Surface-integral formalism for transfer reactions:

Applications and applicability for (d,p) reactions

TORUS Collaboration Meeting
MSU/NSCL, East Lansing, MI
June 25-26, 2012

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LLNL-PRES-562315
This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC. Support was provided by the DOE through the topical collaboration TORUS.

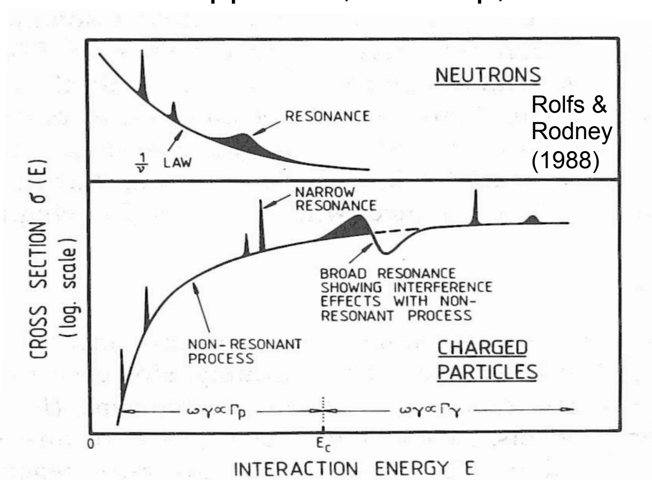
Studying nuclear structure with (d,p) one-nucleon transfers

(d,p) reactions:

- Simplest mechanism for adding a neutron
- Traditionally used to study stable nuclei
- Used in inverse kinematics at RIB facilities, for studying weakly-bound systems

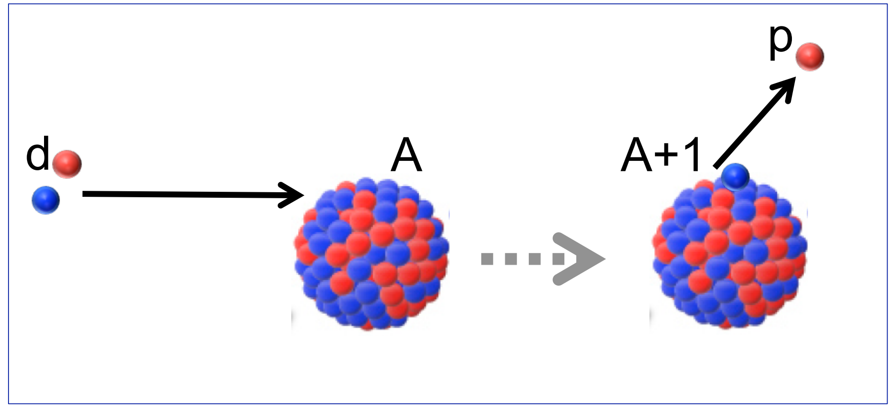
Theoretical descriptions of (d,p) reactions:

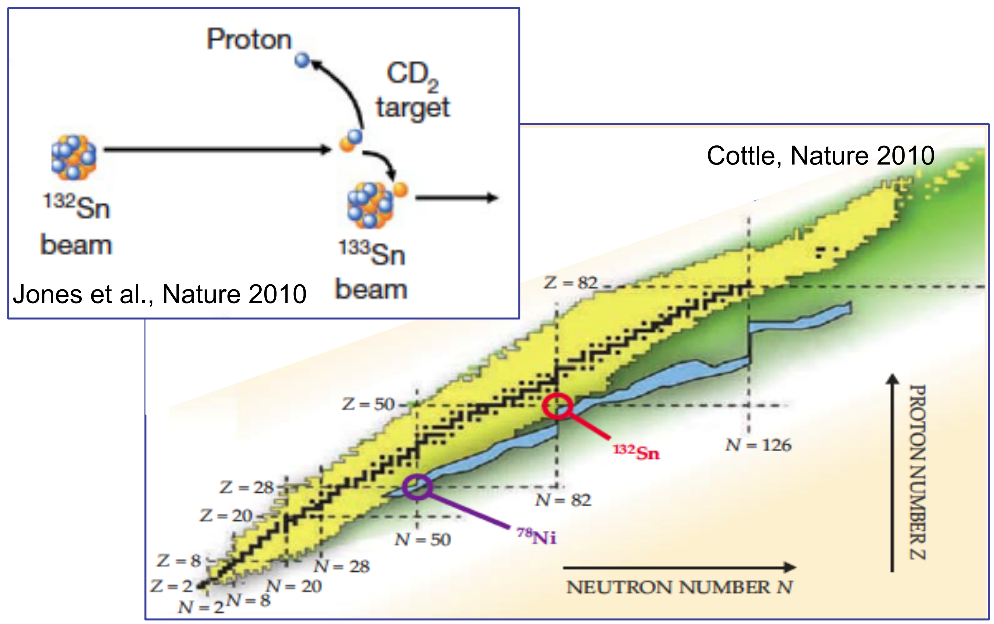
- Progress over the years: Plane-wave theory, DWBA (zero-range & finite-range), coupled-channels approach, breakup, etc.



But: Current theories of (d,p) reactions not very useful for transfers to resonance states:

- Conceptual: extracting spectroscopic information
- Practical: convergence issues



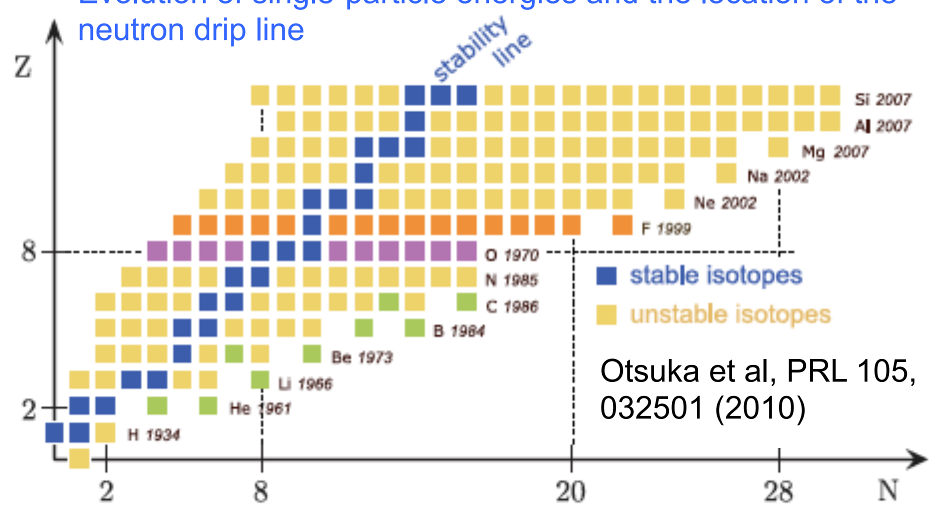


Resonances in low-energy nuclear physics

Resonances:

