Separable nucleon-nucleus optical potentials

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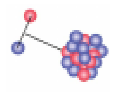
MOTIVATION

- Deuteron stripping (d, p) reactions are a tool to study exotic nuclei
- (d,p) reactions can be treated as a three-body problem with effective two-body interactions:

  neutron-proton
  neutron-nucleus } optical potentials
  proton-nucleus



- Three-body problem: Faddeev techniques in momentum space with two- body separable interactions
- Current calculations are limited up to calcium isotopes (Z= 20) due to numerical complications in treating the Coulomb force via screening techniques

OBJECTIVE

- Develop separable effective interactions
- Show that they reproduce two-body observables
- No screening of the Coulomb force

SUMMARY

- Starting from a Woods-Saxon type phenomenological optical (complex) potentials

  (i) analytic and numeric Fourier transform to momentum space

  (ii) generalized Ernst-Shakin-Thaler (EST) to (a) complex potentials (L. Hlophe, Ch. Elster, et. al., Phys.Rev. **C88**, 064608 (2013).) and (b) charged particles (L. Hlophe, V. Eremenko , et. al., C **90,** 061602(R) (2014).)

  (ii) succeeded in calculating Coulomb distorted form factors in momentum space (N. Upadhyay, V. Eremenko, et. Phys. Rev. **C90,** 014615 (2014).)

- Ingredients are ready for implementation in three-body calculations
- Work in progress : EST scheme for multichannel scattering with application to core excitations of nucleus

RESULTS

Proton-nucleus form factors

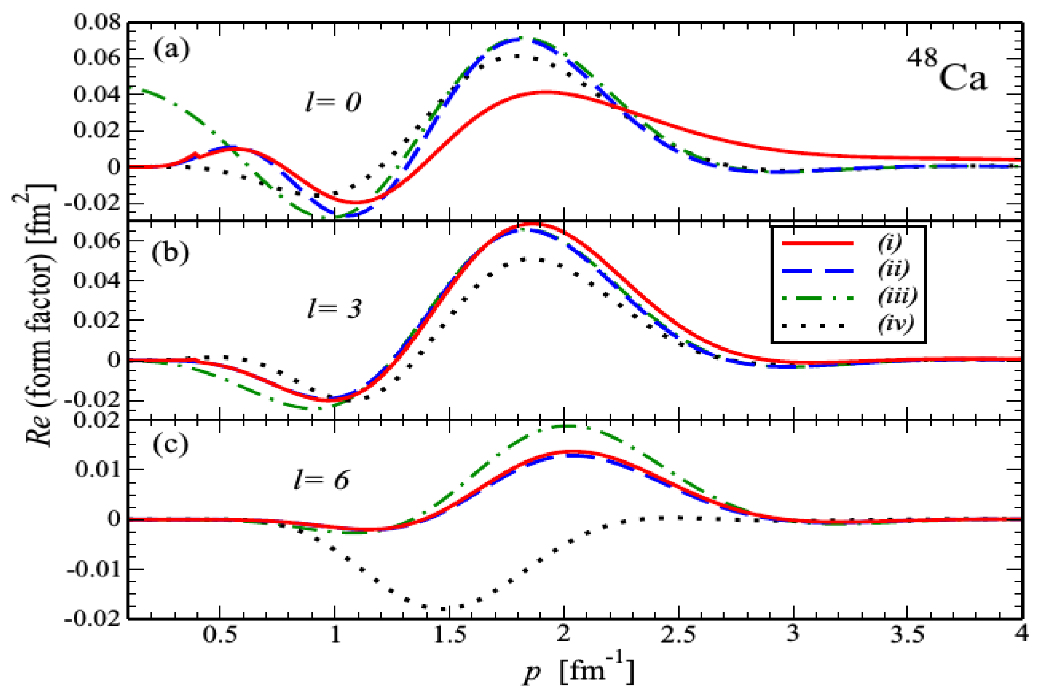


FIG. 1. The real parts of the partial wave **proton-nucleus form factor** for ^{48}Ca as function of the momentum for selected angular momenta The form factors are calculated at $E_{c.m}$ = 36 MeV and based on the CH89 global optical potential: full calculations (i) are compared to omitting the short range Coulomb (ii), the neutron-nucleus form factor (iii) and the Coulomb distorted neutron-nucleus form factor (iv).

