Comparing CDCC, Faddeev and Adiabatic Model

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- 3-body Hamiltonian:

$$H_{3b} = \hat{T}_R + \hat{T}_r + U_{\mathrm{pA}} + U_{\mathrm{nA}} + V_{\mathrm{pn}}$$

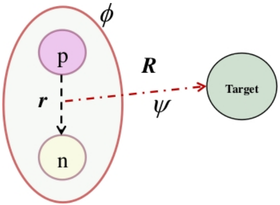
- Obtain 3-body wave function by solving Schrödinger Equation:

$$(H_{\mathrm{3b}} - E)\,\Psi^{\mathrm{3b}}(\mathbf{r},\mathbf{R}) = 0$$

- Use Ψ^{3b} in exact T-matrix

$$T = \langle\, \chi^{(-)}_{\mathrm{pB}}\, \phi^{(-)}_{\mathrm{nA}} \,|\, V_{\mathrm{pn}} + U_{\mathrm{pA}} - U_{\mathrm{pB}} \,|\, \Psi^{\mathrm{3b}} \,\rangle$$

where, U_{pB} is auxillary potential.



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1. **F**inite **R**ange **Ad**iabatic **W**ave **A**pproximation

Ref.: **Johnson and Tandy, Nucl. Phys. A235, 56 (1974).**

- The 3-body wave function is expanded in terms of deuteron Weinberg states, $S_i(\mathbf{r})$.

$$\Psi^{+}(\mathbf{r},\mathbf{R}) = \sum_{i=1}^{\infty} S_i(\mathbf{r})\,\chi_i(\mathbf{R})$$

where, $$(T_r + \alpha_i V_{\rm pn})\,S_i(\mathbf{r}) = -\,\epsilon_d\,S_i(\mathbf{r})$$

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$$\Psi^+_{\rm AD}(\mathbf{r},\mathbf{R}) = S_0(\mathbf{r})\,\chi_0^{\rm AD}(\mathbf{R})$$

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$$\Psi^+_{\rm AD}(\mathbf{r},\mathbf{R}) = S_0(\mathbf{r})\,\chi_0^{\rm AD}(\mathbf{R})$$

Coupled-channel equation simplifies to optical model type equation with distorting potential

$$U_{\rm AD}(R) = -\,\langle\, S_0(\mathbf{r})|V_{\rm pn}\,(U_{\rm nA} + U_{\rm pA})|S_0(\mathbf{r})\,\rangle$$

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2. T-matrix **C**ontinuum **D**iscretized **C**oupled **C**hannels Method

Ref.: **N. Austern** ***et al.*****, Phys. Rep. 154, 125 (1987).**

- The 3-body wave function is expanded in terms of deuteron bound and continuum states.

$$\Psi^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) = \sum_{\alpha} \phi_\alpha(\mathbf{r})\,\psi_\alpha(\mathbf{R})$$

$\phi_\alpha(\mathbf{r})$: eigenstates of deuteron

$$\phi_\alpha(\mathbf{r}) = i^l \frac{u_{\alpha l}(r)}{r} Y_l(\hat{\mathbf{r}})$$

$\psi_\alpha(\mathbf{R})$: relative wave function between deuteron and target

$$\psi_\alpha(\mathbf{R}) = i^L \chi_\alpha(R)\, Y_{L_\alpha}(\hat{\mathbf{R}})$$

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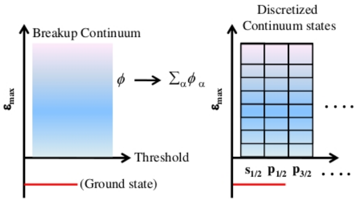
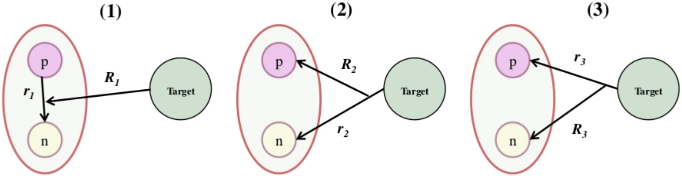
- Discretize the continuum
- Solve CDCC equation



Ref.: **Deltuva and Fonseca, Phys. Rev. C79, 014606 (2009).**

Exact Method



- Explicitly includes elastic, breakup & transfer channels to all orders.
- 3-particle scattering is described in terms of transition operators,

$$T_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma=1}^{3} \bar{\delta}_{\beta\gamma} t_\gamma G_0 T_{\gamma\alpha}$$

- Coulomb interaction is treated using screening & renormalization techniques.