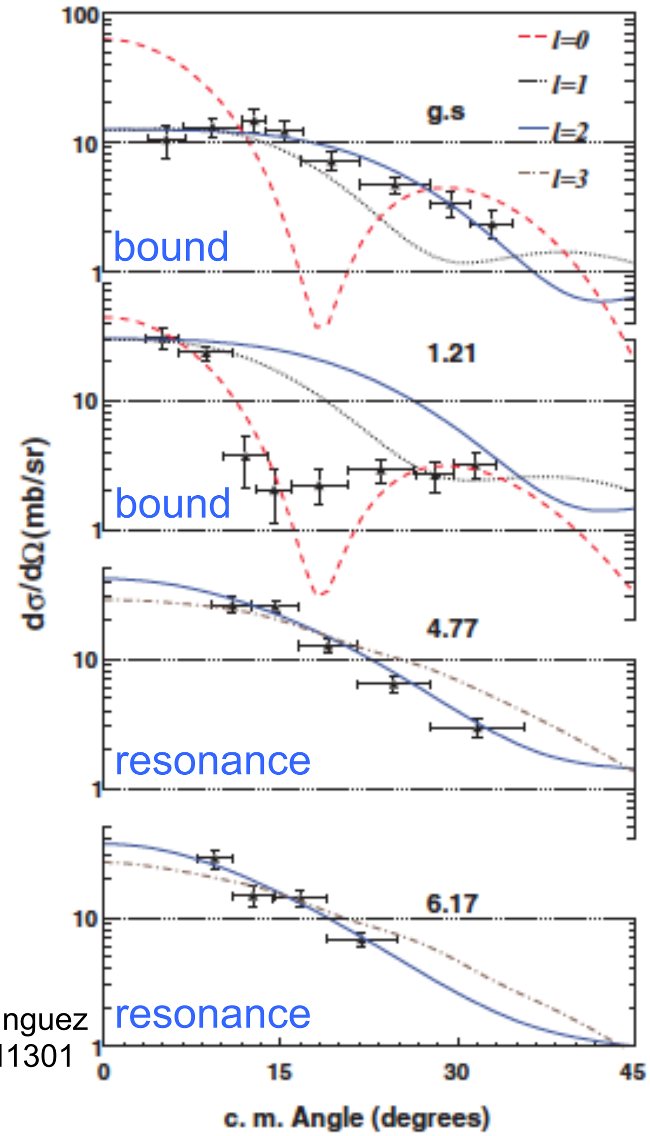
- Unstable quantum-mechanical states
- Occur in light, medium-mass, and heavy nuclei
- Crucially affect astrophysical reaction rates
- Abundant in weakly-bound nuclei

Evolution of single-particle energies and the location of the neutron drip line



Otsuka et al, PRL 105, 032501 (2010)

$^{20}O(d,p)^{21}O$ inverse-kinematics experiment at GANIL to determine N=16 shell gap

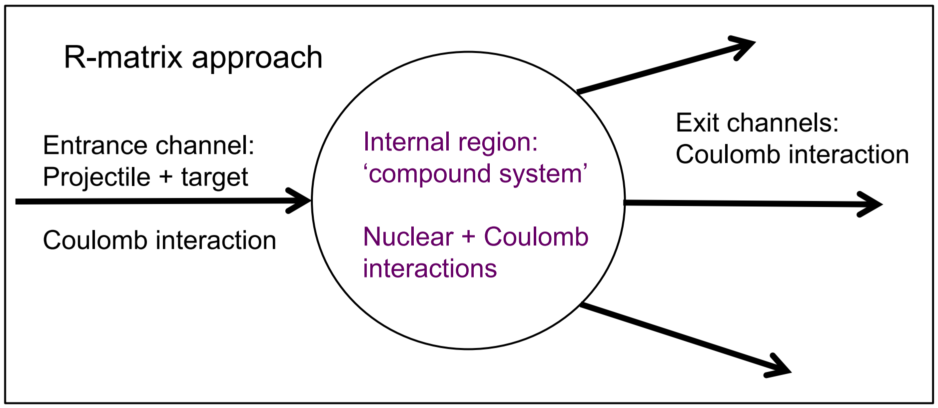


Fernandez-Dominguez et al, PRC 84, 011301 (R) (2011)

Current approach:

- Apply standard (d,p) descriptions to resonances
- Increase model space to achieve convergence

Describing resonances in binary reactions

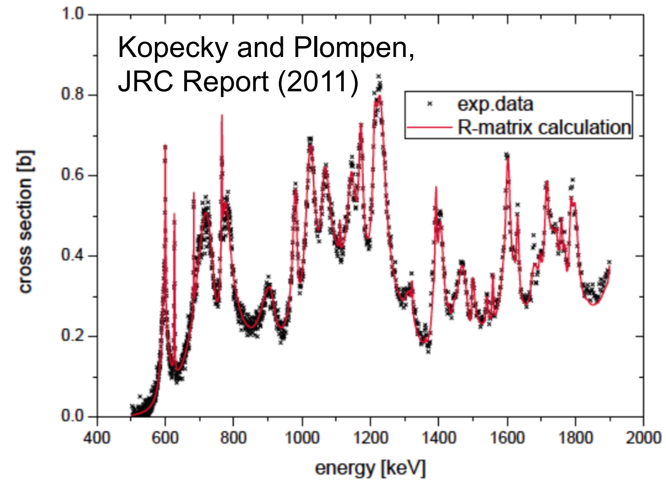


R-matrix approach:

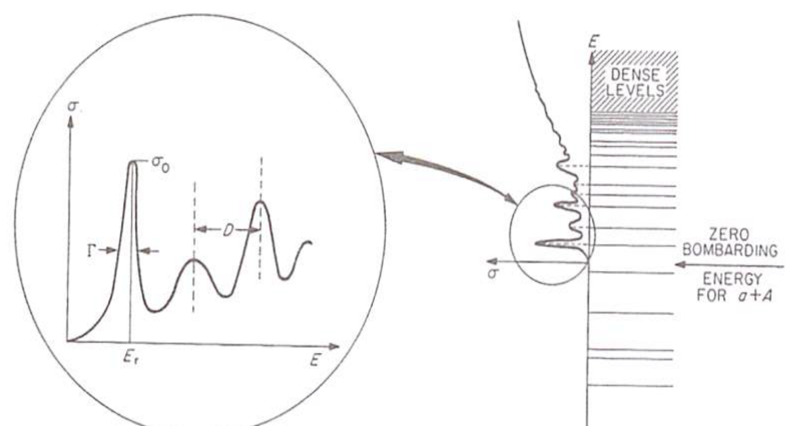
- Main idea: divide space into 2 regions:
  - $r \leq a$ -- interior: nuclear and Coulomb interactions
  - $r > a$ -- exterior: Coulomb only
- Formalism:
  - Interior: expand nuclear wave function in set of basis functions
  - Exterior: scattering wave function
  - Surface: matching conditions allow to parameterize collision matrix -> expressions for cross sections
- Connect observed parameters (E_R, Γ) to formal parameters ($\check{E}_R$, γ^2)
- Typical applications adjust parameters to reproduce measured cross sections

Experimental studies of resonances:

- Elastic & inelastic scattering, capture, etc.



- Characterization of resonances: position & widths



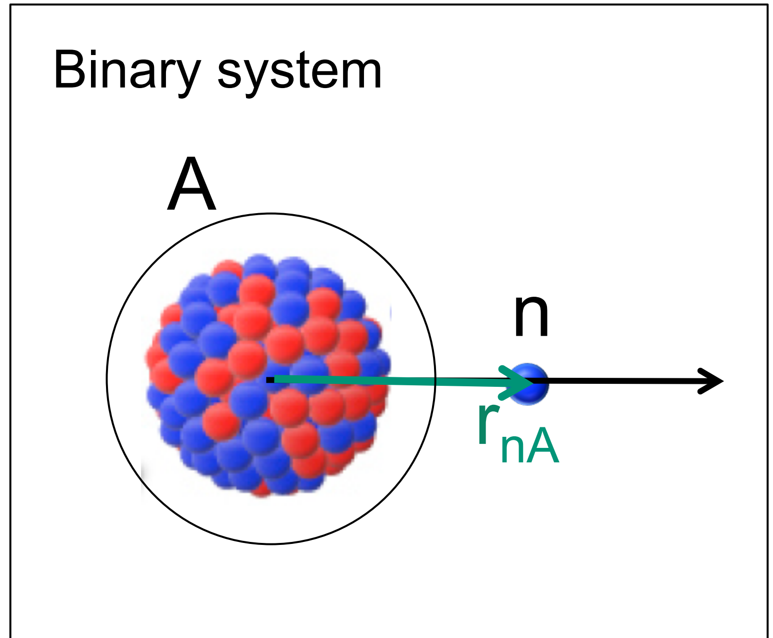
Satchler, Introduction to Nuclear Reactions (1980)

