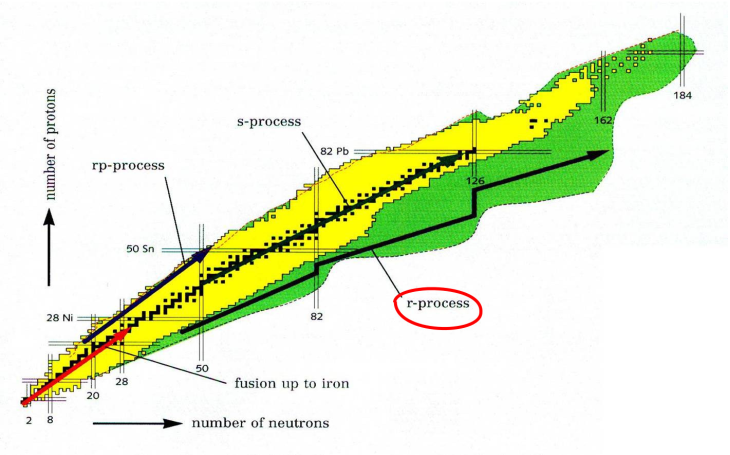
How the distributions of the elements were formed?

$\Downarrow$

How the elements (nuclei) were produced in the Universe?

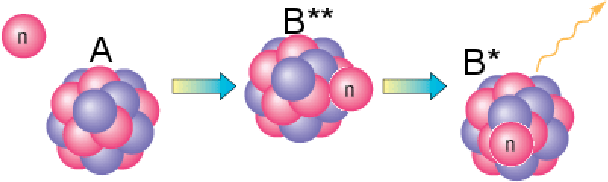
From the Nucleosynthesis to the Nuclear Physics



Need models of the neutron-rich nuclei and (n, γ) reactions!

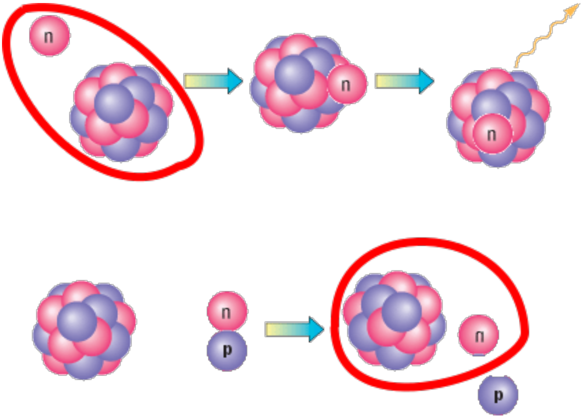
* http://www.phys.utk.edu/expnuclear/nucastro.html

Separation of time scales in (n, γ) reaction



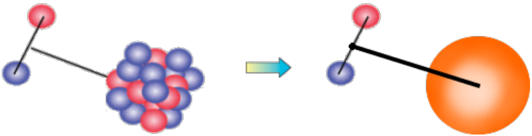
- Neutron capture is driven by the strong interaction ('fast' $\sim 10^{-21}$ sec.).
- Photon emission is the electromagnetic process ('slow' $\gtrsim 10^{-12}$ sec.).

Deuteron as 'Trojan horse'



The way to study the neutron capture process via (d, p) reactions.

(d, p) reactions as three-body problem



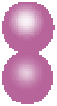
- First approximation: internal degrees of freedom of the nucleus are not resolved.
- Later, take into account some core nucleus' degrees of freedom:
  - collective core excitations,
  - nucleon-level core's internal dynamics.

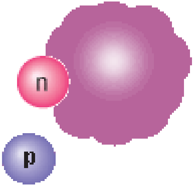
DWBA for $A(d,p)B$ reaction

Simple ansatz: use only two-body scattering.