3-body Hamiltonian

- For pertinent comparison, we construct a simple 3-body Hamiltonian

$$H_{3b} = \hat{T}_R + \hat{T}_r + U_{\mathrm{pA}} + U_{\mathrm{nA}} + V_{\mathrm{pn}}$$

$\hat{T}_R$, $\hat{T}_r$: kinetic energy operators

V_{pn}: Deuteron binding potential $\rightarrow$ Gaussian Potential

U_{pA}: proton-target optical potential Chapel-Hill Global Parametrization

U_{nA}: neutron-target optical potential (spin-orbit neglected)

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- Spins are neglected.

- Binding Potentials for neutron-target in final state ($r_0 = 1.25$ fm & $a_0 = 0.65$ fm)

Nucleus	nl	S_n (MeV)	V_{nA} (MeV)
^{10}Be	$2s$	0.504	57.064
^{12}C	$1p$	4.947	39.547
^{48}Ca	$2p$	5.146	48.905

No direct comparison to data, as we neglect spin.

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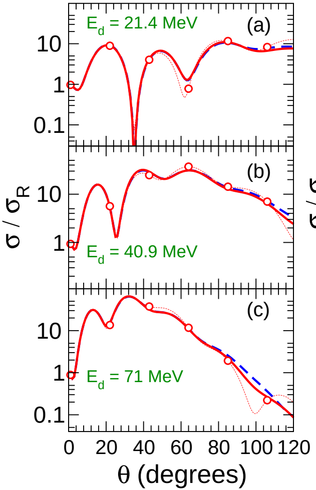
Elastic Cross sections

Various Calculations

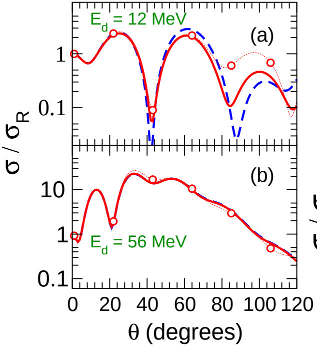
- CDCC: U_{pA} and U_{nA} @ $\mathrm{E_d}/2$.
- Faddeev-AGS (FAGS): U_{pA} and U_{nA} @ $\mathrm{E_d}/2$, producing no nA bound state.
- Faddeev-AGS (FAGS1): FAGS + transfer channel to produce nA bound state.

PRC **85**, 054621 (2012)

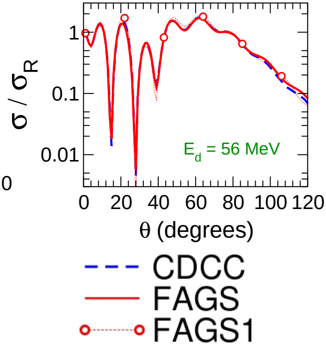
^{10}Be(d,d)^{10}Be



^{12}C(d,d)^{12}C



^{48}Ca(d,d)^{48}Ca



Breakup Cross sections

1. Computationally most demanding calculations
2. For Faddeev-AGS calculations, sufficiently accurate results at forward angles were not obtained with inclusion of Coulomb interaction.
3. Coulomb interaction switched off for both the methods.

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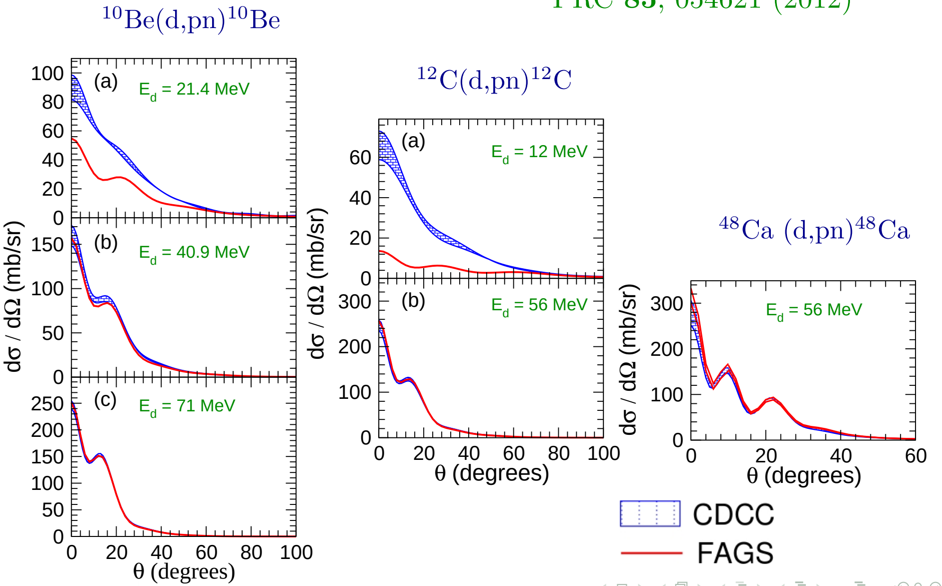
PRC **85**, 054621 (2012)



Breakup cross sections: Energy Distribution

^{10}Be(d,pn)^{10}Be

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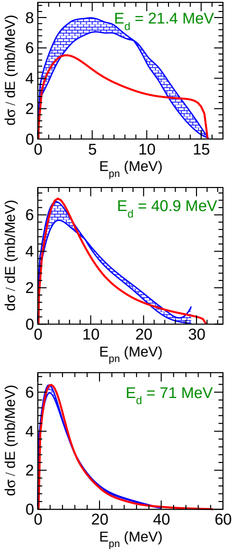

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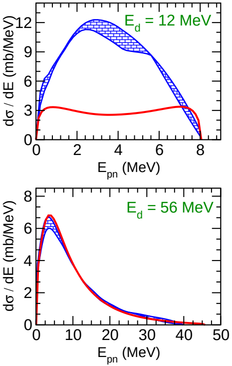

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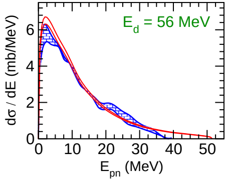

Transfer Cross sections

Various Calculations

- CDCC: U_{pA}, U_{nA} @ $\mathrm{E_d}/2$ in entrance channel, while U_{pB} @ $\mathrm{E_p}$ in exit channel.
- Faddeev-AGS (FAGS1): U_{pA}, U_{pB} @ $\mathrm{E_d}/2$ and U_{nA} @ $\mathrm{E_d}/2$ for all partial waves except for one corresponding to bound state.
- Adiabatic Wave Approximation (ADWA): Same Hamiltonian as in CDCC calculations.

Transfer cross sections: CDCC-Faddeev-ADWA

$^{10}Be(d,p)^{11}Be(g.s.)$

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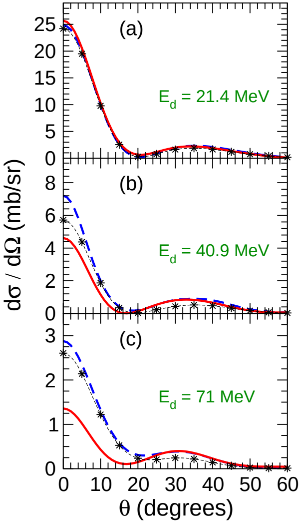
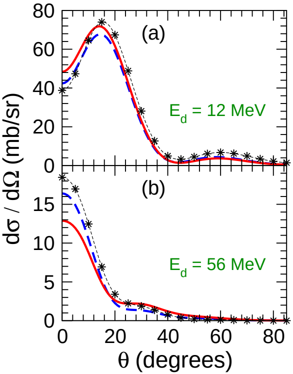
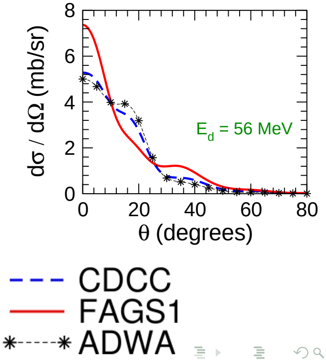