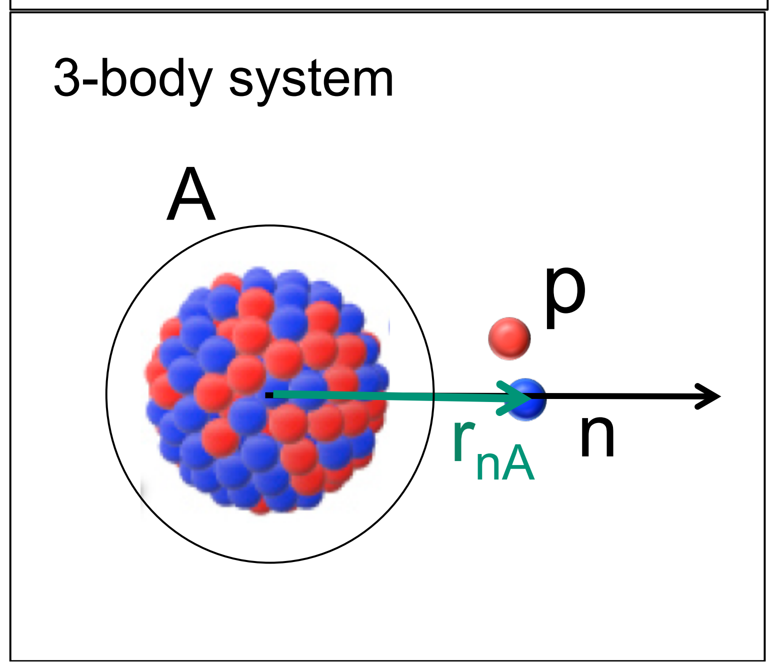
Exploring R-matrix ideas for (d,p) one-nucleon transfers

Proposed new formalism (Mukhamedzhanov, 2011):

- R-matrix concepts:
    - surface separating internal and external regions
    - cross sections expressed in terms of reduced widths, logarithmic derivatives, surface radii
- Goals for (d,p):
    - useful for resonances
    - reduce dependence on model for interior
    - extract useful spectroscopic quantities from comparison to experiment (widths)
- Formalism:
    - applicable to stripping to bound and resonance states
    - general enough to include deuteron breakup contributions via CDCC (continuum-discretized coupled-channels method)
    - bonus: resolves practical issues related to numerical convergence

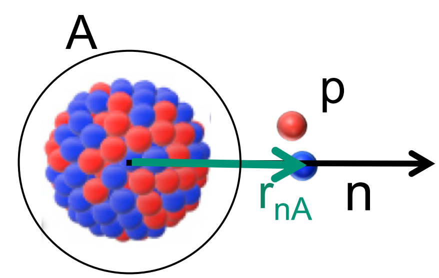
Formalism:
Mukhamedzhanov, PRC 84, 044616 (2011)





Exploring R-matrix ideas for (d,p) one-nucleon transfers II

3-body system



Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(post)} = < \Phi_f^{(-)} \,|\, \Delta V_{pF} \,|\, \Psi_i^{(+)} >$$

DWBA

$$< \varphi_F \chi_{pF}^{(-)} \,|\, \Delta V_{pF} \,|\, \varphi_d \varphi_A \chi_{dA}^{(+)} >$$

$$< I_A^F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \varphi_d \chi_{dA}^{(+)} >$$

3-body

$$< \varphi_F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \varphi_A \Psi_i^{3B(+)} >$$

$$< I_A^F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \Psi_i^{3B(+)} >$$

CDCC

$$< I_A^F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \Psi_i^{CDCC(+)} >$$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = < \varphi_A \,|\, \varphi_F >$

One-body overlap of A and A+1 systems
- carries structure information
- typically approximated by single-particle function

$$M^{(prior)} = < \Psi_f^{(-)} \,|\, \Delta V_{dA} \,|\, \Phi_i^{(+)} >$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Generalized R-matrix formalism for (d,p) reactions I

Splitting the transition matrix element M:

- Interior and exterior with respect to r_{nA}

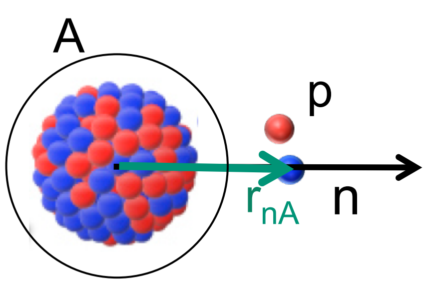
$$M^{(post)} = < \Phi_f^{(-)} \,|\, \Delta V_{pF} \,|\, \Psi_i^{(+)} >$$

<u>DWBA</u>

$$< \varphi_F\, \chi_{pF}^{(-)} \,|\, \Delta V_{pF} \,|\, \varphi_d \varphi_A\, \chi_{dA}^{(+)} >$$

$$< I_A^F\, \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \varphi_d\, \chi_{dA}^{(+)} >$$

3-body system



<u>Interior + exterior</u>

$$M^{(post)} = M^{(post)}(0,a) + M^{(post)}(a,\infty)$$

$$I_A^F = < \varphi_A \,|\, \varphi_F > = I_A^F\,(r_{nA})$$

<u>Mukhamedzhanov</u>

$$M^{(post)}(a,\infty) = M_{surf}(a) + M^{(prior)}(a,\infty)$$

$$M_{surf}(a) = < I_A^F\, \chi_{pF}^{(-)} \,|\, [\overleftarrow{T} - \overrightarrow{T}\,] \,|\, \varphi_d\, \chi_{dA}^{(+)} >_{ext}$$

$$\int_{r\leqslant R} \mathrm{d}\mathbf{r}\, f(\mathbf{r})[\overleftarrow{T} - \overrightarrow{T}\,]g(\mathbf{r})$$

$$= -\frac{1}{2\mu}\oint_{r=R} \mathrm{d}\mathbf{S}[g(\mathbf{r})\nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r})\nabla_{\mathbf{r}} g(\mathbf{r})]$$

$$= -\frac{1}{2\mu}R^2 \int \mathrm{d}\Omega_{\mathbf{r}} \left[g(\mathbf{r})\frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r})\frac{\partial g(\mathbf{r})}{\partial r}\right]_{r=R}$$

<u>Surface term</u>

$M_{surf}(a) = f(a,\, C_A^F,\, B_{nA})$

B_{nA} = log derivative of I_A^F at surface radius a

ANC: C_A^F defined through: $I_A^F\,(r_{nA}) \rightarrow C_A^F\, W(kr_{nA})$

related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

Generalized R-matrix formalism for (d,p) reactions II

DWBA matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

$$M_{(surf)}(a) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{j_{nA} m_{j_{nA}} m_{l_{nA}} M_n} \langle J_A M_A j_{nA} m_{j_{nA}} | J_F M_F \rangle \langle J_n M_n l_{nA} m_{l_{nA}} | j_{nA} m_{j_{nA}} \rangle \langle J_p M_p J_n M_n | J_d M_d \rangle \gamma_{nA j_{nA} l_{nA}}$$

$$\times \int d\mathbf{r}_{pF}\, \chi^{(+)}_{-\mathbf{k}_{pF}}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{nA}}\, Y^{*}_{l_{nA} m_{l_{nA}}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA}=R_{nA}}$$

Assessing the approach:

- Internal – external separation sensible?
- Dominant surface term? Size of corrections?
- Study cross sections arising from different terms
- Start with DWBA and bound states
- Investigate resonances