(Pre)

$\langle dA| s^{\dagger}_{dA} = \langle \Psi_{(pn)A}|$





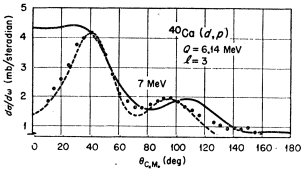
$U_{pA} + V_{np} - U_{pB}$

(Post)

$E_{p(lab)} = E_{beam(lab)}/2 - B_d$

$|\Psi_{p(nA)}\rangle = s_{pB}|pB\rangle$

DWBA results for ^{40}Ca(d, p) reaction at $E_d = 7$ MeV

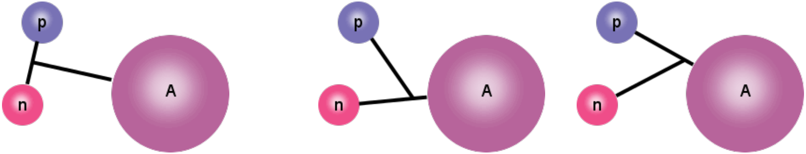


- Only peripheral direct reactions.
- $E_{beam} > B_d$.
- No 3-body dynamics of npA system.

* Lee *et. al.* // Phys. Rev. **136B**, 971 (1964).

Apn system

Channels (configurations):



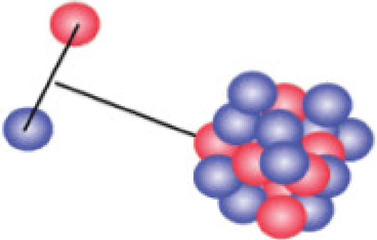
$$A(pn) = d + A \qquad p(nA) \qquad n(Ap)$$

Faddeev formalism treats all channels on the same footing.

CDCC method (shortcut)

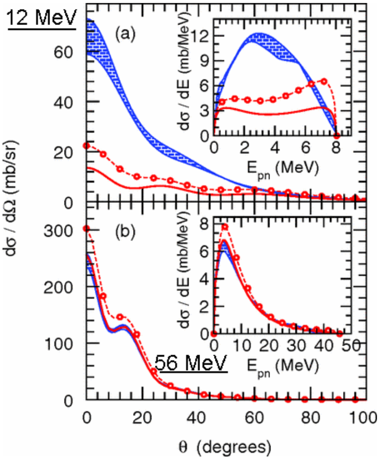
Continuum-Discretised Coupled Channel method

* Austern *et al.* // Physics Reports **154**, No. 3, 125–204 (1987).



- Pick one channel and expand it in some basis.
- Other channels are expressed in this basis.
- Convergence in other three-body channels.

CDCC results for breakup ^{12}C$(d, pn)^{12}$C vs. FAGS



- CDCC fails for low energy.

* Upadhyay *et. al.* // Phys. Rev. C **85**, 054621 (2012).

Coulomb interaction in momentum space

- Due to the boundary conditions, momentum space is preferrable for 3-body problem.
- Coulomb interaction has infinite range and have to be treated with great care.

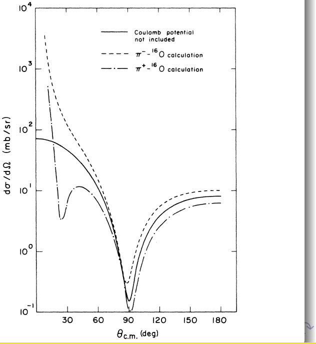
Early work

* Vincent and Phatak. // Phys. Rev. C **10**, 391 (1974).

- $\pi^{\pm} + {}^{16}\mathrm{O}$ at $E_{\pi(lab)} = 30$ MeV,
- got the momentum-space results by using the two-potential formula (textbook):

$$f(\theta) = f_C(\theta) + f_n^C(\theta),$$

- plus boundary matching.



