About Faddeev-AGS equations for (d, p) reactions on heavy nuclei

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Focus on deuteron-induced reactions

Deuteron-induced reactions allow to study the structure of niclei

Experiments with exotic nuclear beams with deuterated targets in inverse kinematics.

Low energy: E_{lab} from few to 50 MeV/A

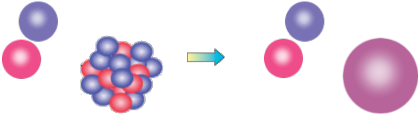
Broad range of nuclei: from C to Pb isotopes

Main focus is on (d, p) reactions

Emphasis: All three-body reaction channels (e.g. deuteron breakup) should be treated on the same footing.

Deuteron-induced reactions as three-body problem

- Treat the core nucleus as one large particle
  $\Rightarrow$ 3-body problem
- Use Faddeev-AGS approach in momentum space



Approximations:

- internal degrees of freedom of the nucleus are neglected
- genuine 3-nucleon forces (3NF) are not taken into account

Faddeev approach to the three-body problem

Free state $|\phi\rangle = |\phi_1\rangle$ is initial state $A + (pn)$

Coulomb Green's function

$$g_0^C(E) = \left(E - H_0 - V_{pA}^C + i0\right)^{-1} \qquad |\Psi\rangle = |\phi\rangle + \sum_{a=1}^{3} g_0^C(E) V_a^S |\Psi\rangle$$

Faddeev components ψ_a

$$|\Psi\rangle \equiv \sum_a |\psi_a\rangle = \sum_a \underbrace{\delta_{a,1}|\phi_1\rangle + g_0^C(E) V_a^S |\Psi\rangle}_{|\psi_a\rangle}$$

Faddeev-type equations

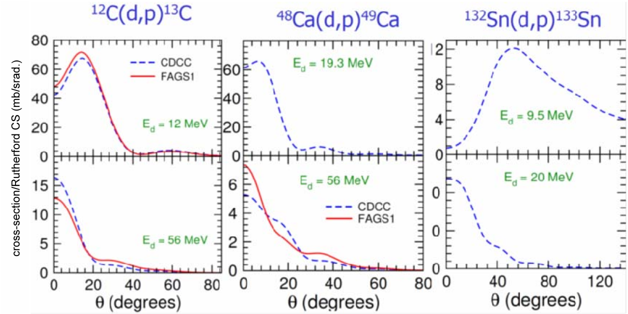
$$\begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) V_1^S \sum_a |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) V_2^S \sum_a |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) V_3^S \sum_a |\psi_a\rangle \end{cases} \Rightarrow \begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) t_1 \sum_{a\neq 1} |\psi_a\rangle \\ |\psi_2\rangle = \qquad g_0^C(E) t_2 \sum_{a\neq 2} |\psi_a\rangle \\ |\psi_3\rangle = \qquad g_0^C(E) t_3 \sum_{a\neq 3} |\psi_a\rangle \end{cases}$$

$A(d,p)B$ reactions with screened Coulomb interaction

$$\widetilde{V}^C = \frac{Z_1 Z_2 \alpha^2}{r} \exp(-\mu r^n) \qquad H_0 \equiv \mathbf{q}_a/2M_a + \mathbf{p}_a/2\mu_a$$

$$g_0 = \frac{|\mathbf{k}\rangle\langle\mathbf{k}|}{E - H_0 + i0} \qquad g_0 \equiv [E - H_0 + i0]^{-1}$$

‘Sommerfeld wall’



* Results obtained in collaboration with N. Upadhyay and A. Deltuva.

Faddeev-AGS method hits a ‘Sommerfeld wall’: implemented algorithms (Deltuva) do not converge for large $\eta = Z_A e^2 \mu_p / k$.

$A(d,p)B$ reactions using Coulomb Green's function

$$\widetilde{V}^C = \frac{Z_1 Z_2 \alpha^2}{r} \exp(-\mu r^n) \qquad H_0 \equiv \mathbf{q}_a/2M_a + \mathbf{p}_a/2\mu_a$$

$$g_0 = \frac{|\mathbf{k}\rangle\langle\mathbf{k}|}{E - H_0 + i0} \qquad g_0 \equiv [E - H_0 + i0]^{-1}$$

* Deltuva *et al.* Phys. Rev. C **71** (2005), 054005.

Coulomb-modified Green's function

$$V^C = \frac{Z_1 Z_2 \alpha^2}{r} \qquad g_0^C \equiv \left[E - H_0 + i0 - V_{pA}^C\right]^{-1}$$

$$g_0^C = \frac{|\psi^C_{\mathbf{k},\eta}\rangle\langle\psi^C_{\mathbf{k},\eta}|}{E - H_0 + i0}$$

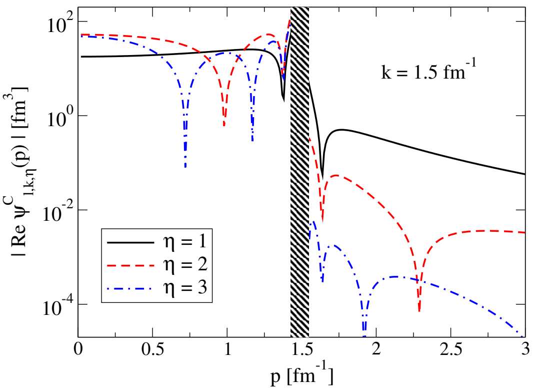
* É.I.Dolinskiĭ and A.M.Mukhamedzhanov. Sov. J. of Nucl. Phys. **3** (1966), 180.
* C.R.Chinn *et al.* Phys. Rev. C **44** (1991), 1569.

Needed: Coulomb function in momentum space

- 'Regular' representation for $p \neq k$.
- 'Pole-proximity' representation for $p \approx k$:

$$\psi^C_{l,k,\eta}(p) \sim \frac{1}{(p-k+i0)^{1+i\eta}}.$$

$\psi^C_{l=0,k,\eta}(p)$



* Eremenko *et al.* Comp. Phys. Comm. **187**, 195 (2015).

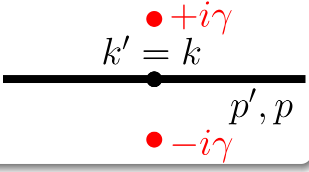
Two-body t-matrix in Coulomb momentum basis

The kernel of the Faddeev equations contains the two-body t-matrix in the Coulomb basis:

$$t^{C}_{a,l}(k',k,E) = \int dp'\,dp\,\psi^{C}_{l,k',\eta'}(p')^{\dagger}\,t_{a,l}(p',p,E)\,\psi^{C}_{l,k,\eta}(p)$$

Pinch singularity in the elastic channel:

- $E = 2\mu k^2 = 2\mu {k'}^2$
- Since $\gamma \to +0$, singularities of $\psi^{C\dagger}_{l,k',\eta'}$ and $\psi^{C}_{l,k,\eta}$ are pinching the integration contour



If t-matrix has separable representation

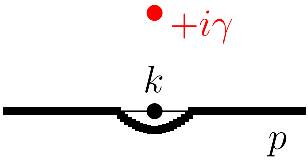
$$t_l^C(k,k',E) = \sum_{zy} u_{l,z}^C(k)\,\lambda_{l,zy}(E)\,u_{l,y}^C(k')^\dagger$$

$$u_{l,z}^C(k) = \int \frac{dp\,p^2}{2\pi^2} u_{l,z}(p) \psi_{l,k,\eta}^C(p)^*$$

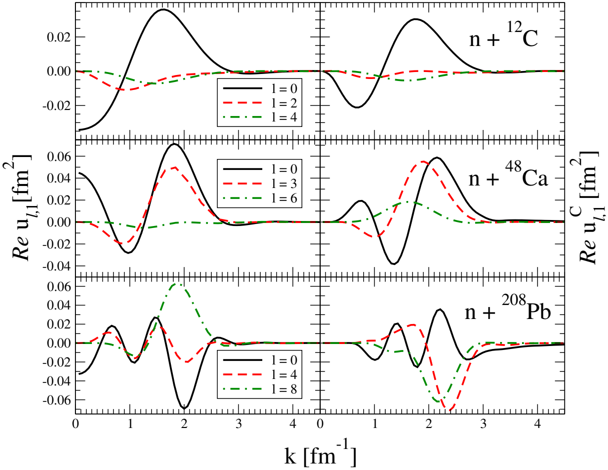
* Linda Hlophe's poster at the Poster Session.

No pinch singularity!

- Two independent integrals over p and p'
- Cauchy's theorem
- Numerical calculations uses Gel'fand-Shilov regularization



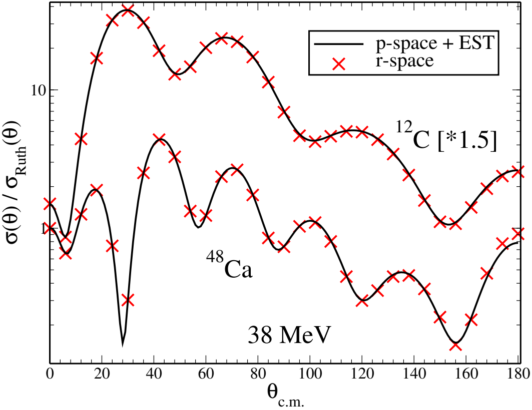
Results: form-factors in Coulomb basis $(n + A)$



* Upadhyay *et al.*
Phys. Rev. C **90**, 014615 (2014).

$$u^C_{l,z}(k) = \int \frac{dp\, p^2}{2\pi^2} u_{l,z}(p) \psi^C_{l,k,\eta}(p)^*.$$

Results: $p + A$ scattering in Coulomb momentum basis



- Special treatment required! Presented on L. Hlophe poster.
- Hlophe *et al.* Phys. Rev. C **90**, 061602(R) (2014).

Summary & Outlook

- Faddeev formalism is the theoretical tool to study deuteron-induced reactions.
  - Treats all possible three-body channels on the same footage.
  - Momentum space is preferable due to the boundary conditions.
- Coulomb interaction can be treated exactly by using the Coulomb basis in momentum space.
- Pinch singularity is avoided by choosing the two-body interactions in separable form.
- Mathematics and machinery are developed to compute Coulomb functions and matrix elements in Coulomb basis in momentum space, e.g. form-factors of np, nA, and pA two-body interactions.
- Implementing the Faddeev-AGS equations:
  - Coulomb basis,
  - spin degrees of freedom,
  - 3-body singularity.