Cases considered so far:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs, 1st excited state, $2f_{7/2}$ resonance
- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs, 1st excited state
- ^{20}O(d,p) for E_d=21 MeV
  - ^{21}O gs, 1st excited state, $1d_{3/2}$ and $1f_{7/2}$ resonances
- ^{12}C(d,p) for E_d=30 MeV
- ^{40}Ca(d,p) for E_d=34.4 MeV
- ^{209}Pb(d,p) for E_d=52 MeV
- Planned: ^{48}Ca(d,p) for E_d=19.3 and 56 MeV

Assessing the R-matrix ideas Ia

1. Interior vs exterior contributions

$M = M(0,a) + M(a,\infty)$

This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+) ← bound
  - 1^{st} excited state (1/2+)
  - $2f_{7/2}$ resonance

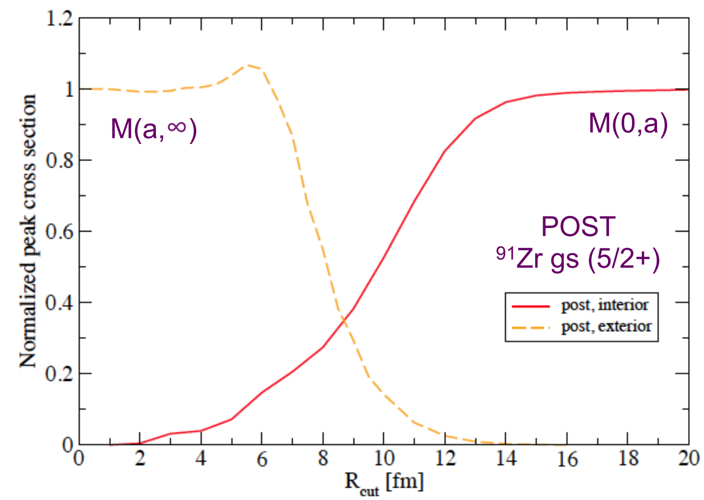
Observations

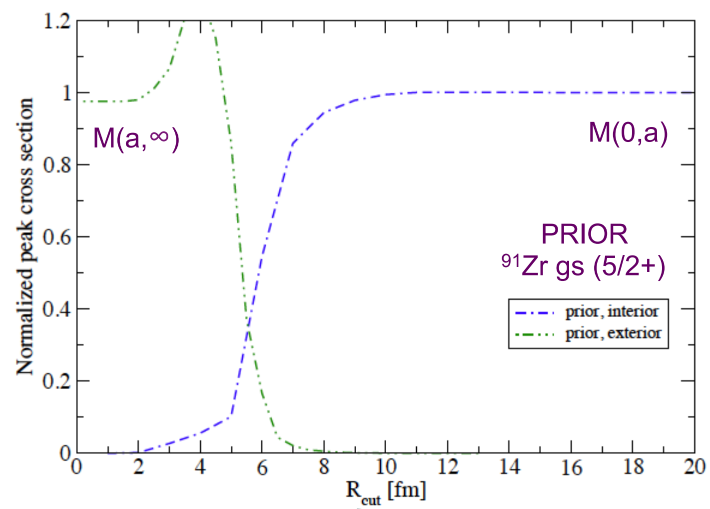
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

$$M^{(post)} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Peak cross section relative to full calculation





surface radius with respect to r_{nA}

Assessing the R-matrix ideas Ib

1. Interior vs exterior contributions

$$M = M(0,a) + M(a,\infty)$$

This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1^{st} excited state (1/2+) ⟵ bound
  - $2f_{7/2}$ resonance

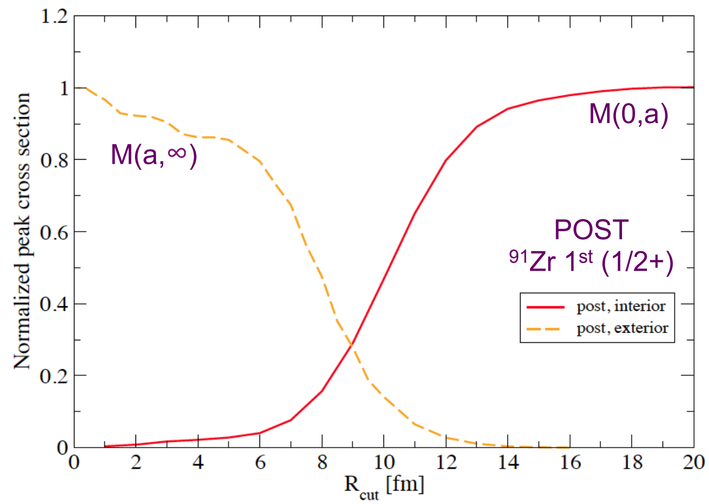
Observations

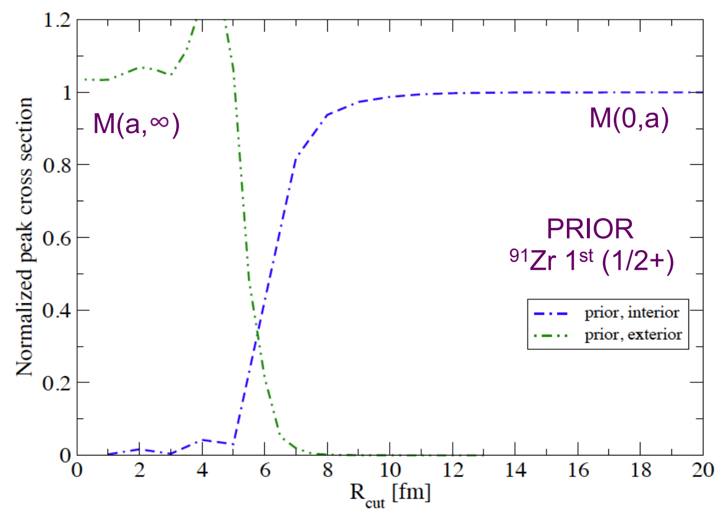
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

$$M^{(post)} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Peak cross section relative to full calculation





Assessing the R-matrix ideas Ic

1. Interior vs exterior contributions

$M = M(0,a) + M(a,\infty)$

This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance ← resonance

Observations

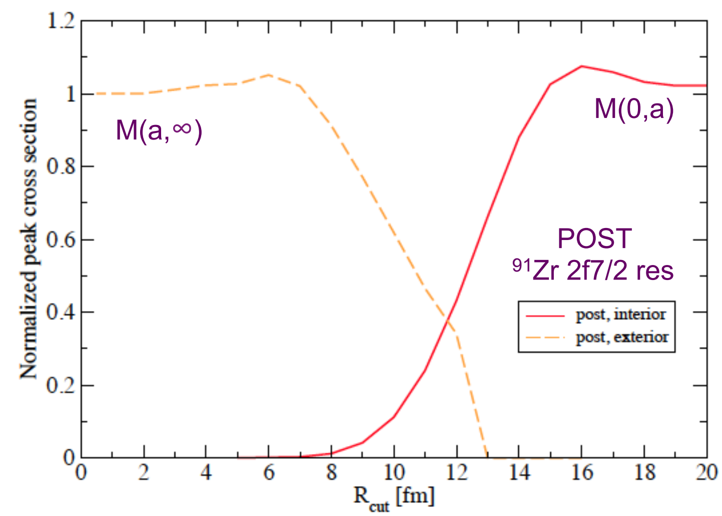
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