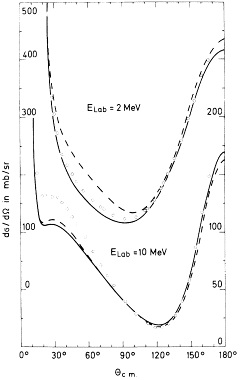
dp scattering with screened Coulomb interaction

The screening factor:

$$V_C = \frac{Z_1 Z_2 \alpha^2}{r} \quad \to \quad \widetilde{V}_C = \frac{Z_1 Z_2 \alpha^2}{r} \exp(-\mu r).$$

Alt *et al.* // Phys. Rev. Lett. **37**, 1537 (1976)

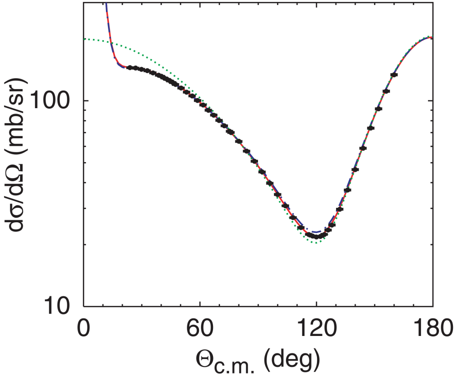
- Screening technique works for the *pd* scattering.
- Various strong amplitude corrections required.



dp scattering with screened Coulomb interaction (continued)

Deltuva // Phys. Rev. C **80**, 064002 (2009)

- pd at $E_{p(lab)} = 9$ MeV
- The most advanced screening techniques to the date.
- AV18 and AN18+UIX potentials.



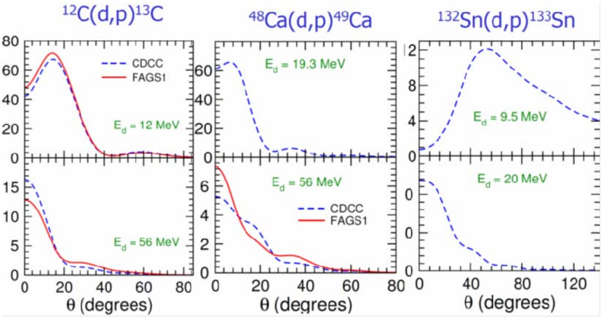
$A(d,p)B$ reaction with screened Coulomb interaction

By using more complicated techniques, screening procedure extended to the nuclei up to $^{48}_{20}$Ca (Deltuva).

* Deltuva. // Phys. Rev. C **80**, 064002 (2009).

* Upadhyay, Deltuva, Nunes. // Phys. Rev. C **85**, 054621 (2012).

Hitting the 'charge wall'



Courtesy: F.M. Nunes

Towards highly-charged nuclei

Task

Treat all (d, p) reactions on the same footage for all the nuclei from $^1_1$H up to $^{208}_{82}$Pb.

Towards highly-charged nuclei

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Suggestion

Switch to the basis of Coulomb functions, i.e. include Coulomb potential in the Green's function.

Problems to be solved

- Calculation of Coulomb function in momentum space.
- Calculation of the two-body t-matrices and other quantities in Coulomb basis in momentum space.

Coulomb function in momentum space

- 3D: Guth and Mullin. // Phys. Rev. **83**, 667 (1951).
- In partial waves: Dolinskii and Mukhamedzhanov. // Sov. Journ. of Nucl. Phys., vol. 3, No. 2, p. 180 (1966).

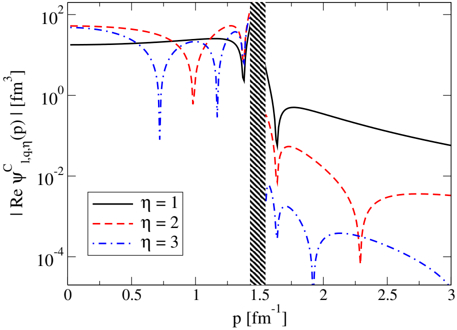
$$\psi^C_{l,q,\eta}(p) \propto \lim_{\gamma \to +0} \frac{d}{d\gamma} \left[\cdots Q_l^{i\eta}(\zeta) \right],$$

$$\eta = Z_1 Z_2 \alpha^2 \mu / q; \qquad \zeta = (q^2 + p^2 + \gamma^2)/2qp.$$

Coulomb function in momentum space (continued)

$$\psi^C_{l,q,\eta}(p):$$

- two representations, depending on the value of ζ,
- special functions of complex arguments, including hypergeometric function ${}_2F_1$,
- algorithms to choose the correct representation,
- numerical issues...

