Reactions with deuterons within the CDCC formalism

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TORUS COLLABORATION

Preface

- **The Continuum Discretized Coupled Channel Method (CDCC)**:
  - ✓ Well established theory for breakup reactions.
  - ✓ Includes breakup to all orders but assumes breakup-transfer couplings are small.

- **Faddeev Approach:**
  - ✓ Explicitly includes breakup and transfer channels to all orders.

- **Is Ψ^{CDCC} good representation of full 3-body wave function to calculate the transfer matrix element?**
- **What are the limitations of the CDCC method?**

- **Aim of this work:** To seek answer to these questions.

CDCC Method

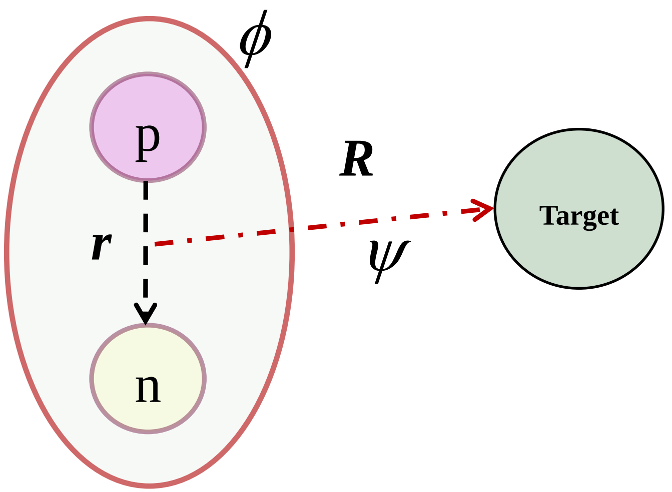
- **Schrödinger Equation**:

$$(\mathrm{H}_{3b} - \mathrm{E})\,\Psi(\mathbf{r},\mathbf{R}) = 0$$

where, $\mathrm{H}_{3b} = T_R + \mathrm{H}_{int}(\mathbf{r}) + U_{nT} + U_{pT}$

$$\mathrm{H}_{int}(\mathbf{r}) = T_r + V_{pn}(\mathbf{r})$$

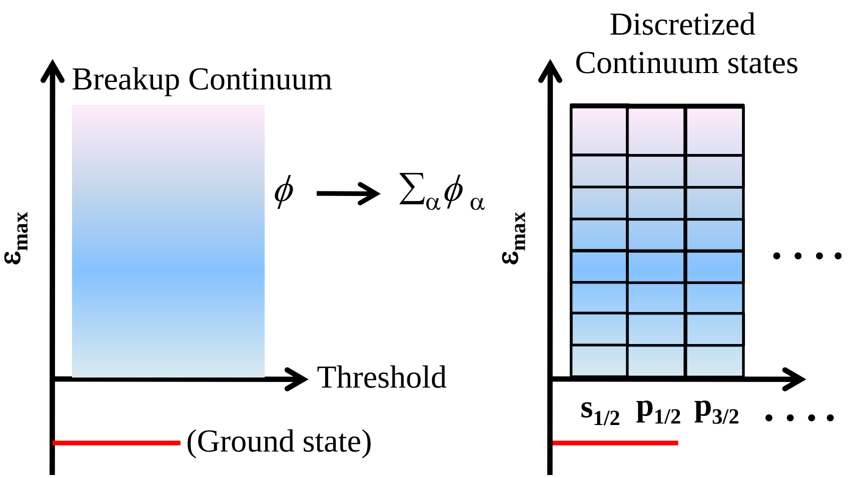
U_{nT}: neutron-target optical potential
U_{pT}: proton-target optical potential
V_{pn}: proton-neutron binding potential



- **The CDCC wave function**, Ψ^{CDCC} is expanded in terms of bound & continuum states of a given pair subsystem, as

$$\Psi^{CDCC}(\mathbf{r},\mathbf{R}) = \sum_{\alpha} \phi_{\alpha}(\mathbf{r})\,\psi_{\alpha}(\mathbf{R})$$