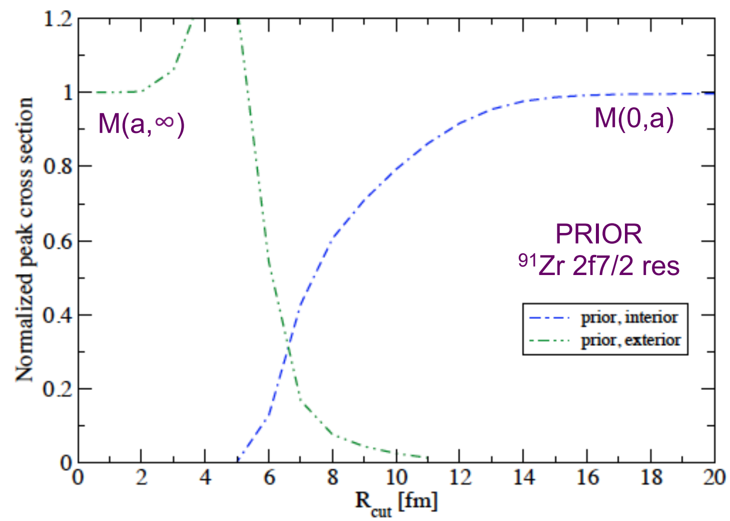
$$M^{(post)} = \langle \Phi_f^{(-)} | \Delta V_{pF} | \Psi_i^{(+)} \rangle$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = \langle \Psi_f^{(-)} | \Delta V_{dA} | \Phi_i^{(+)} \rangle$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

- Resonance: reduced contribution from interior, more pronounced surface effect

Peak cross section relative to full calculation





Assessing the R-matrix ideas IIa

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

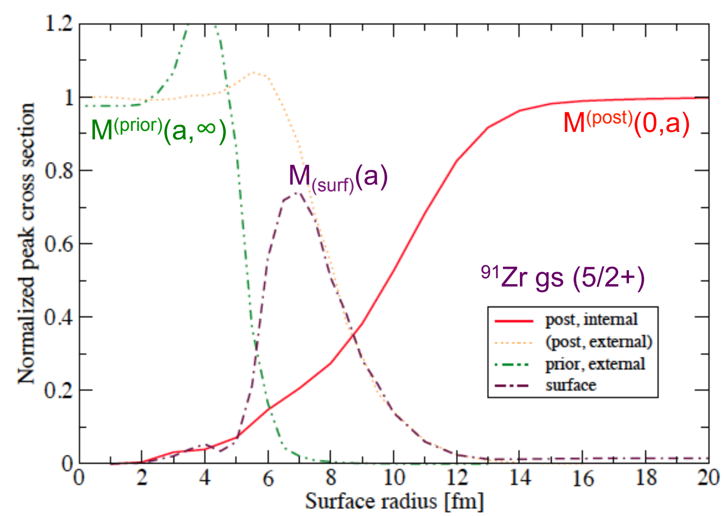
This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+) ← bound
  - 1^{st} excited state (1/2+)
  - $2f_{7/2}$ resonance

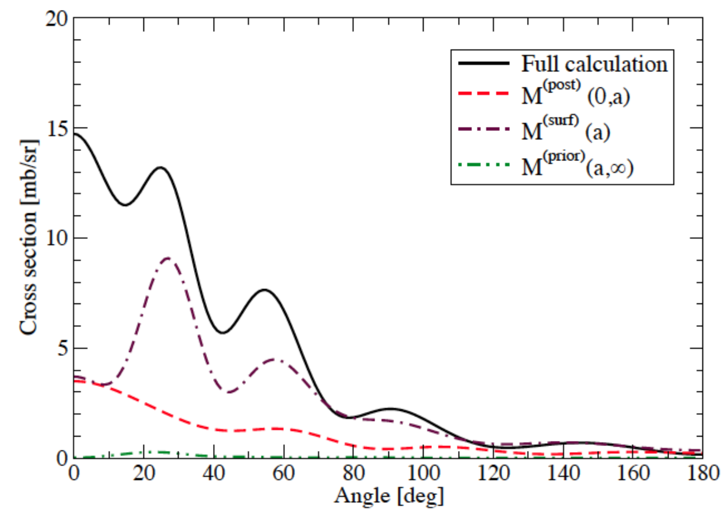
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation



Angular cross sections for a=7 fm



Assessing the R-matrix ideas IIb

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

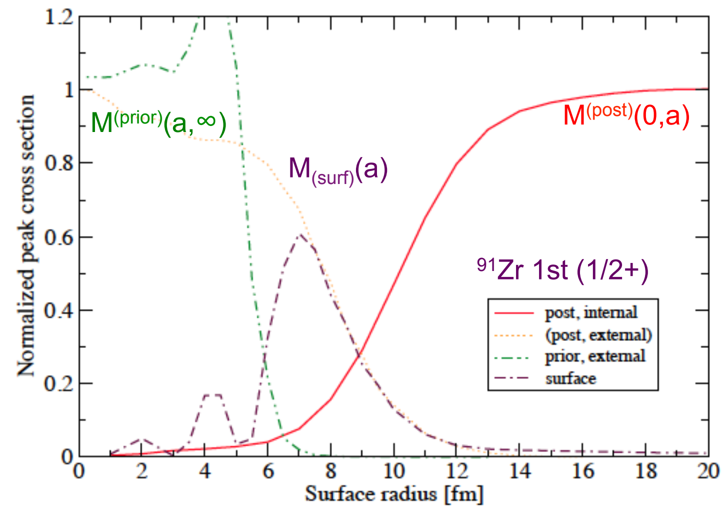
This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1^{st} excited state (1/2+) ⟵ bound
  - $2f_{7/2}$ resonance

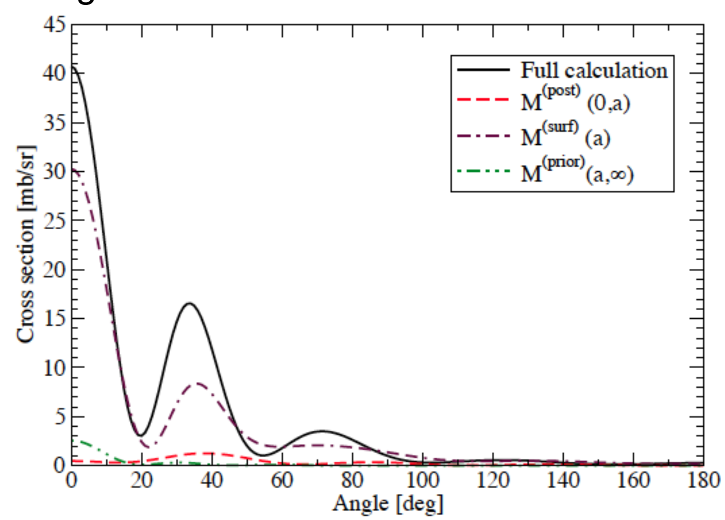
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation



Angular cross sections for a=7 fm



Assessing the R-matrix ideas IIc

<u>2. Surface contribution</u>

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

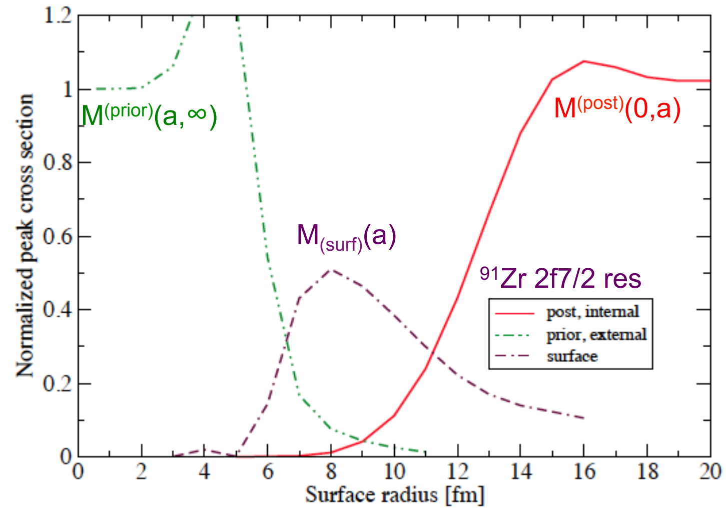
This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance ← resonance

