Exploring R-matrix ideas for the description of one-nucleon transfer reactions

Jutta Escher, Ian Thompson (LLNL)
Akram Mukhamedzhanov (Texas A&M)
TORUS Collaboration

NHEP Section talk

May 30, 2012

Lawrence Livermore National Laboratory

LLNL-PRES-XXXXXX

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC

TORUS Collaboration

ReactionTheory.org

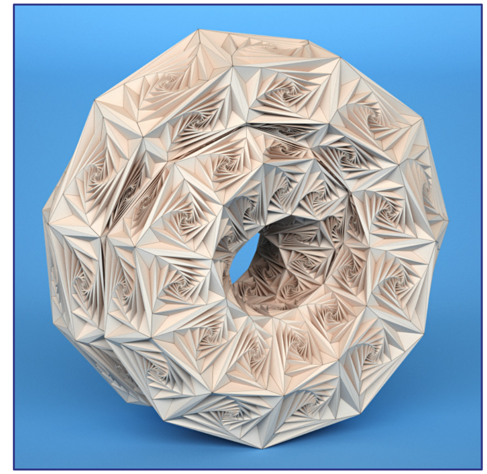
TORUS: Theory of Reactions for Unstable iSotopes
A Topical Collaboration for Nuclear Theory

- Collaboration
- Research Proposal
- Research Papers
- Research Talks
- TORUS internal
- Workshops
- Laboratories
- Experiments
- Site Details

Theory of Reactions for Unstable iSotopes

A Topical Collaboration to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations and by developing a new partial-fusion theory to integrate descriptions of direct and compound-nucleus reactions. This multi-institution collaborative effort is directly relevant to three areas of interest identified in the solicitation: (b) properties of nuclei far from stability; (c) microscopic studies of nuclear input parameters for astrophysics and (e) microscopic nuclear reaction theory.

www.ReactionTheory.org   Webmaster: IJT@ReactionTheory.org



©Torolf Sauermann

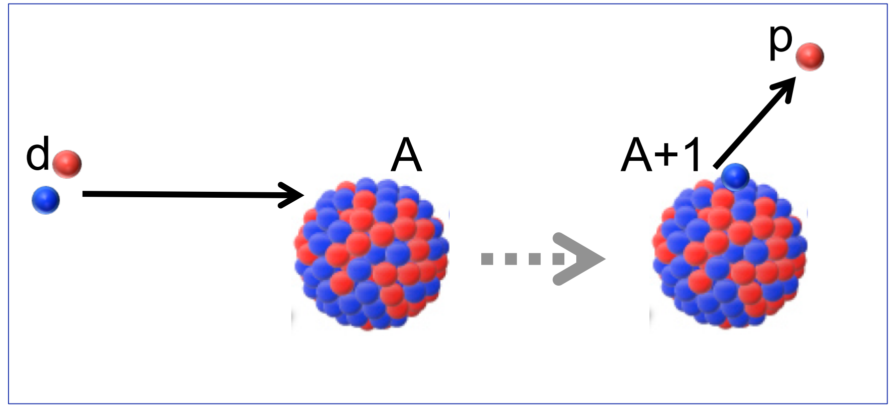
TORUS members

Ian Thompson, LLNL
Jutta Escher, LLNL
Filomena Nunes, MSU
Neelam Upadhyay (PD)
Akram Mukhamedzhanov (TAMU)
V. Eremenko (PD)
Charlotte Elster (OU)
Goran Arbanas (ORNL)

Studying nuclear structure with (d,p) one-nucleon transfers

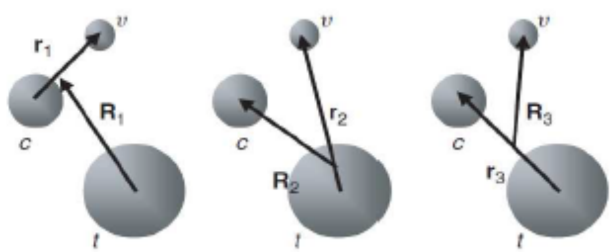
(d,p) reactions:

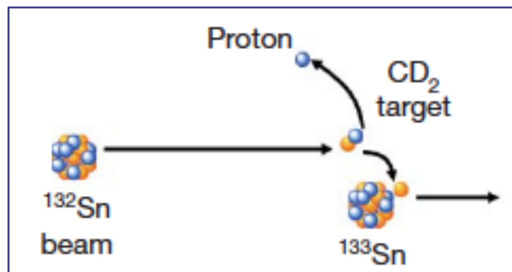
- Simplest mechanism for adding a neutron
- Traditionally used to study stable nuclei
- Used in inverse kinematics at RIB facilities, for studying weakly-bound systems



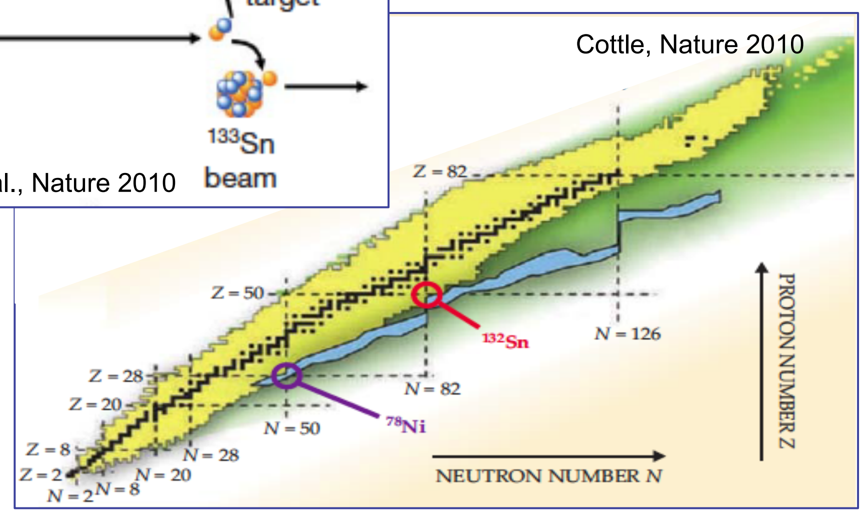
Theoretical descriptions of (d,p) reactions:

- Progress over the years: Plane-wave theory, DWBA (zero-range & finite-range), coupled-channels approach, breakup, etc.





Jones et al., Nature 2010



Cottle, Nature 2010

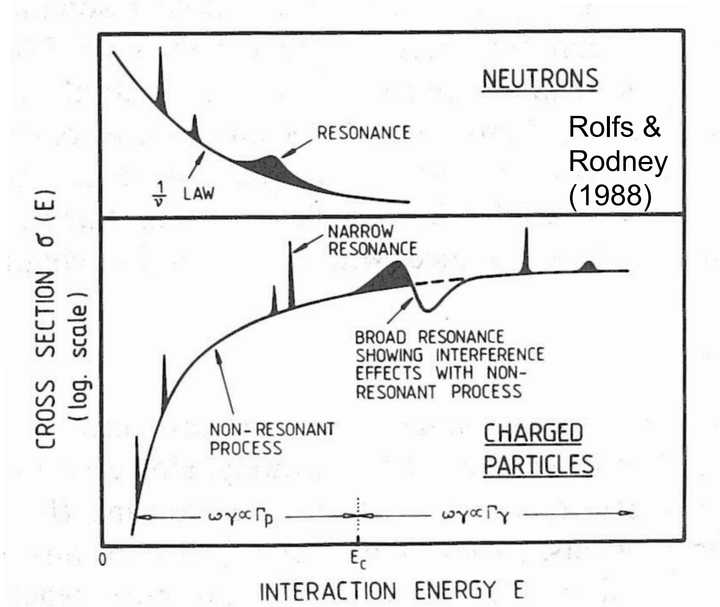
Current status of (d,p) direct-reaction theories:

- Developing Fadeev techniques to better account for 3-body effects (TORUS collab.)
- Conceptual work needed: rethinking spectroscopic factors
- Not very useful for transfers to resonance states.