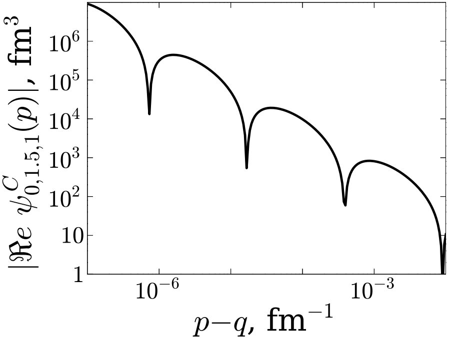


* Eremenko *et al.* // Comp. Phys. Comm. **187**, 195 (2015).

Singularity at the on-shell point

$$\psi^C_{l,q,\eta}(p)\Big|_{p\sim q} \propto \frac{1}{(p-q\pm i0)^{1\pm i\eta}}.$$

Infinetely rapid oscillatory singularity.

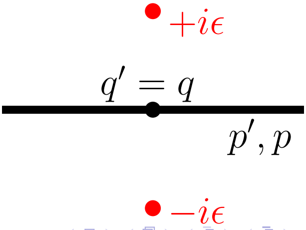


* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

T-matrix elements in Coulomb basis

$$t_l^C(q', q) = \int dp'\, dp \times \psi_{l,q',\eta'}^{C(-)}(p')^*\, t_l(p', p)\, \psi_{l,q,\eta}^{C(+)}(p).$$

Pinch singularity in the elastic channel:



Separable representation of the optical potentials

$$t_l(p', p) = \sum_{ky} |\chi_{l,k}(p')\rangle \lambda_{l,ky} \langle \chi_{l,y}(p)|.$$

* Hlophe *et al.* // Phys. Rev. C **88**, 064608 (2013).

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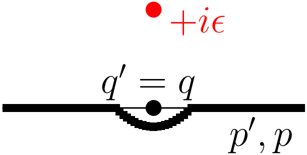
* Hlophe *et al.* // Phys. Rev. C **88**, 064608 (2013).

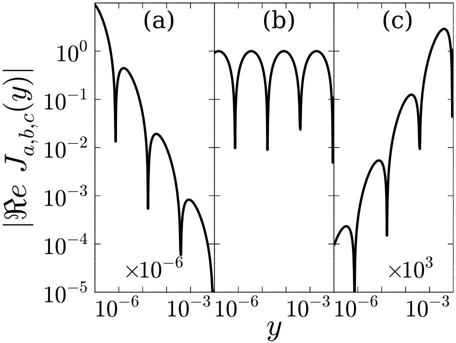
- Two independent integrals over p and p'.
- Cauchy's theorem.
- No pinch singularity!



Gel'fand-Shilov regularization for complex form-factors

$$u_l^C(q) \propto \int_{a<0}^{b>0} \frac{f(y)\,dy}{y^{1+i\eta}} \equiv \int_a^b dy\; J_a(y), \qquad (\text{e.g. } f(y) = y^2 + y + 1).$$

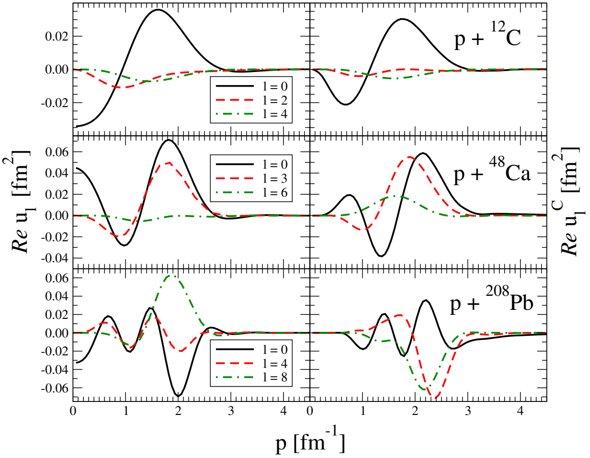
Subtract as many terms of Laurent expansion of $f(y)$ around the integrand's special point $y = 0$ as needed to split the integral and get the regular term, plus the analytically calculated terms.