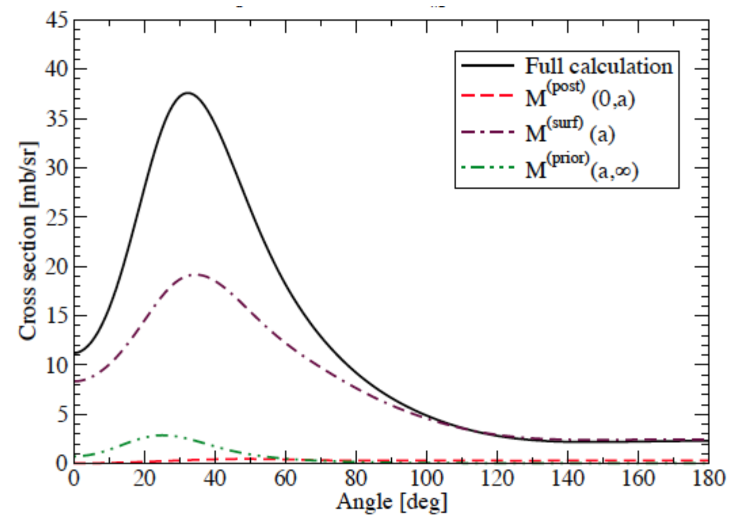
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external
- Reduced interior contribution at peak for surface term

Peak cross section relative to full calculation



Angular cross sections for a = 8 fm



Assessing the R-matrix ideas - ^{48}Ca

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

This case:

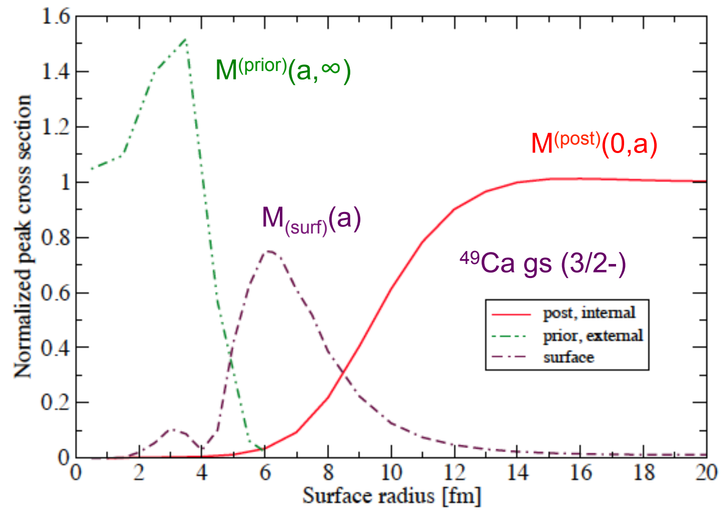
- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs (3/2-)
  - 1st excited state (1/2-)

← bound

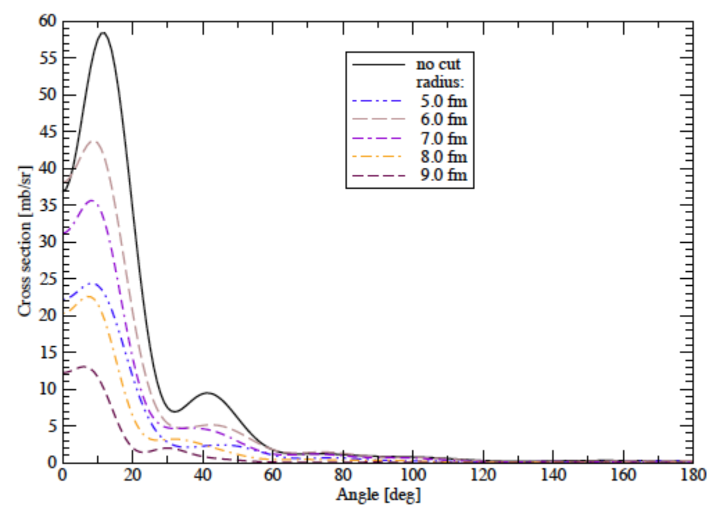
Observations

- Surface term indeed dominant 5-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external
- Cross section calculated from surface term sensitive to surface radius

Peak cross section relative to full calculation



Surface angular cross sections for various a values



Assessing the R-matrix ideas - ^{48}Ca

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

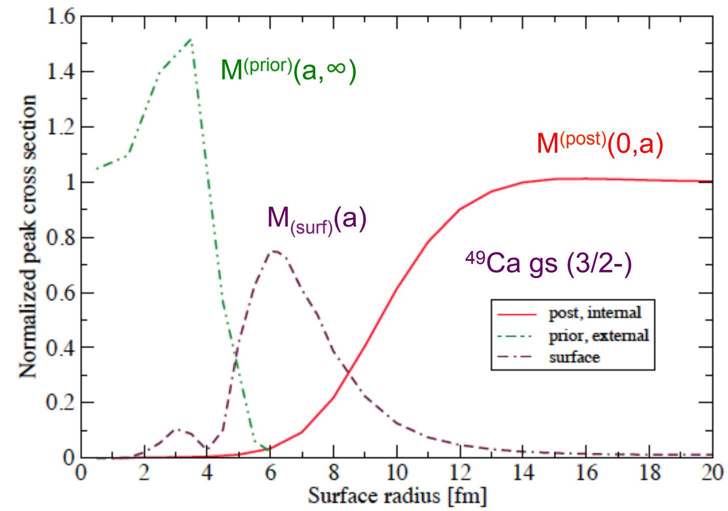
This case:

- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs (3/2-)
  - 1st excited state (1/2-) ← bound

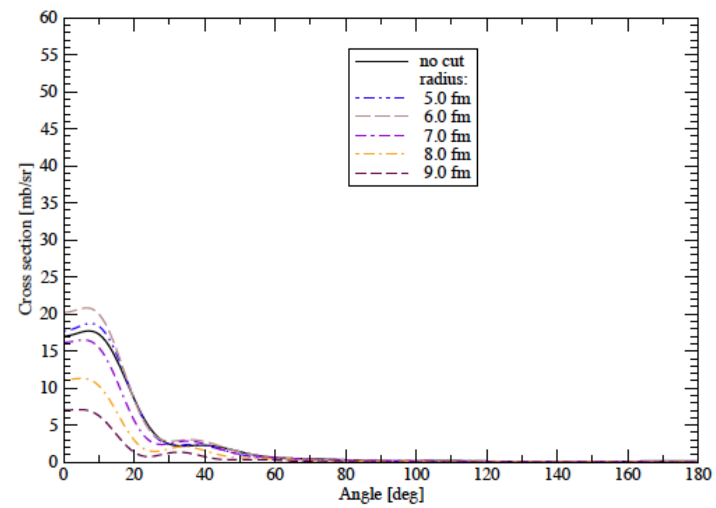
Observations

- Surface term indeed dominant 5-7 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external
- Cross section calculated from surface term sensitive to surface radius

Peak cross section relative to full calculation



Surface angular cross sections for various a values



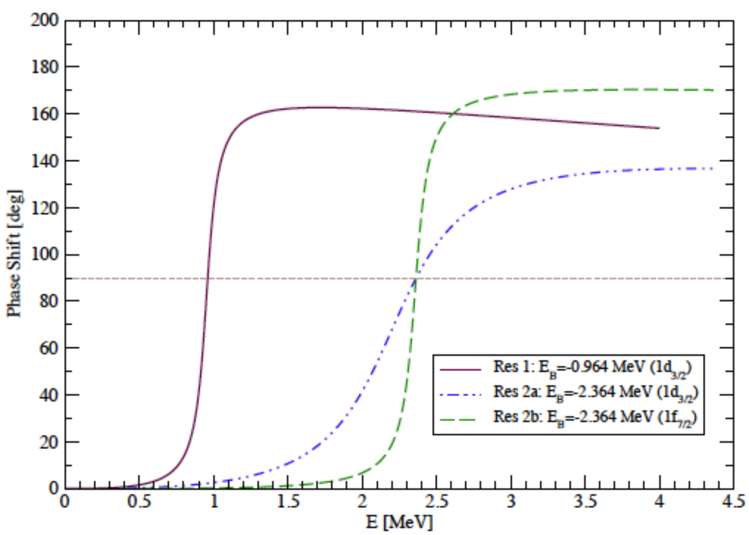
Assessing the R-matrix ideas – ^{20}O

<u>In progress</u>
Data for both bound and resonance states available!