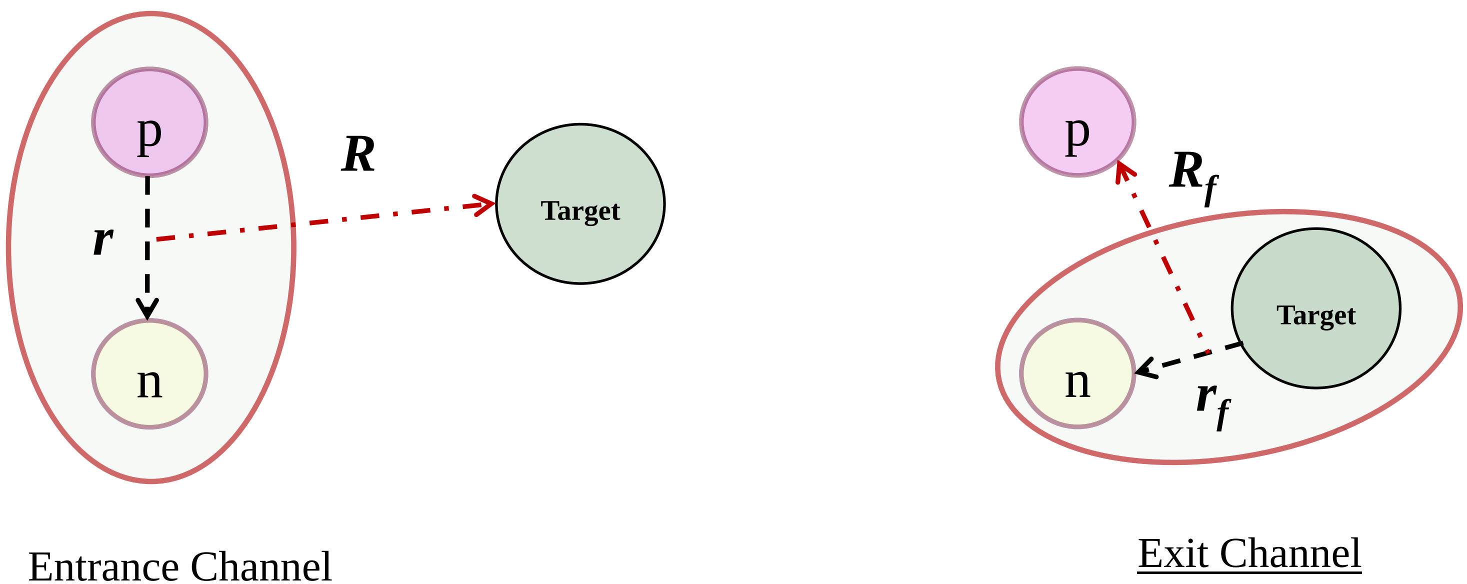
with $\mathrm{H}_{int}\,\phi_{\alpha} = \epsilon_{\alpha}\,\phi_{\alpha}$



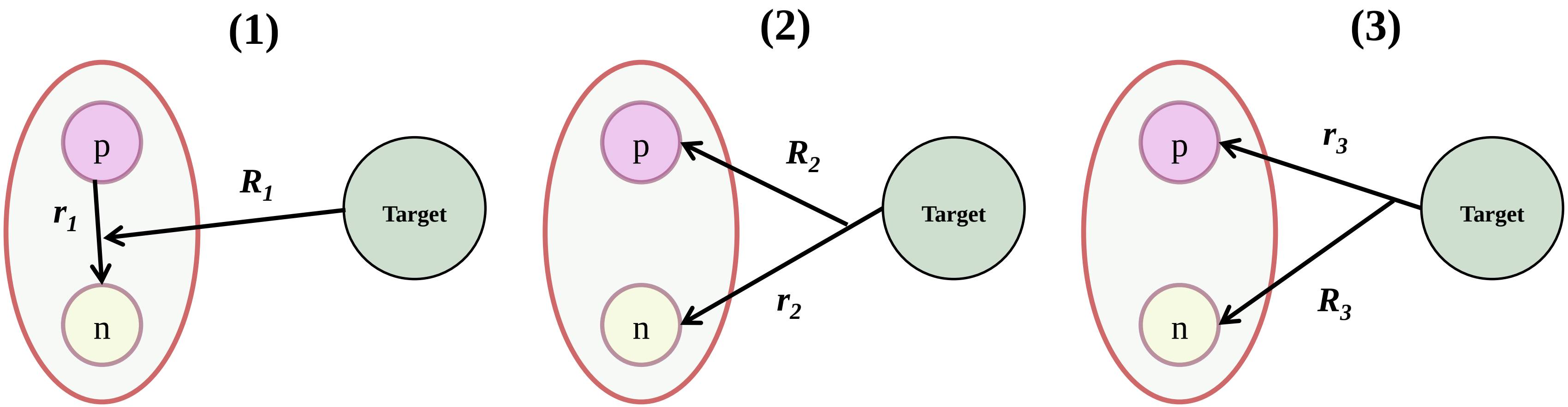
• **Transfer Matrix Element:**

$$\mathbf{T} = \langle\, \Phi(\mathbf{r}_f)\, \chi^{(-)}(\mathbf{R}_f)\,|\, V_{\rm pn} + U_{\rm pT} - U^{f}\, |\, \Psi^{\rm CDCC}\, \rangle$$

Remnant



Faddeev Formalism



$$(E - T_1 - V_{\rm np})\,\Psi^{(1)} = V_{\rm np}\,(\Psi^{(2)} + \Psi^{(3)})$$

$$(E - T_2 - V_{\rm pT})\,\Psi^{(2)} = V_{\rm pT}\,(\Psi^{(3)} + \Psi^{(1)})$$

$$(E - T_3 - V_{\rm Tn})\,\Psi^{(3)} = V_{\rm Tn}\,(\Psi^{(1)} + \Psi^{(2)})$$

❖ Few examples, where the formalism is used to study (d,p) reactions:

- A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, & A. C. Fonseca, Phys. Rev. C**76**, 064602 (2007).
- A. Deltuva & A. C. Fonseca, Phys. Rev. C**79**, 014606 (2009).
- A. Deltuva, Phys. Rev. C**79**, 021602 (2009).

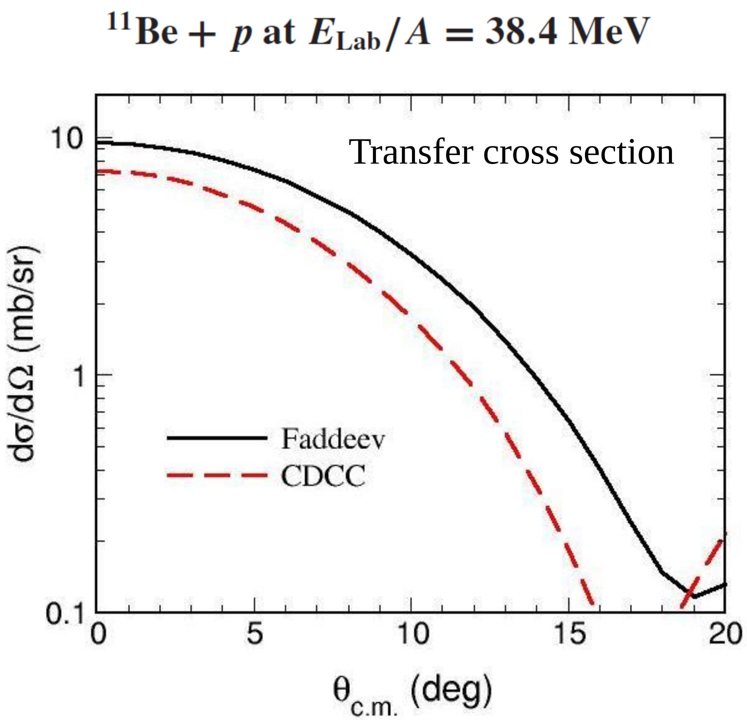
CDCC v/s Faddeev

Ref.: A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, & A. C. Fonseca, PRC**76**, 064602 (2007).

❖ Studied for three reactions:

- ✓ d + ^{12}C @ E_d = 56 MeV
- ✓ d + ^{58} Ni @ E_d = 80 MeV (elastic & breakup observables)
- ✓ p + ^{11}Be @ E_{11Be} = 38.4 MeV/A (elastic, breakup & transfer observables)

CDCC in agreement with Faddeev!



For breakup & transfer:
CDCC underestimates Faddeev!

Calls for better understanding of limits of CDCC methods.

Our Approach

- We start with studying (d, p) reactions on ^{10}Be, as a function of beam energy.
- Compare CDCC and Faddeev calculations starting from the same 3-body Hamiltonian.
- In this talk, calculations for the reaction of deuteron on ^{10}Be are presented at

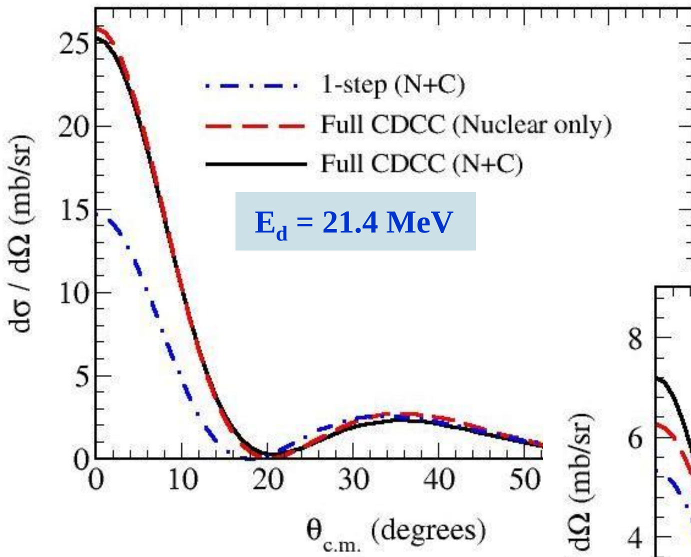
   1. E_d = 21.4 MeV (Data from ORNL [under analysis])
   2. E_d = 40.9 MeV (Data from GANIL [NPA683, 48 (2001)])
   3. E_d = 71 MeV

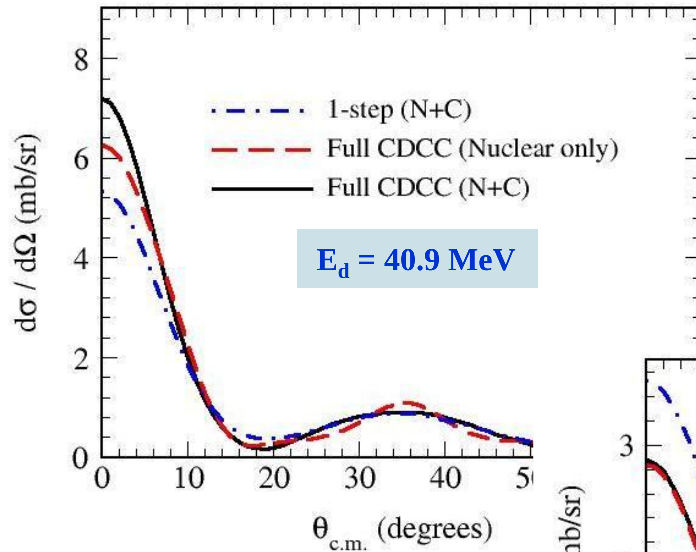
Inputs to 3-body Hamiltonian

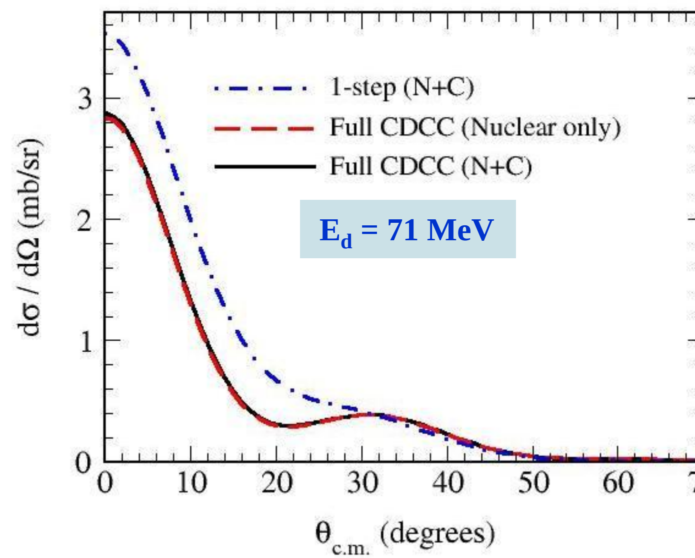
- CDCC calculations are performed using deuteron breakup states.
- Potentials:
  1. For neutron-proton (n-p) bound & continuum states:
     A Gaussian Potential
     $$V(r) = -V_0\, e^{-(r/r_0)^2}$$
     where, V_0 = 72.15 MeV and r_0 = 1.484 fm
  2. Optical potentials:
     Chapel-Hill Global Parameterization (spin-orbit neglected)
  3. n-^{10}Be binding potential: V = 57.07 MeV, r = 1.25 fm, a = 0.65 fm
- The interactions between all pairs are spin independent.
- Model space convergence is checked.

Transfer Cross section

$^{10}Be + d \rightarrow {}^{11}Be + p$







Results indicate that:

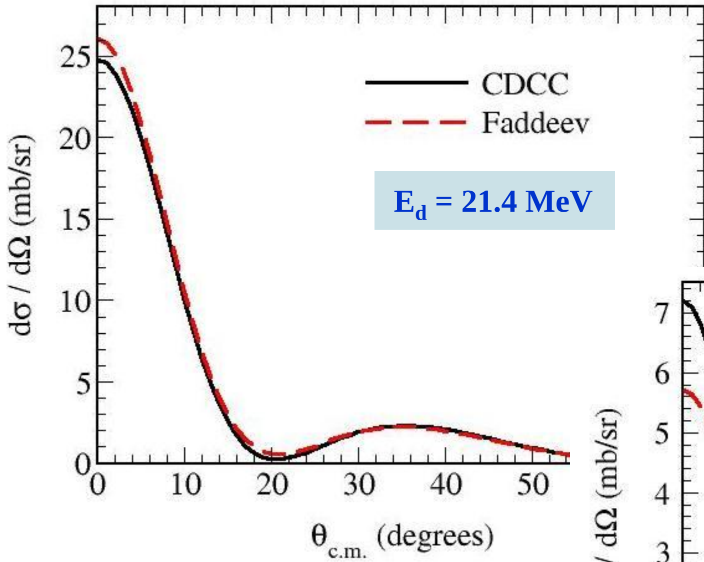
- Coulomb effects are significant in angular distributions for $\theta < 10^o$.
- Continuum has strong influence on Transfer process.

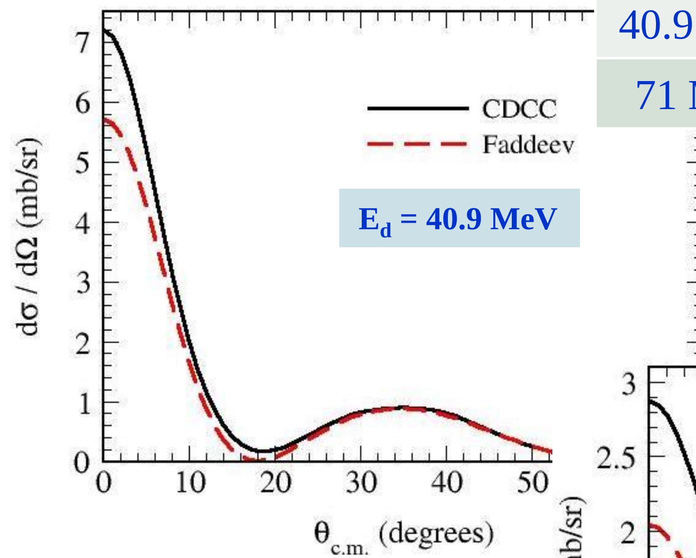
Comparing CDCC with Faddeev

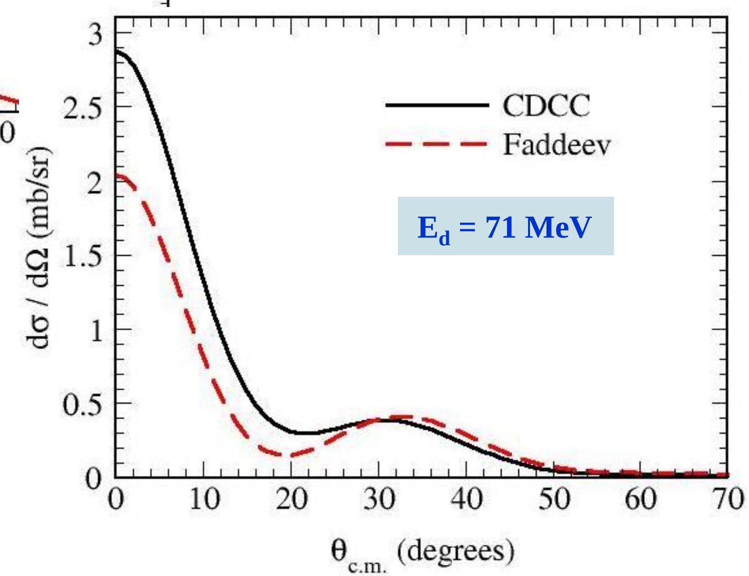
$^{10}Be + d \rightarrow {}^{11}Be + p$

Estimate of discrepancy in two methods

E_d	$\Delta(\text{CDCC-FAD})_{\theta = 0^o}$
21.4 MeV	5.12%
40.9 MeV	–20.72 %
71 MeV	–29.06%







Conclusions

- **Comparison of CDCC with Faddeev:**
  - At low energy, *i.e.*, E_d = 21.4 MeV, we get better agreement.
  - Discrepancy in two methods increases with beam energy.

- Study for other cases is in progress.

Thank You!

- My Audience
- Ian Thompson (LLNL, USA)

Converged CDCC Model Space

E_d = 21.4 MeV & 40.9 MeV

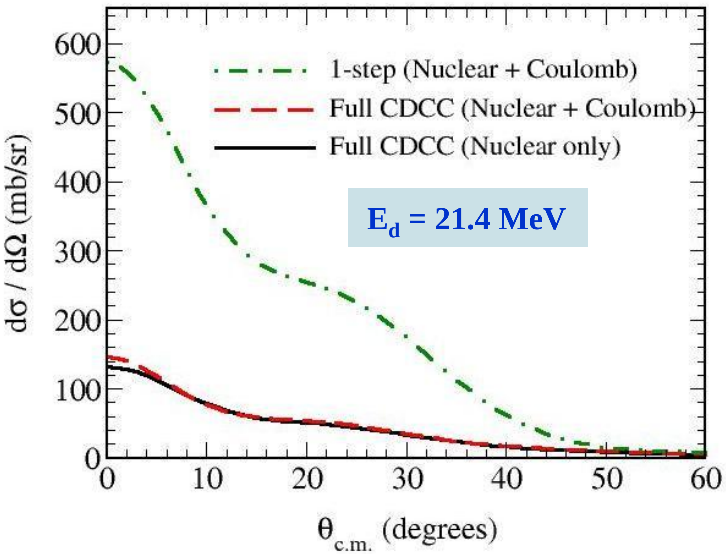
- ✓ Partial waves for p-n relative motion: $l_{max} \leq 4$
- ✓ Maximum excitation energy in the continuum: E_{max} = 17 MeV (E_d = 21.4 MeV)
  E_{max} = 29.1 MeV (E_d = 40.9 MeV)
- ✓ Coupled equations are integrated up to R_{max} = 60 fm with total angular momentum, J_{max} = 40
- ✓ Multipoles $Q \leq 4$ are included for the CDCC coupling potentials.

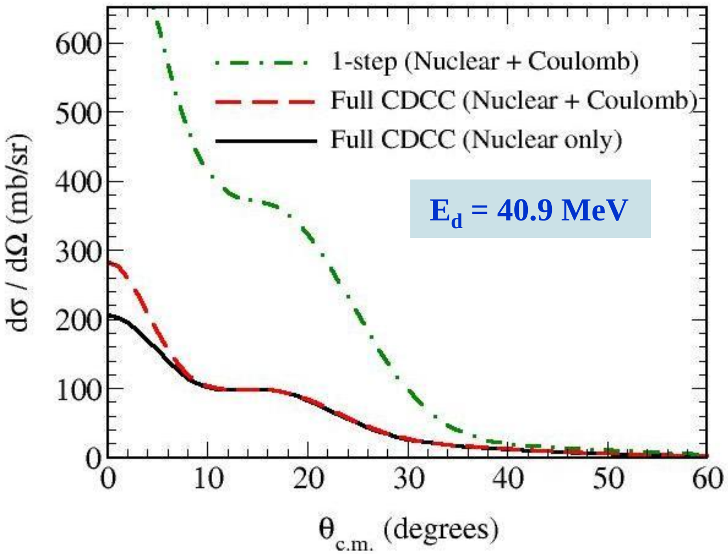
E_d = 71 MeV

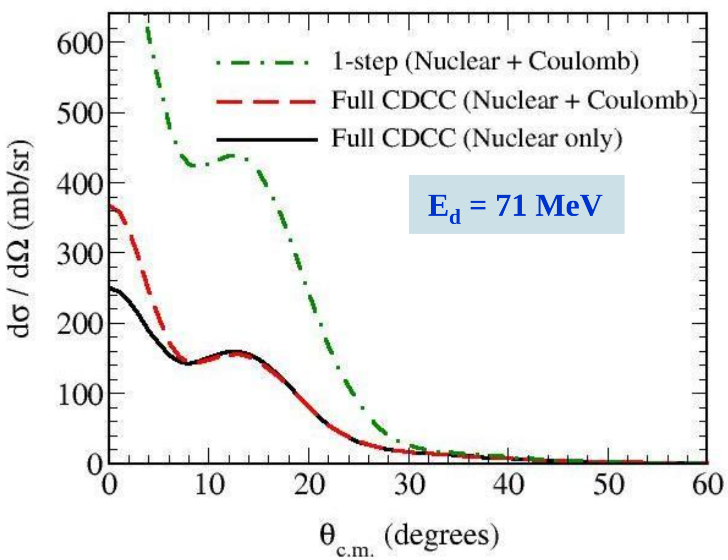
- ✓ Partial waves for p-n relative motion: $l_{max} \leq 4$
- ✓ Maximum excitation energy in the continuum: E_{max} = 40.1 MeV
- ✓ Coupled equations are integrated up to R_{max} = 60 fm with total angular momentum, J_{max} = 50
- ✓ Multipoles $Q \leq 6$ are included for the CDCC coupling potentials.

Breakup Cross section

$^{10}Be + d \rightarrow {}^{10}Be + p + n$

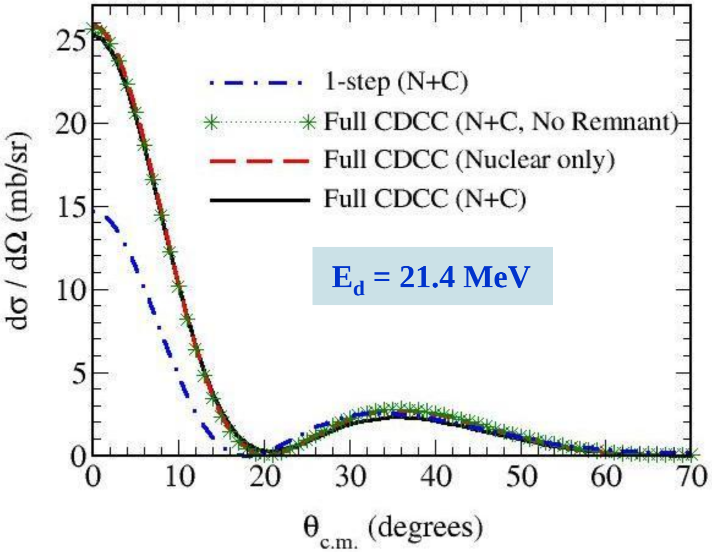


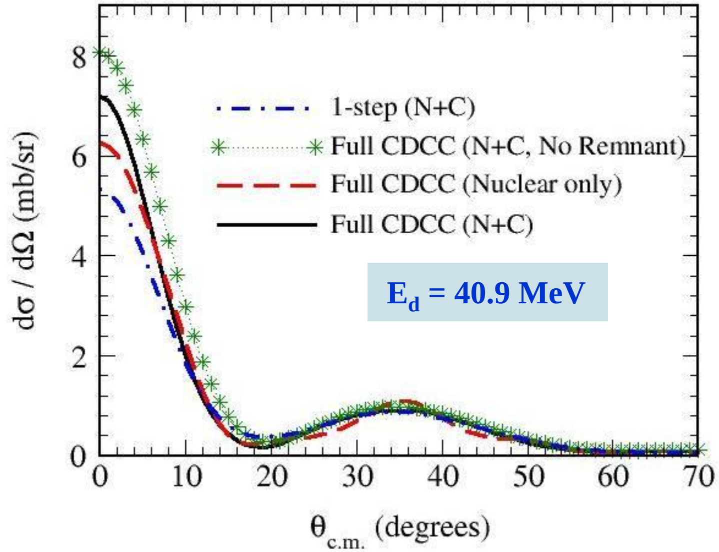


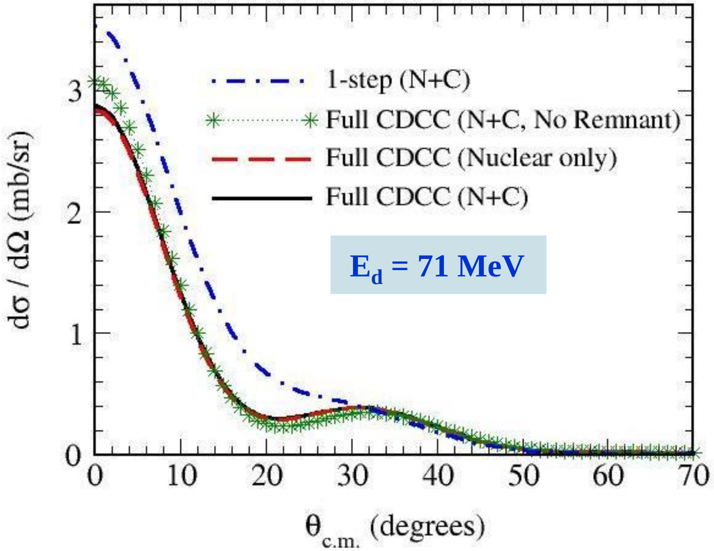


Transfer Cross section

$^{10}Be + d \rightarrow {}^{11}Be + p$







Interaction details for Faddeev

1. n-T optical potentials are *l*-dependent:
   - It is a Binding potential for $l = 0$.
   - It is a Chapel Hill -89 global parameterization for $l \neq 0$.

2. p-T optical potentials can be calculated at energy E_p in the exit channel or ($E_d/2$) in the entrance channel.