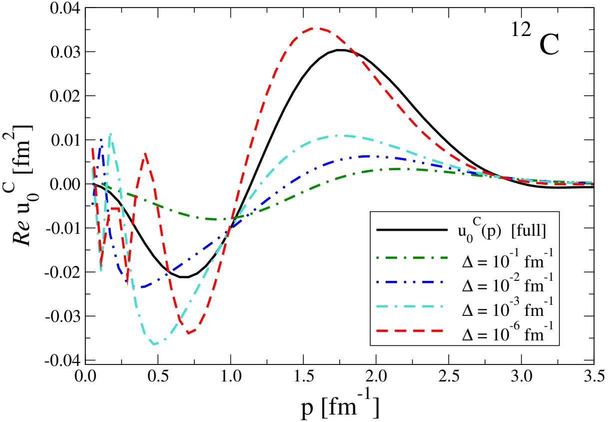
* Upadhyay *et al.* // Phys. Rev. C **90**, 014615 (2014).

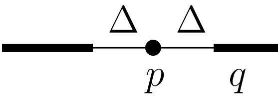
Results: form-factors in Coulomb basis

$$u_l^C(p) = \frac{1}{2\pi^2}\int dq\, q^2 u_l(q)\psi^C_{l,p,\eta}(q).$$

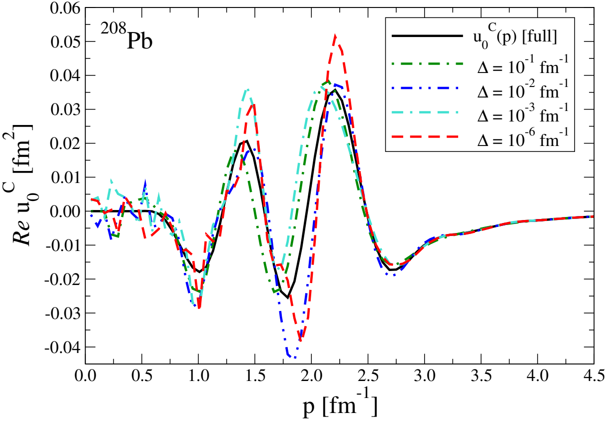


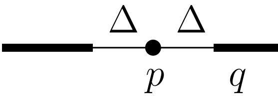
Singularity contribution for $p + {}^{12}C$





Singularity contribution for $p + {}^{208}\text{Pb}$





Faddeev-AGS equations in Coulomb basis (in progress)

Mukhamedzhanov *et al.* // Phys. Rev. C **86**, 034001 (2012)

Faddeev-AGS equations in Coulomb basis for real two-body t-matrices and spinless particles.

- Take spin degrees of freedom into account.
- Generalize the equations for the complex potentials.
- Develop the codes to solve the equations.

Summary & Outlook

- Faddeev formalism is the theoretical tool to study (d, p) reactions.
- This formalism treats all possible three-body channels on the same footage.
- Momentum space is preferrable due to the boundary conditions.
- Coulomb interaction can be treated properly by using the Coulomb basis in momentum space.
- Pinch singularity is avoided by choosing the two-body interactions in separable form.
- Mathematics and machinery are developed to compute Coulomb functions and matrix elements in Coulomb basis in momentum space.
- Work is in progress to cast Faddeev-AGS equations in Coulomb basis taking into account spin degrees of freedom and complex two-body t-matrix form-factors.