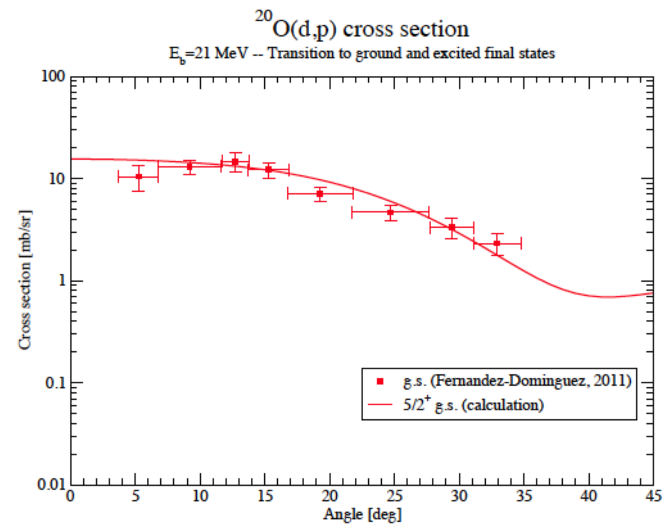
This case:

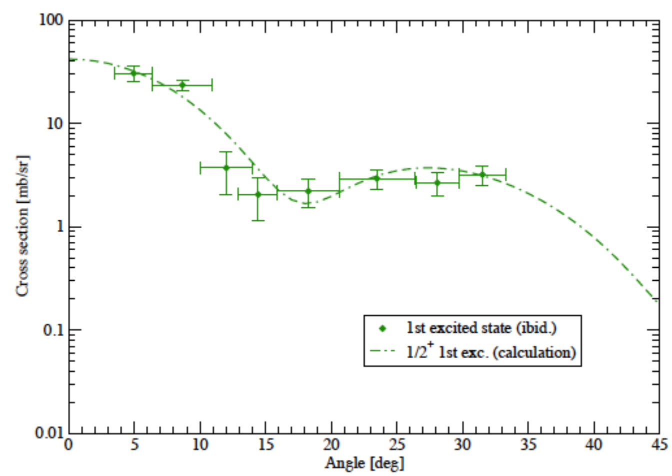
- $^{20}O(d,p)$ for E_d=21 MeV
  - ^{20}O gs (3/2-) ⟵ bound
  - 1st excited state (1/2-) ⟵ bound
  - $1d_{3/2}$ and $1f_{7/2}$ resonances ⟵ resonance

Phase shifts for resonances considered



Cross sections for bound states compared to data





Next: Extension of the formalism to include breakup

DWBA matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

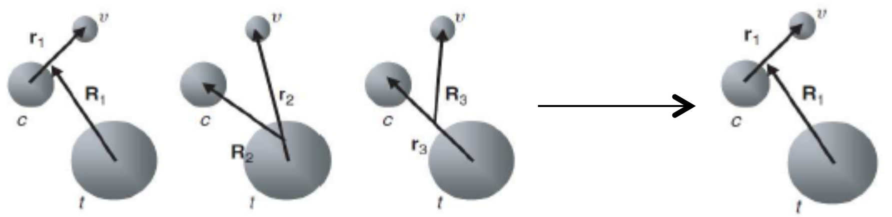
CDCC matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a)$$

$M^{(prior)}(a,\infty) = 0$ (is included in breakup)

CDCC (Continum-discretized coupled channels)

- Approximate treatment of 3-body problem
- Describes breakup of deuteron



- Successfully used for describing data
- Currently revisited via comparison with Fadeev

CDCC extension of R-matrix formalism

- Simultaneous calculation of breakup and transfer cross sections
- Exterior term included in breakup, convergence issues removed
- More peripheral, reduce interior contribution
- Surface term dominant

Conclusions

Studying resonances with (d,p):

- Already underway at RIB facilities
- Conceptual and practical problems have to be overcome

New formalism:

- Builds on ideas from successful R-matrix approach
- Separation into interior and exterior regions works formally well, surface term emerges as important contributor, can be expressed in terms of familiar R-matrix parameters -> meaningful spectrosopic information
- Test cases show that the surface term is dominant; other contributions may not be negligible, but resonances less affected by interior contributions
- Including breakup via CDCC removes exterior prior contribution, thus eliminates convergence problem for resonances

Further studies will clarify conditions where the surface formalism will work well.

Promising approach for transfers to resonances.

TORUS Collaboration

ReactionTheory.org

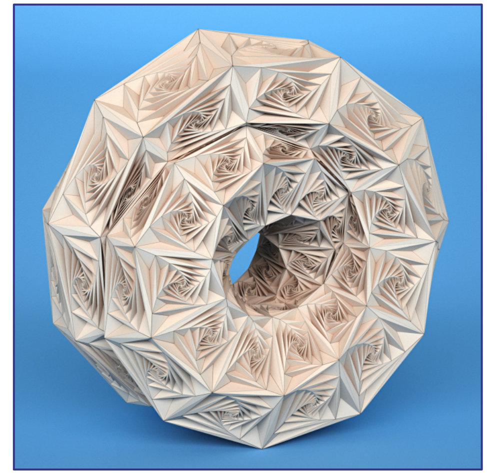
TORUS: Theory of Reactions for Unstable iSotopes
A Topical Collaboration for Nuclear Theory

- Collaboration
- Research Proposal
- Research Papers
- Research Talks
- TORUS internal
- Workshops
- Laboratories
- Experiments
- Site Details

Theory of Reactions for Unstable iSotopes

A Topical Collaboration to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations and by developing a new partial-fusion theory to integrate descriptions of direct and compound-nucleus reactions. This multi-institution collaborative effort is directly relevant to three areas of interest identified in the solicitation: (b) properties of nuclei far from stability; (c) microscopic studies of nuclear input parameters for astrophysics and (e) microscopic nuclear reaction theory.

www.ReactionTheory.org Webmaster: IJT@ReactionTheory.org



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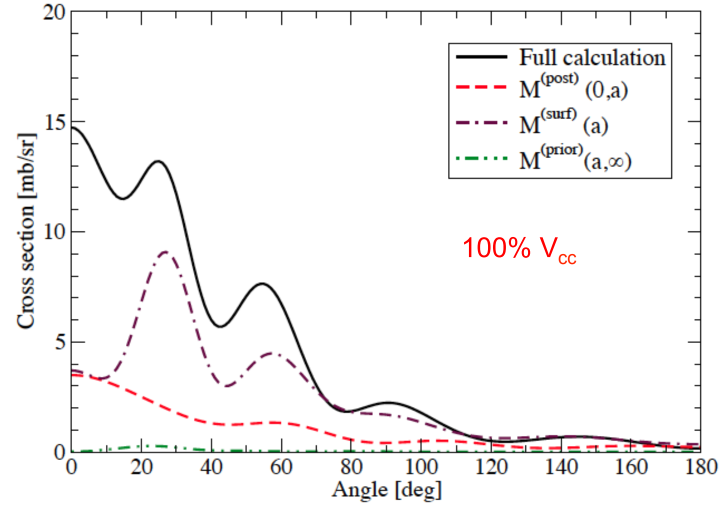
Appendix