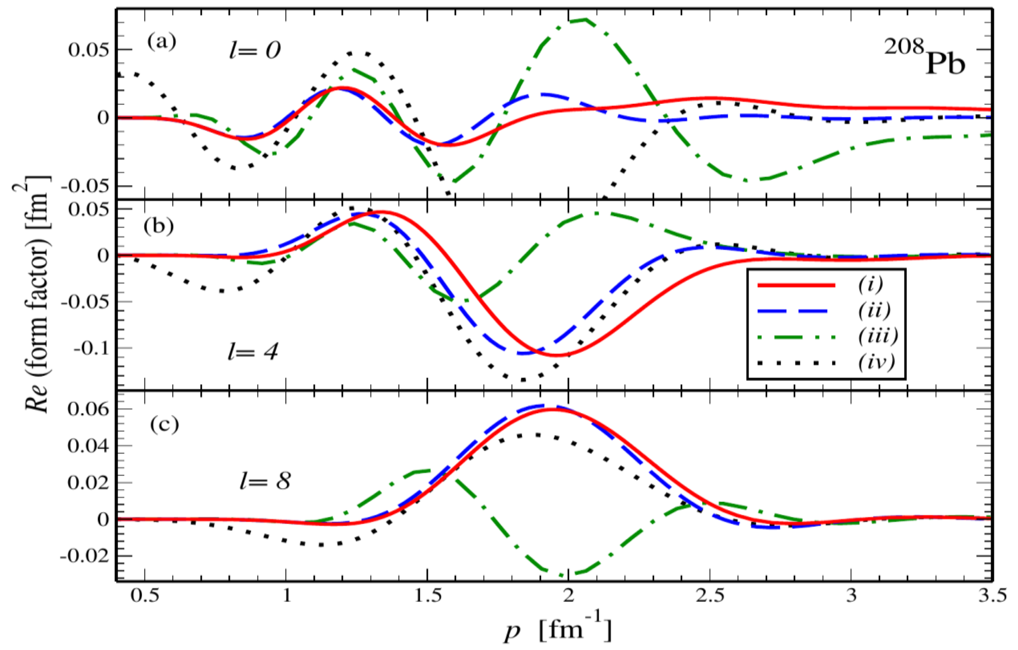


FIG. 2. Same as Fig. 1 but for ^{208}Pb. The form factors for l = 0 (a) and l = 4 are calculated at E = 21 MeV but for l = 8 (c) these are calculated at $E_{c.m.}$ = 36 MeV.

neutron-nucleus cross sections

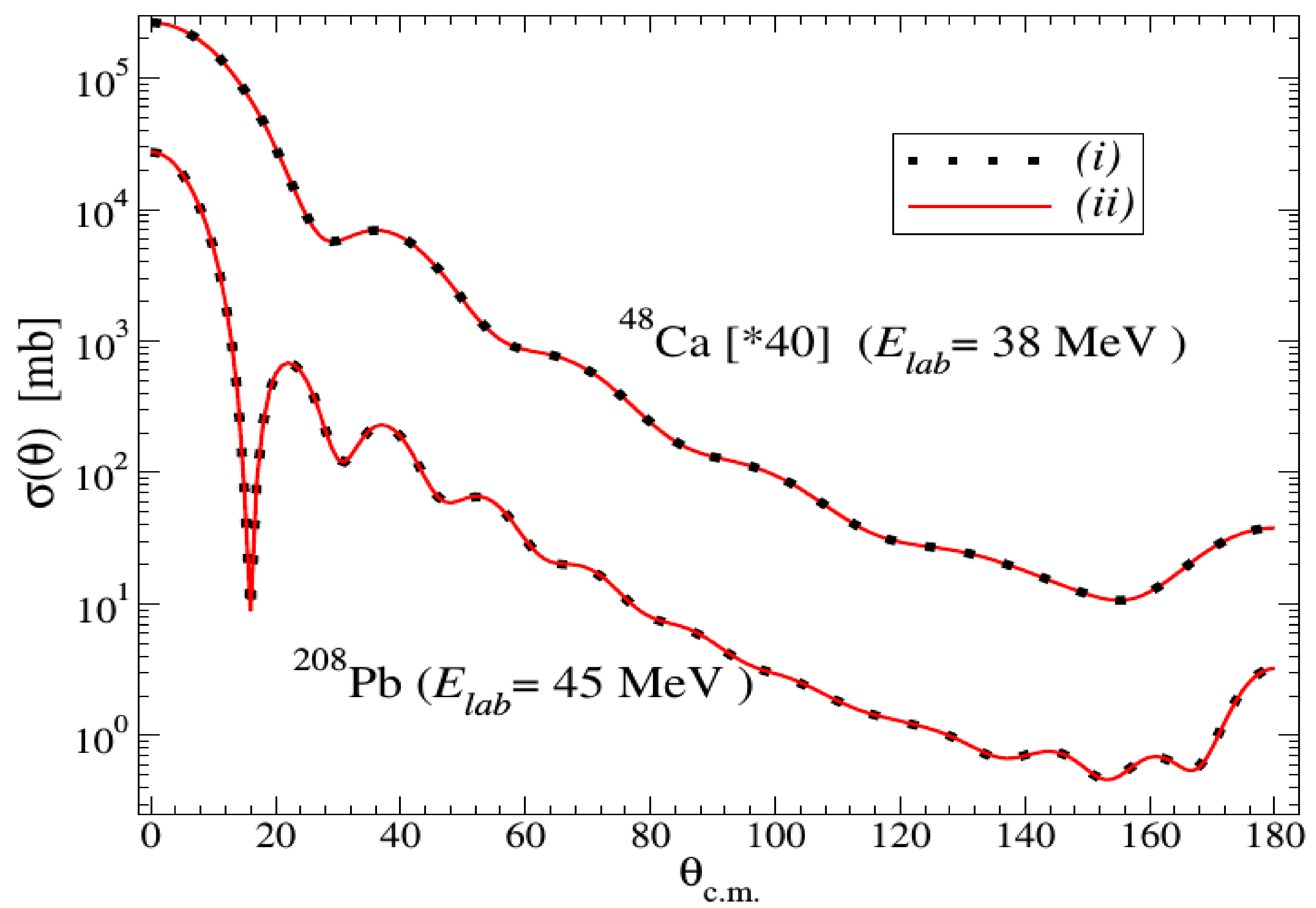


FIG. 3 The unpolarized cross section for **neutron-nucleus elastic scattering** obtained from the CH89 global optical potential as function of the center of momentum angle $\theta_{c.m.}$.The elastic cross section is calculated at a laboratory kinetic energy of 38 MeV (scaled up by a factor of 40) for n+^{48}Ca and 45 MeV for n+^{208}Pb. The red solid line (i) gives the elastic neutron-nucleus cross section calculated from the momentum space separable representation of the CH89 global optical potential, while the black dotted line (ii) depicts the corresponding coordinate space calculation.

Off-shell transition matrix

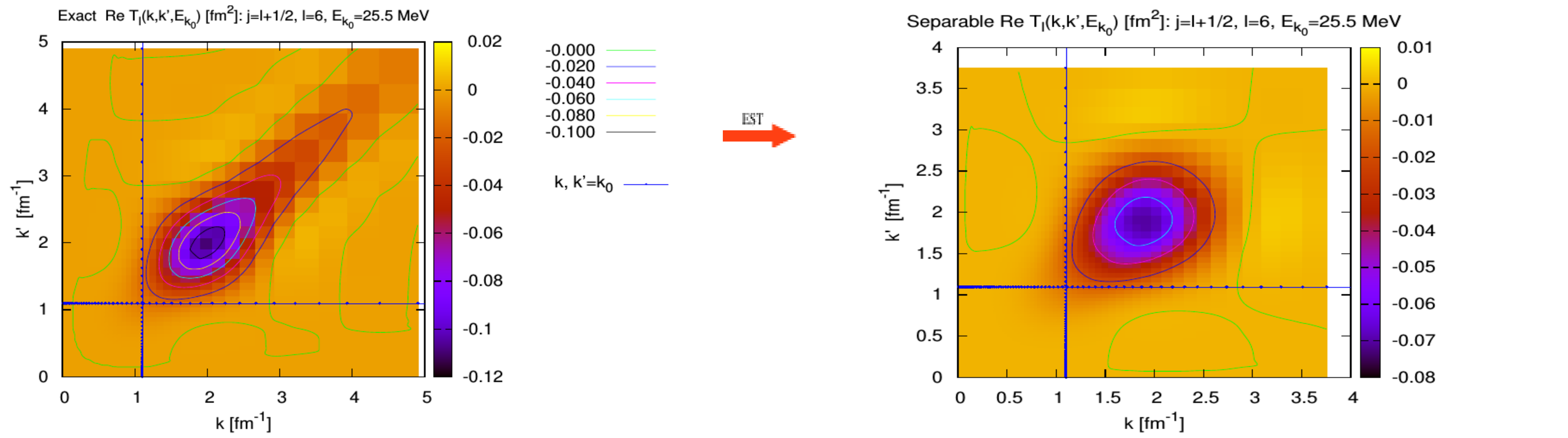


FIG. 5 Real part of the off-shell t-matrix calculated using the exact CH89 potential. This t-matrix is symmetric and exhibits high momentum components along the diagonal.

FIG. 6 : Separable representation of the real part of the off-shell t-matrix calculated using the exact CH89 potential. The high momentum components present in the exact t-matrix have been projects out.

proton-nucleus cross sections

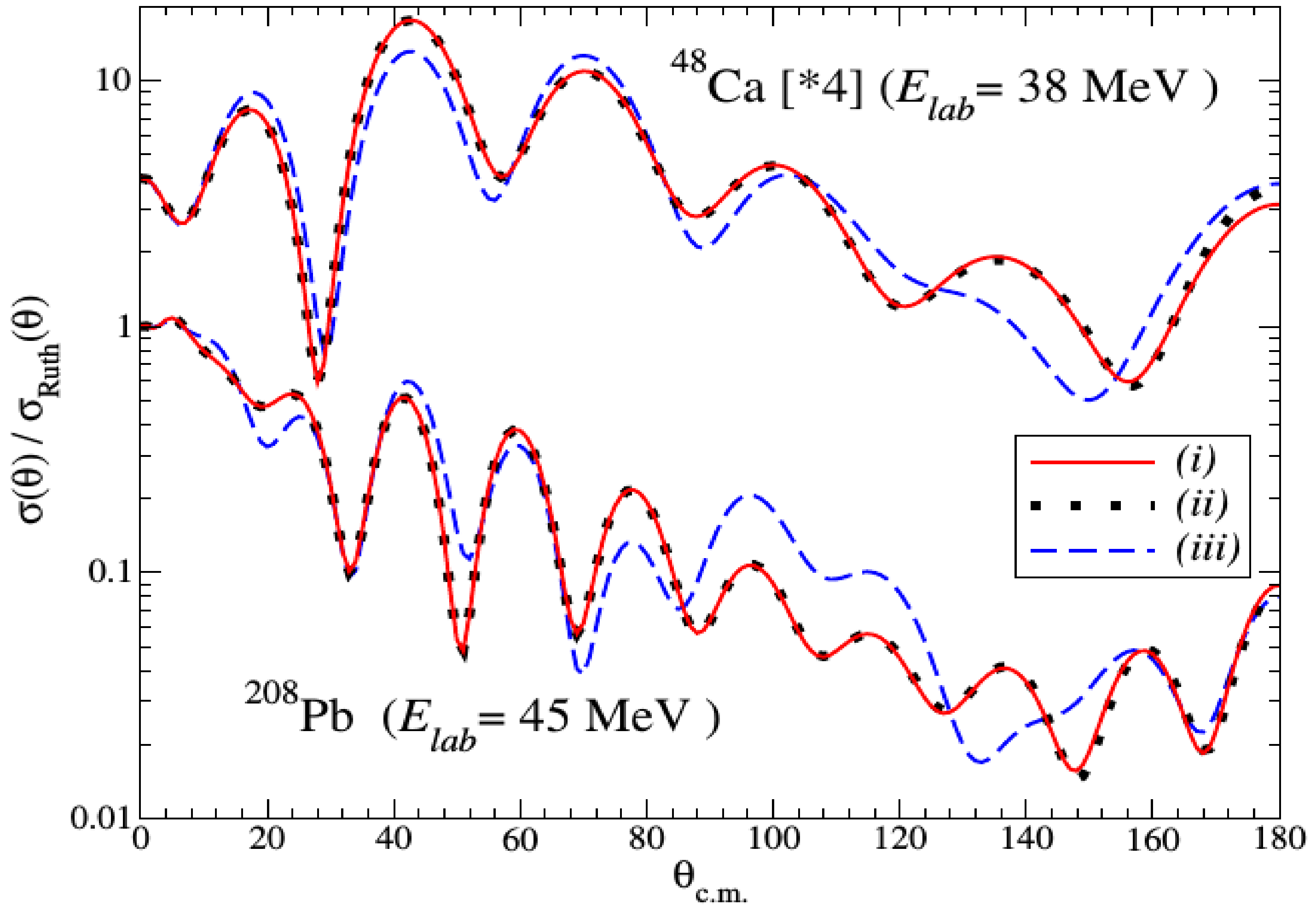


FIG. 4 The unpolarized cross section for **proton-nucleus elastic scattering** obtained from the CH89 global optical potential as function of the center of momentum angle $\theta_{c.m.}$. The elastic cross section is calculated at a laboratory kinetic energy of 38 MeV (scaled up by a factor of 4) for p+ and 45 MeV for p+^{208}Pb. The red solid line (i) gives the elastic proton-nucleus cross section calculated from the momentum space separable representation of the CH89 global optical potential, while the black dotted line (ii) depicts the corresponding coordinate space calculation. The blue dash-dotted line shows the calculation in which the short-range Coulomb potential is omitted.

METHODS

- The starting point for developing separable interactions is the EST scheme which requires that at fixed energies scattering wavefunctions from the original potential and its separable representation be identical
- Complex potentials u^s : generalize EST scheme to use 'in' and 'out' scattering states to preserve reciprocity

$$\mathbf{U} = \sum_{i,j} u|f_{l,k_{E_i}}\rangle\langle f_{l,k_{E_i}}|M|f^*{}_l, k_{E_j}\rangle\langle f^*_{l,k_{E_j}}|u$$

$$\delta_{ik} = \sum_j \langle f_{l,k_{E_i}}|M|f^*_{l,k_{E_j}}\rangle\langle f^*_{l,k_{E_j}}|u|f_{l,k_{E_k}}\rangle$$

- Charged particles: EST scheme is generalized to use Coulomb scattering states instead of plane wave

$$\tau_l^{CN}(E_{p_0}) = \sum_{i,j} u^s|f^c_{l,k_{E_i}}\rangle\, \tau^c_{ij}(E_{p_0})\, \langle f^{c*}_{l,k_{E_j}}|u^s,$$

with the free propagator replaced by the Coulomb propagator g_c so that

$$\sum_j \tau_{ij}^{CN}(E_{p_0})\, \langle f^{c*}_{l,k_{E_j}}|u^s - u^s g_c(E_{p_0}) u^s|f^c_{l,k_{E_k}}\rangle = \delta_{ik}$$

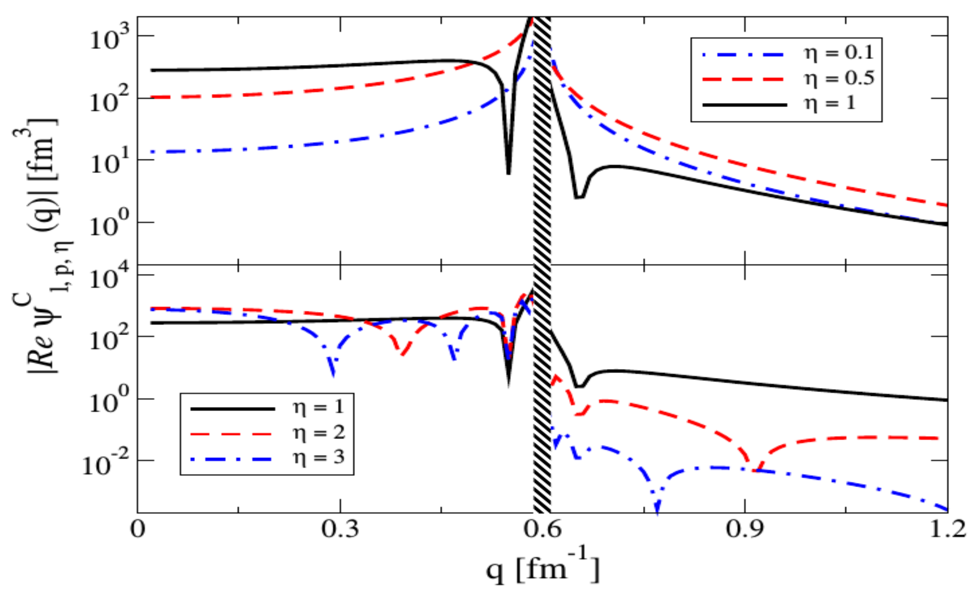
where the Coulomb distorted half-shell t-matrix is obtained via

$$\langle p|\tau_l^{CN}(E_{p_0})|p_0\rangle = \langle p|u_l^s|p_0\rangle + \int p'^2 dp' \langle p|u_l^s|p'\rangle\langle p'|g_c(E_{p_0}+i\varepsilon)|p'\rangle\langle p'|\tau_l^{CN}(E_{p_0})|p_0\rangle$$

and the potential matrix elements are evaluated as

$$\langle \Phi^c_{l,p'}|u_l^s|\Phi^c_{l,p}\rangle = \frac{2}{\pi}\int_0^\infty \langle \Phi^c_{l,p'}|r'\rangle\, r'^2 dr'\, \langle r'|u_l^s|r\rangle\, r^2 dr\, \langle r|\Phi^c_{l,p}\rangle$$

Coulomb scattering wavefuntion



- Gel'fand-Shilov regularization of oscillatory singularity in folding integrals of form factors with Coulomb wavefunctions

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