$^{12}C(d,p)^{13}C(g.s.)$



$^{48}Ca(d,p)^{49}Ca(g.s.)$



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- Faddeev-AGS (FAGS2): U_{pA}, U_{pB} @ $\mathrm{E_p}$ and U_{nA} @ $\mathrm{E_d}/2$ for all partial waves except for one corresponding to bound state.

Transfer cross sections

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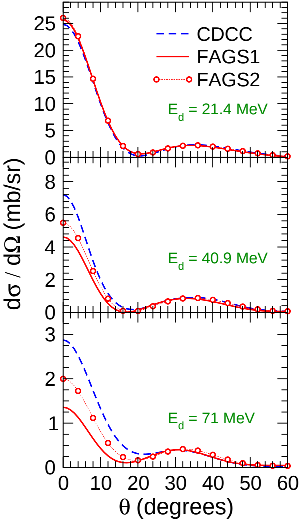

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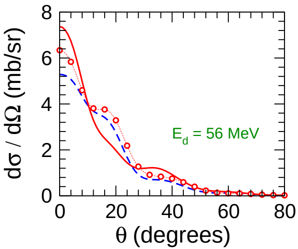

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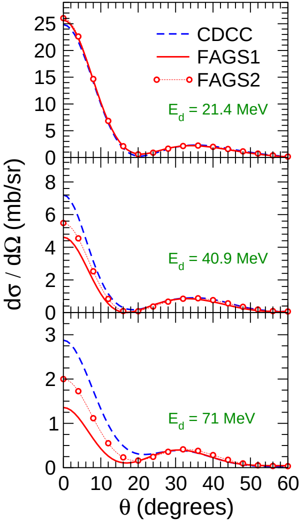

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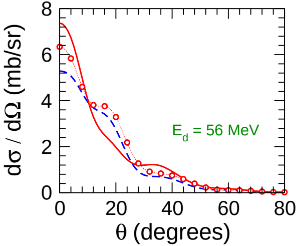
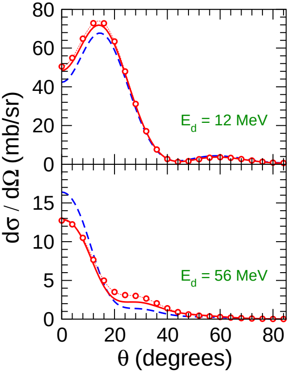


^{12}C(d,p)^{13}C(g.s.)



Conclusions

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3. Breakup Process
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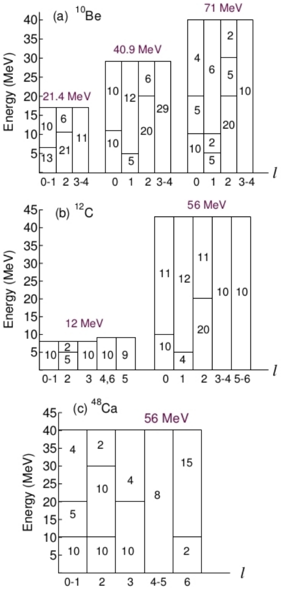
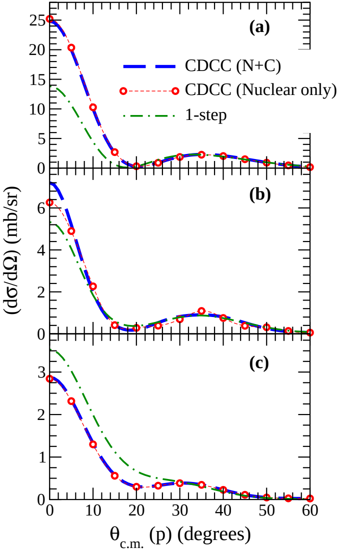
4. Transfer Process
   - ADWA is a good approximation to CDCC/FAGS1 at low energy ∼10 MeV/A.
   - Sensitivity of cross sections to the choice of the energy at which the proton interaction is calculated in the Faddeev method makes the comparison of methods ambiguous for ^{10}Be and ^{48}Ca but robust for ^{12}C.