Investigating the role of the core-core interaction V_{pA}

3. V_{pA} dependence of the various contributions

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

Angular cross sections for a=7 fm

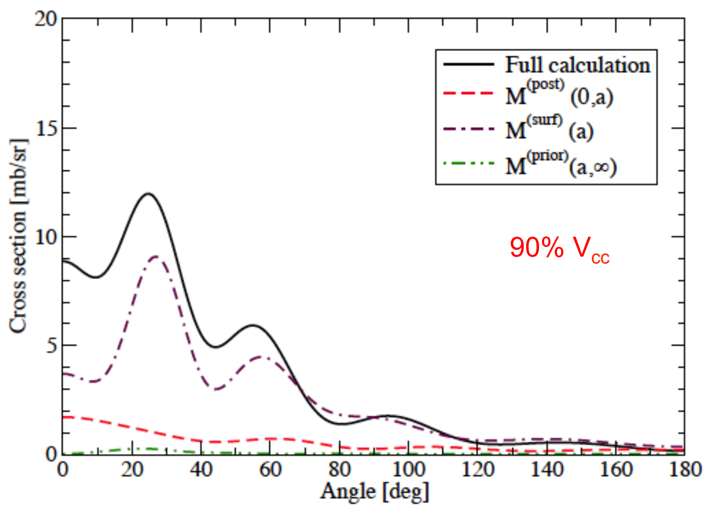


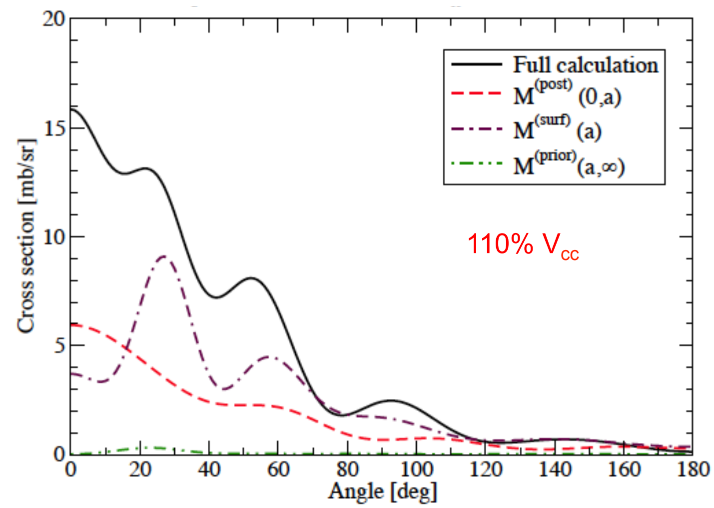
This case:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+) ← bound
  - $2f_{7/2}$ resonance

Observations

- Overall cross section and relative strength of contributions varies with the strength of the core-core interaction





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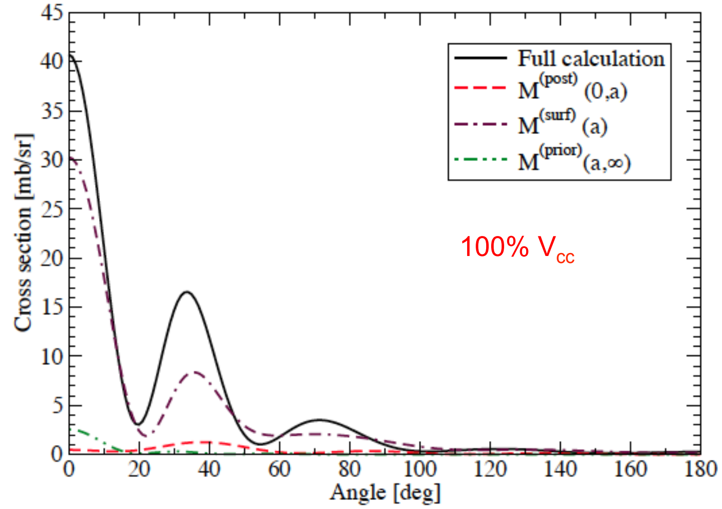
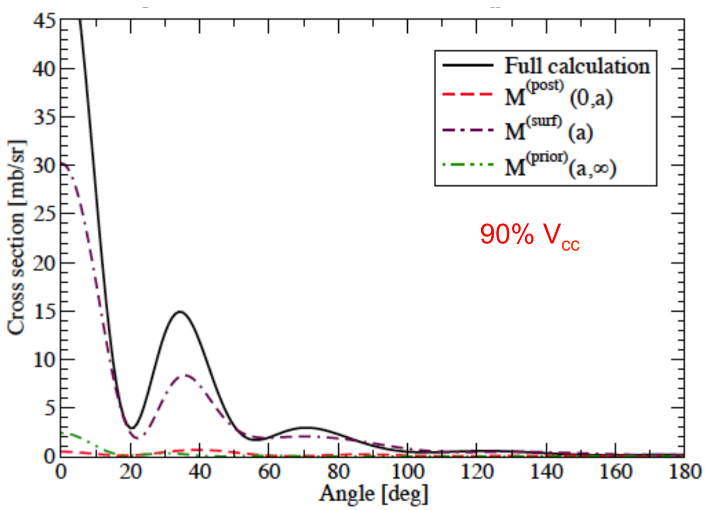
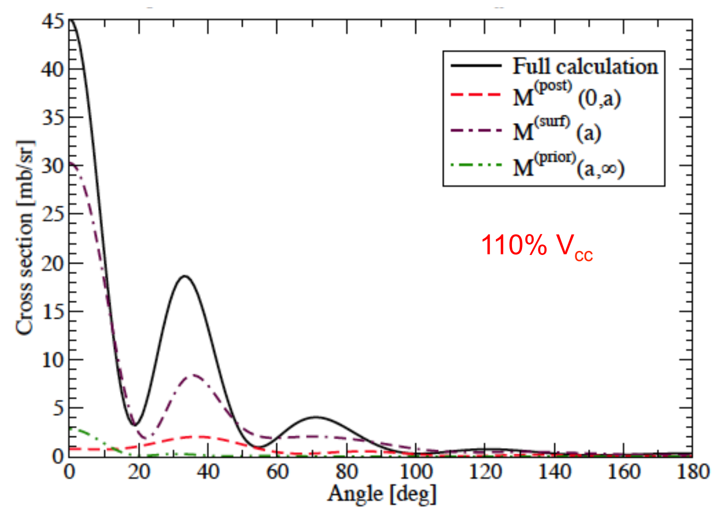






Exploring R-matrix ideas for (d,p) one-nucleon transfers II

Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(post)} = \langle \Phi_f^{(-)} \,|\, \Delta V_{pF} \,|\, \Psi_i^{(+)} \rangle$$

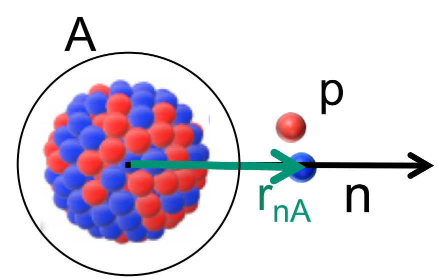
DWBA

$$\langle \varphi_F \chi_{pF}^{(-)} \,|\, \Delta V_{pF} \,|\, \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \,|\, \Delta V_{pF} \,|\, \varphi_d \chi_{dA}^{(+)} \rangle$$

3-body ≈ CDCC

3-body system



$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = \langle \varphi_A | \varphi_F \rangle = I_A^F (r_{nA})$

One-body overlap of A and A+1 systems

- carries structure information
- typically approximated by single-particle function

$$M^{(prior)} = \langle \Psi_f^{(-)} \,|\, \Delta V_{dA} \,|\, \Phi_i^{(+)} \rangle$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Assessing the R-matrix ideas - ^{48}Ca

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

This case:

- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs (3/2-) ← bound
  - 1st excited state (1/2-) ← bound

Observations

- Surface term indeed dominant 5-7 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation