Testing Formalisms for Deuteron Breakup and Transfer Reactions

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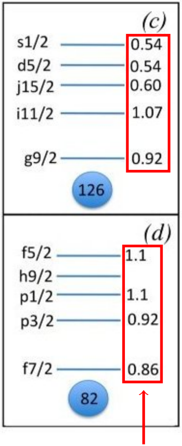
Single-particle states



Spectroscopic factors

^{208}Pb(d,p)^{209}Pb

^{132}Sn(d,p)^{133}Sn

- ⋆ **Study nature of single-particle state**
- ⋆ **Tool: (d,p) reactions**
- ⋆ **Important to have reaction theory providing accurate description.**

K. Jones *et al.*, Nature **465** (2010), 454.

- Deuteron → loosely bound system
- Theories including deuteron breakup:
  1. T-matrix **C**ontinuum **D**iscretized **C**oupled **C**hannels Method

     Includes breakup to all orders in complete basis of projectile bound and continuum states, but replaces exact 3-body wave function by CDCC wave function in the transfer amplitude.
  2. Alt, Grassberger, Sandhas Formalism (Faddeev-AGS)

     Explicitly includes breakup and transfer channels to all orders.
- Our aim: To quantify accuracy of CDCC.
- We study three test cases as a function of beam energy.
  1. ^{10}Be(d,p)^{11}Be(g.s.) @ E_d = 21.4, 40.9 & 71 MeV
  2. ^{12}C(d,p)^{13}C(g.s.) @ E_d = 12 & 56 MeV
  3. ^{48}Ca(d,p)^{49}Ca(g.s.) @ E_d = 56 MeV

3-body Hamiltonian

- For pertinent comparison, we construct a simple 3-body Hamiltonian

$$H_{3b} = \hat{T}_R + \hat{T}_r + U_{\rm pA} + U_{\rm nA} + V_{\rm pn}$$

$\hat{T}_R$, $\hat{T}_r$: kinetic energy operators

$V_{\rm pn}$: Deuteron binding potential → Gaussian Potential

$U_{\rm pA}$: proton-target optical potential Chapel-Hill Global Parametrization

$U_{\rm nA}$: neutron-target optical potential (spin-orbit neglected)

- The interactions between all pairs are spin independent.

- Binding Potentials for neutron-target in final state ($r_0 = 1.25$ fm & $a_0 = 0.65$ fm)

Nucleus	nl	S_n (MeV)	$V_{\rm nA}$ (MeV)
^{10}Be	$2s$	0.504	57.064
^{12}C	$1p$	4.947	39.547
^{48}Ca	$2p$	5.146	48.905

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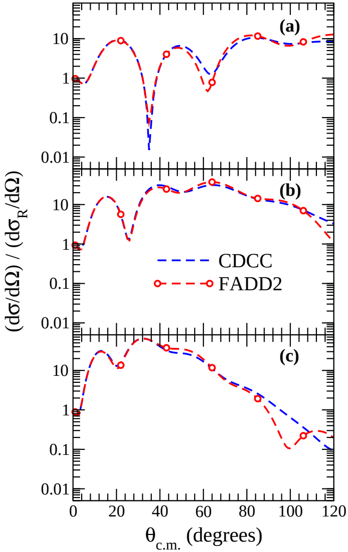
Deuterons on ^{10}Be

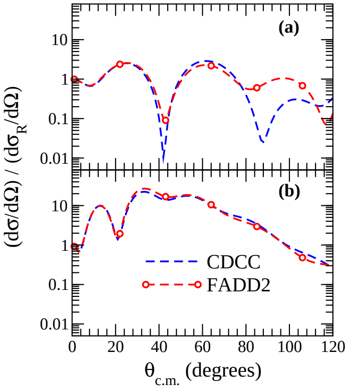


Elastic cross section for deuterons on ^{10}Be at: (a.) $E_d = 21.4$ MeV, (b.) $E_d = 40.9$ MeV and (c.) $E_d = 71$ MeV.

- In CDCC and Faddeev calculations (FADD2), U_{pA} & U_{nA} are calculated at half the deuteron energy (E_d).
- Small disagreement between CDCC & Faddeev at angles $> 80°$.

Elastic cross sections

Deuterons on ^{12}C

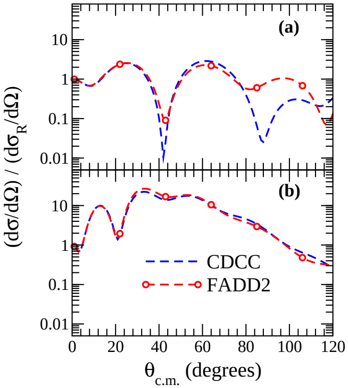


Elastic cross section for deuterons on ^{12}C at: (a.) $E_d = 12$ MeV and (b.) $E_d = 56$ MeV.

- At lowest energy, large disagreement between CDCC & Faddeev at angles $> 70°$.
- Increase in beam energy improves agreement.

Elastic cross sections

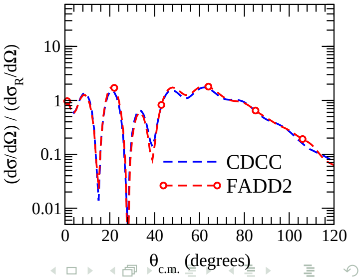
Deuterons on ^{12}C



Elastic cross section for deuterons on ^{12}C at: (a.) $E_d = 12$ MeV and (b.) $E_d = 56$ MeV.

- At lowest energy, large disagreement between CDCC & Faddeev at angles $> 70°$.
- Increase in beam energy improves agreement.
- For ^{48}Ca at $E_d = 56$ MeV, better agreement in two methods → behaviour similar to ^{12}C @ 56 MeV!

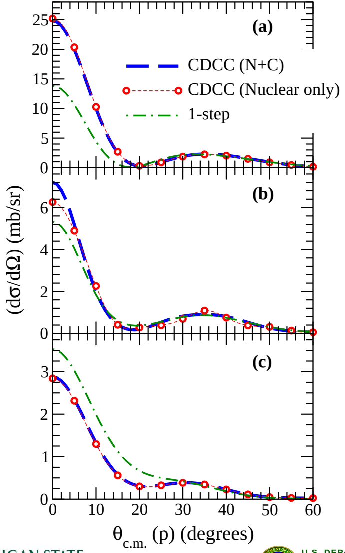
Elastic cross section for deuterons on ^{48}Ca at $E_d = 56$ MeV.



Transfer cross sections: Testing Formalism

Transfer cross sections: Testing Formalism

^{10}Be (d, p) ^{11}Be(g.s.)



Transfer cross section for deuterons on ^{10}Be at: (a.) $E_d = 21.4$ MeV, (b.) $E_d = 40.9$ MeV and (c.) $E_d = 71$ MeV.

Results indicate:

- Small Coulomb effects at very forward angles.
- Continuum has strong influence on Transfer process.

Transfer cross sections: CDCC v/s Faddeev

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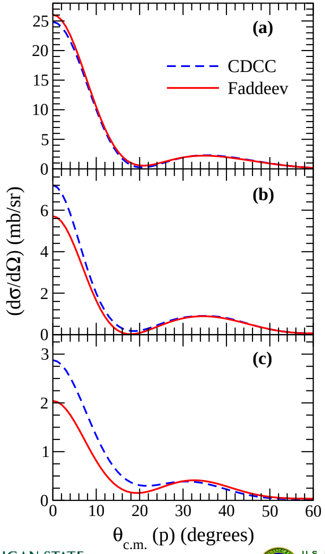
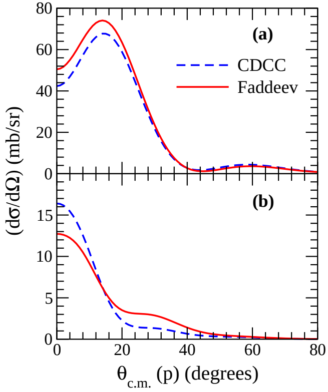
^{10}Be (d, p) ^{11}Be(g.s.)



Transfer cross section for deuterons on ^{10}Be at: (a.) E_d = 21.4 MeV, (b.) E_d = 40.9 MeV and (c.) E_d = 71 MeV.

- In CDCC calculation, U_{pA} & U_{nA} are calculated at half the deuteron energy (E_d).
- In Faddeev calculations, U_{pA} is calculated at proton energy (E_p) in the exit channel. U_{nA} is calculated at $E_d/2$ for all partial waves except for one corresponding to the bound state.
- Disagreement increases with beam energy.

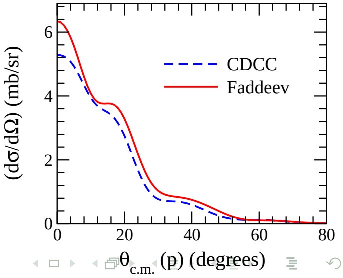
^{12}C (d, p) ^{13}C(g.s.)



Transfer cross section for deuterons on ^{12}C at: (a.) E_d = 12 MeV and (b.) E_d = 56 MeV.

- Disagreement increases with beam energy.

Transfer cross section for ^{48}Ca (d, p) ^{49}Ca(g.s.) at E_d = 56 MeV.



Estimate of Disagreement

Reaction	Energy (MeV)	nl	θ (deg.)	$\Delta_{FADD-CDCC}$ (%)
$^{10}Be(d,p)$	21.4	$2s$	0	5
	40.9	$2s$	0	-31
	71	$2s$	0	-44
$^{12}C(d,p)$	12	$1p$	14	9
	56	$1p$	0	-29
$^{48}Ca(d,p)$	56	$2p$	0	17

Estimate of Disagreement

Reaction	Energy (MeV)	nl	θ (deg.)	$\Delta_{\text{FADD}-\text{CDCC}}$ (%)	$\Delta_{\text{FADD}-\text{ADWA}}$ [1] (%)
	21.4	$2s$	0	5	7
$^{10}Be(d,p)$	40.9	$2s$	0	-31	-4
	71	$2s$	0	-44	-31
$^{12}C(d,p)$	12	$1p$	14	9	1
	56	$1p$	0	-29	-45
$^{48}Ca(d,p)$	56	$2p$	0	17	21

[1] F. M. Nunes & A. Deltuva, PRC**84**, 034607 (2011)

Estimate of Disagreement

Reaction	Energy (MeV)	nl	θ (deg.)	$\Delta_{\mathrm{FADD-CDCC}}$ (%)	$\Delta_{\mathrm{FADD-ADWA}}$ [1] (%)
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	56	$1p$	0	-29	-45
$^{48}Ca(d,p)$	56	$2p$	0	17	21

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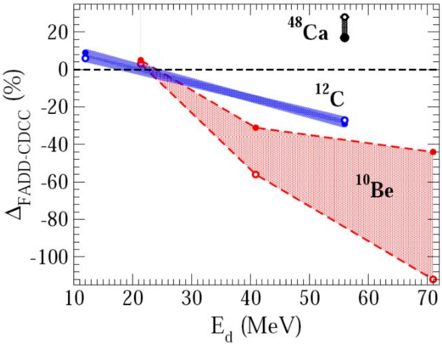

Effect of choice of proton energy at which proton interaction is calculated in Faddeev calculations.

Conclusions & Outlook

1. CDCC and Faddeev calculations are performed for varied test cases spanning large beam energy range.
   $\hookrightarrow$ Good agreement in Elastic cross sections.
   $\hookrightarrow$ Transfer Process: Two methods are in good agreement at low energy. Disagreement increases with the beam energy, however systematic uncertainity also increases.
2. Comparison of two methods for breakup is in progress.
3. Studying Faddeev formalism in momentum space
   - To include Optical potentials in separable form.
   - Solve Faddeev-AGS equations with core excitations.


Thank you very much for your attention
FINISH

- In CDCC:
  - The full 3-body wave function is expanded in terms of a complete basis of the deuteron's bound and continuum states.

$$\Psi = \sum_{\alpha} \phi_\alpha \, \psi_\alpha$$

where, ϕ_α: Deuteron eigen states.
ψ_α: Relative wave function between deuteron and target

  - The transfer matrix element is written as

$$T = \langle \chi^{(-)} \, \phi_{\mathrm{nT}}^{(-)} \, | \, V_{\mathrm{pn}} + U_{\mathrm{pT}} - U^{\mathrm{f}} \, | \, \Psi \rangle$$

- In Faddeev:
  - Set of coupled integral equations in terms of 3-body AGS transition operators are solved.
  - Coulomb is treated using the renormalization and screening techniques.