Backup Slides

CDCC Model Space



^{10}Be (d, p) ^{11}Be(g.s.)



Transfer cross section for deuterons on ^{10}Be at: (a.) $E_d = 21.4$ MeV, (b.) $E_d = 40.9$ MeV and (c.) $E_d = 71$ MeV.

Results indicate:

- Small Coulomb effects at very forward angles.
- Continuum has strong influence on Transfer process.

$$T_{\beta\alpha} = \bar{\delta}_{\beta\alpha}\, G_0^{-1} + \sum_{\gamma=1}^{3} \bar{\delta}_{\beta\gamma}\, t_\gamma\, G_0\, T_{\gamma\alpha}$$

where,
$\bar{\delta}_{\beta\alpha} = (1 - \delta_{\beta\alpha})$ &
$G_0 = (E + i0 - H_0)^{-1}$ is the free resolvent with E being the total energy in 3-body c.m. system.

t_γ is 2-body transition operator for each interacting pair and is derived from the pair potential v_γ via the Lippmann-Schwinger equation

$$t_\gamma = v_\gamma + v_\gamma\, G_0\, t_\gamma.$$

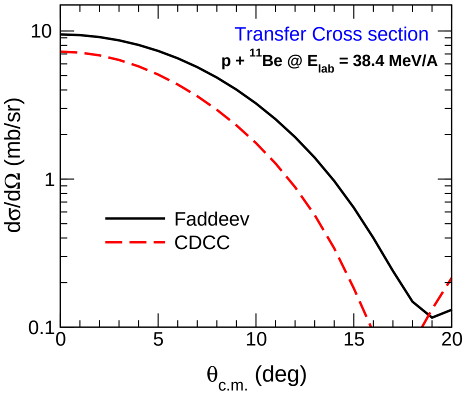
Scattering amplitude: $X_{\beta\alpha} = \langle \phi_\beta | T_{\beta\alpha} | \phi_\alpha \rangle$

CDCC vs Faddeev: First Attempt

Ref.: **A. Deltuva** *et al.*, **Phys. Rev. C76, 064602 (2007)**

- Good agreement for Elastic and Breakup observable for
  1. d + ^{12}C @ E_d = 56 MeV
  2. d + ^{58}Ni @ E_d = 80 MeV
- Disagreement for Breakup and Transfer observable for

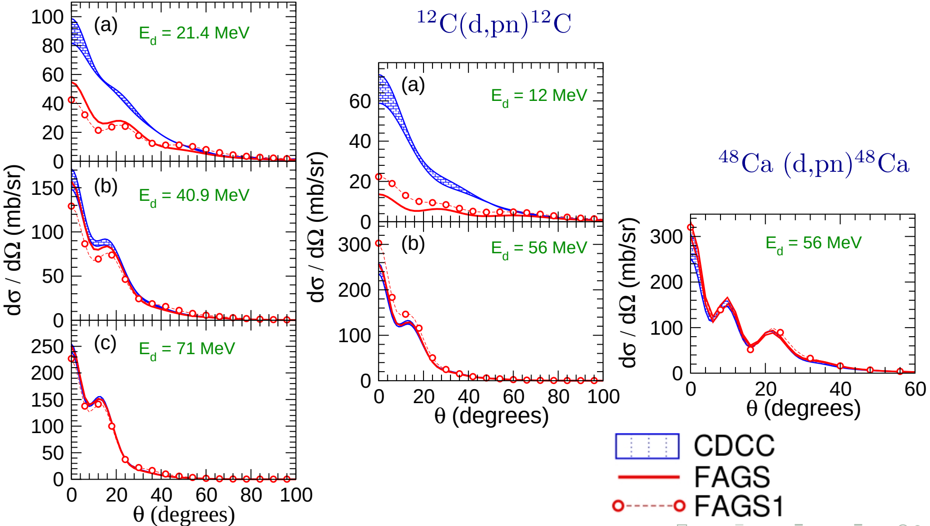
  p + ^{11}Be @ E_{Be} = 38.4 MeV/A



What is the range of validity of CDCC method?

Coulomb interaction switched off

^{10}Be(d,pn)^{10}Be



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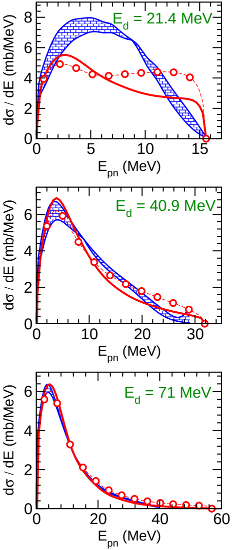
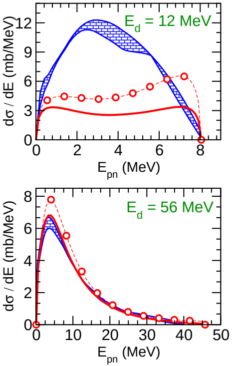
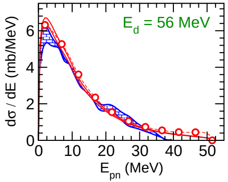


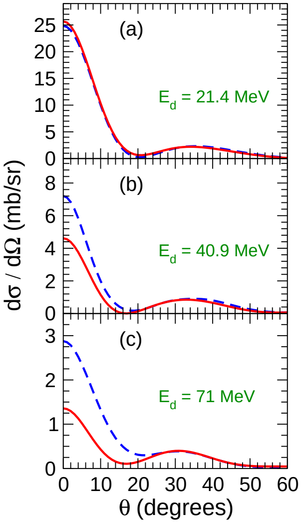
^{12}C(d,pn)^{12}C



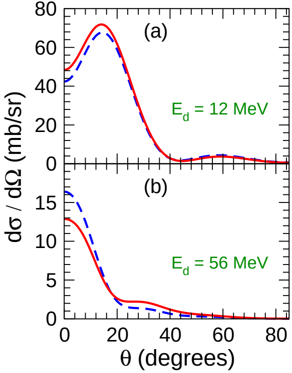
^{48}Ca (d,pn)^{48}Ca



$^{10}Be(d,p)^{11}Be(g.s.)$



$^{12}C(d,p)^{13}C(g.s.)$



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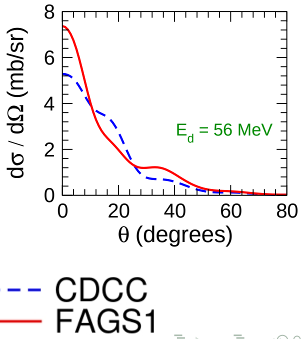

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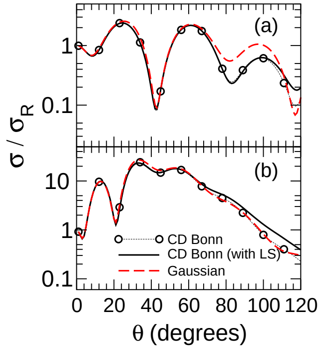
Reaction	Energy (MeV)	nl	θ (deg.)	$\Delta_{FAGS1-CDCC}$ [2] (%)	$\Delta_{FAGS1-ADWA}$ [1] (%)
^{10}Be(d,p)	21.4	$2s$	0	3	6
	40.9	$2s$	0	-36	-19
	71	$2s$	0	-53	-48
^{12}C(d,p)	12	$1p$	14	6	-2
	56	$1p$	0	-21	-30
^{48}Ca(d,p)	56	$2p$	0	39	47

[1] Phys. Rev. C**84**, 034607 (2011)

[2] Phys. Rev. C**85**, 054621 (2012)

Sensitivity to NN interaction

$^{12}C(d,d)^{12}C$



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