Resonances in low-energy nuclear physics

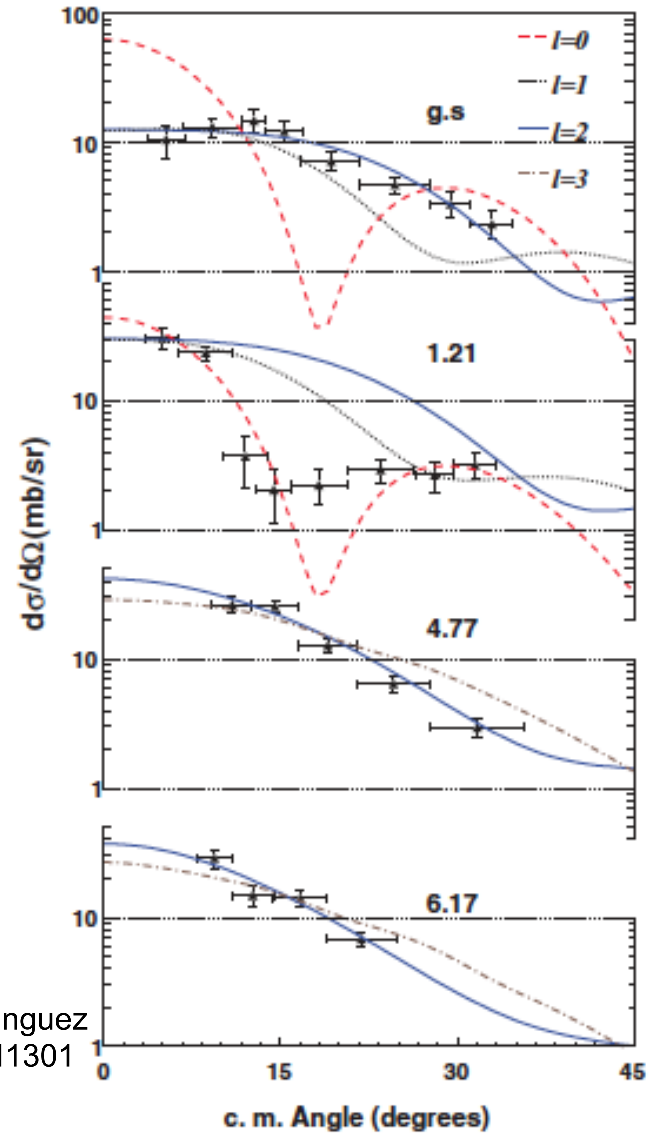
Resonances:

- Unstable quantum-mechanical states
- Occur in light, medium-mass, and heavy nuclei
- Crucially affect astrophysical reaction rates
- Abundant in weakly-bound nuclei



Rolfs & Rodney (1988)

$^{20}O(d,p)^{21}O$ inverse-kinematics experiment at GANIL to determine N=16 shell gap



Fernandez-Dominguez et al, PRC 84, 011301 (R) (2011)

Problems in applying standard method to resonances:

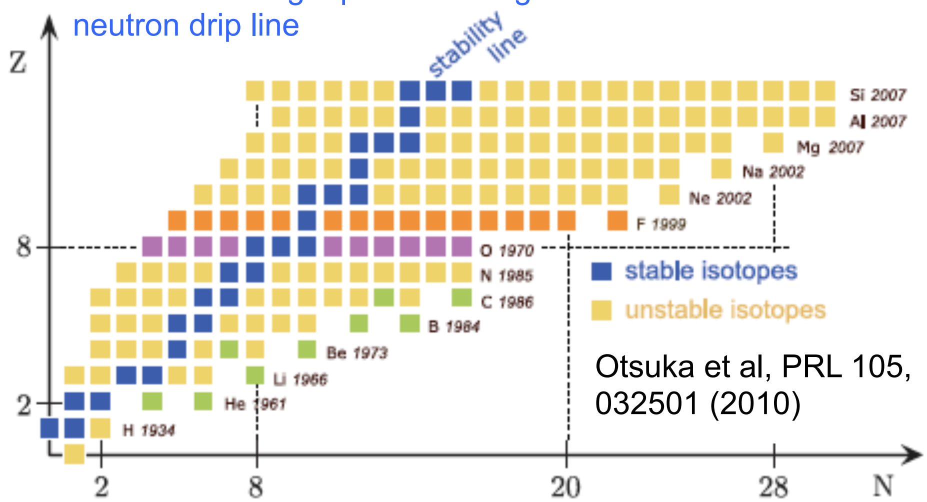
- Conceptual: meaning of spectroscopic factor?
- Practical: convergence issues

Resonances in low-energy nuclear physics

Resonances:

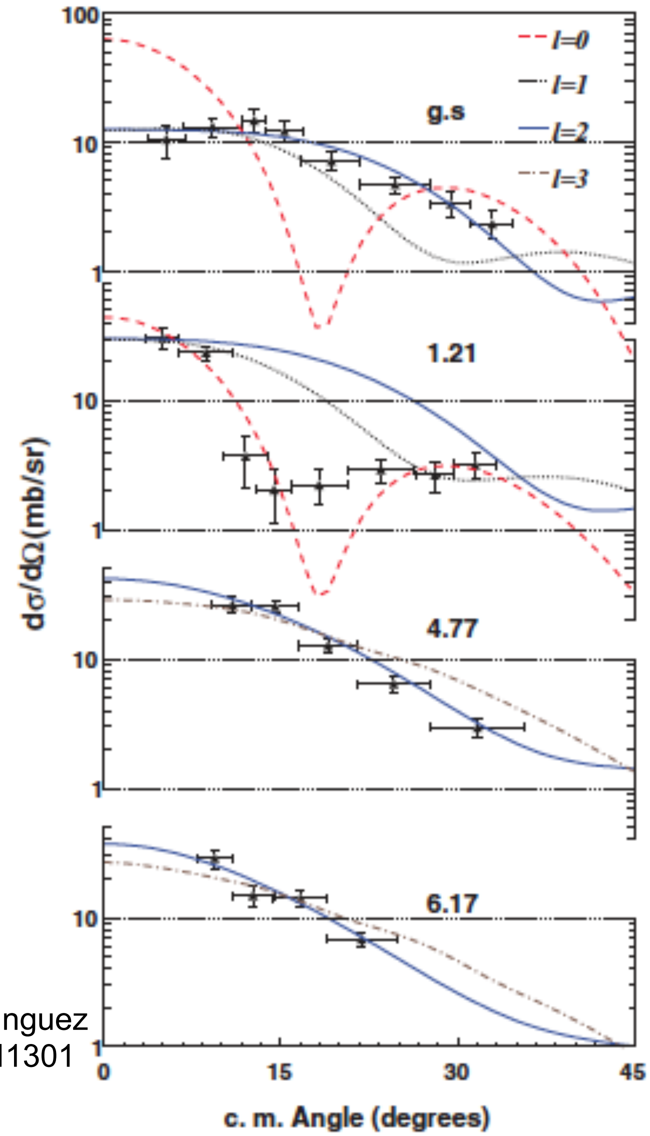
- Unstable quantum-mechanical states
- Occur in light, medium-mass, and heavy nuclei
- Crucially affect astrophysical reaction rates
- Abundant in weakly-bound nuclei

Evolution of single-particle energies and the location of the neutron drip line



Otsuka et al, PRL 105, 032501 (2010)

$^{20}O(d,p)^{21}O$ inverse-kinematics experiment at GANIL to determine N=16 shell gap



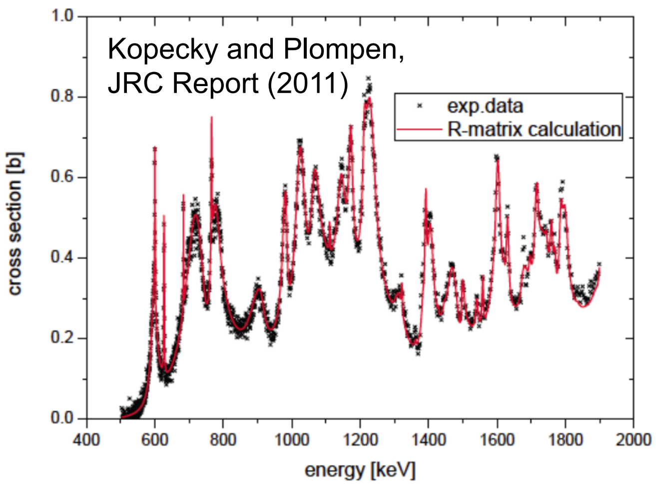
Fernandez-Dominguez et al, PRC 84, 011301 (R) (2011)

Problems in applying standard method to resonances:

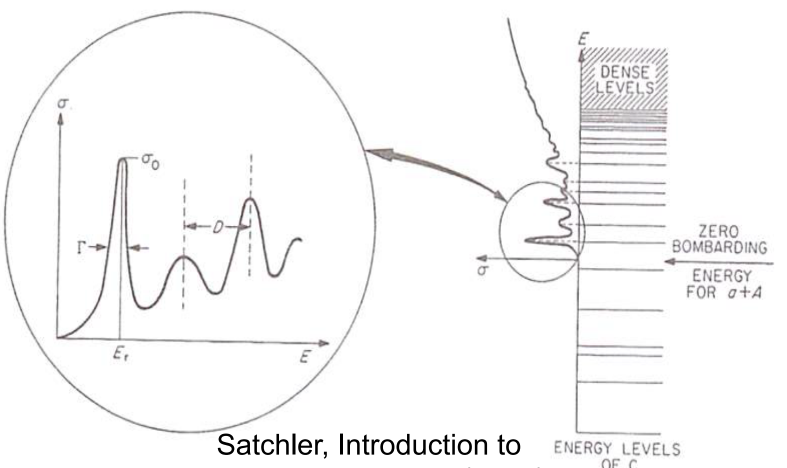
- Conceptual: meaning of spectroscopic factor?
- Practical: convergence issues

Describing resonances in binary reactions

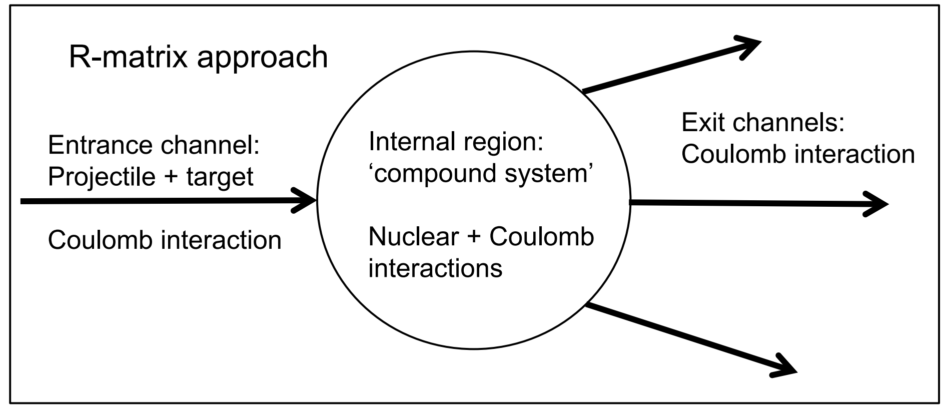
Experimental studies of resonances:
- Elastic & inelastic scattering, capture, etc.



- Characterization of resonances: position & widths



Satchler, Introduction to Nuclear Reactions (1980)



R-matrix approach:

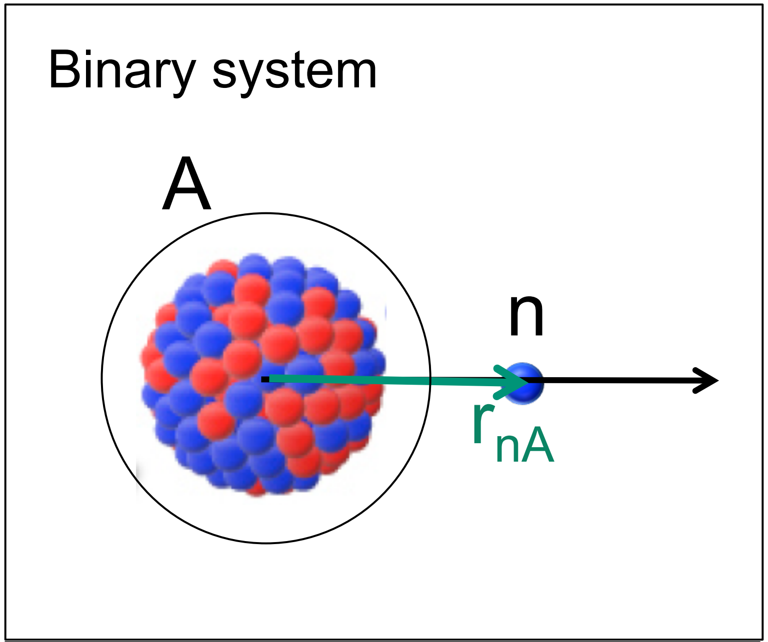
- Main idea: divide space into 2 regions:
  - $r \leq a$ -- interior: nuclear and Coulomb interactions
  - $r > a$ -- exterior: Coulomb only
- Formalism:
  - Interior: set of basis functions to express nuclear wave function
  - Exterior: scattering wave function
  - Surface: matching conditions allow to parameterize collision matrix -> expressions for cross sections
- Connect observed parameters (E_R, Γ) to formal parameters ($\check{E}_R$, γ^2)
- Typical applications adjust parameters to reproduce measured cross sections

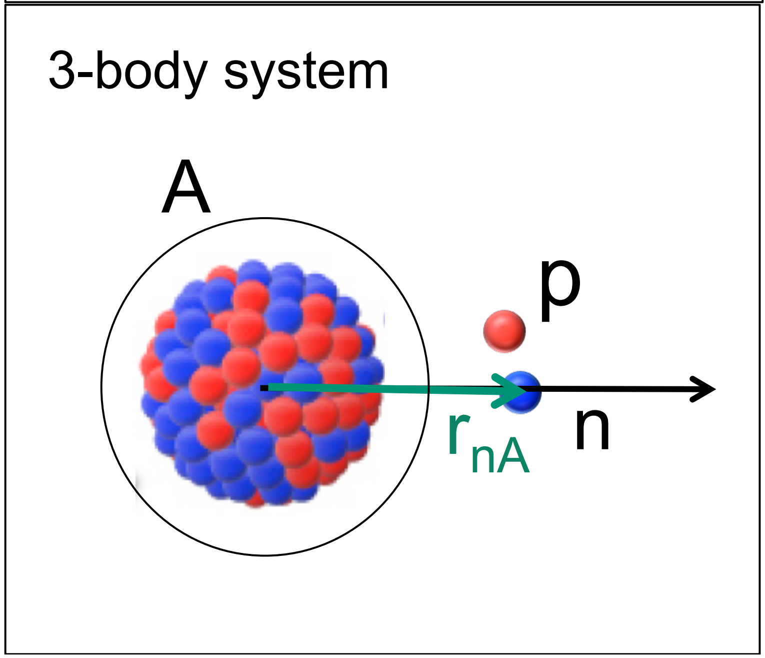
Exploring R-matrix ideas for (d,p) one-nucleon transfers

Proposed new formalism (Mukhamedzhanov, 2011):

- R-matrix concepts:
  - surface separating internal and external regions
  - cross sections expressed in terms of reduced widths, logarithmic derivatives, surface radii
- Applicable to stripping to bound and resonance states
- Provides conceptually improved way to describe (d,p) transfer reactions
- General enough to include deuteron breakup contributions via CDCC
- Resolves practical issues related to numerical convergence

Formalism:
Mukhamedzhanov, PRC 84, 044616 (2011)



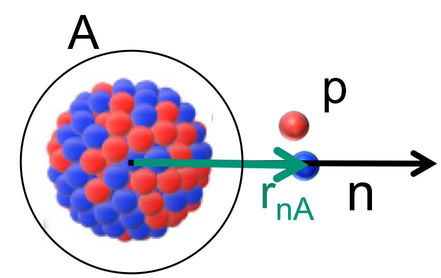


Exploring R-matrix ideas for (d,p) one-nucleon transfers II

Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

3-body system



$$M^{(post)} = < \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} >$$

DWBA

$$< \varphi_F \chi_{pF}^{(-)} \mid \Delta V_{pF} \mid \varphi_d \varphi_A \chi_{dA}^{(+)} >$$

$$< I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_d \chi_{dA}^{(+)} >$$

3-body

$$< \varphi_F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_A \Psi_i^{3B(+)} >$$

$$< I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{3B(+)} >$$

CDCC

$$< I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{CDCC(+)} >$$

$\Psi_i^{(+)}$: exact d+A scattering function

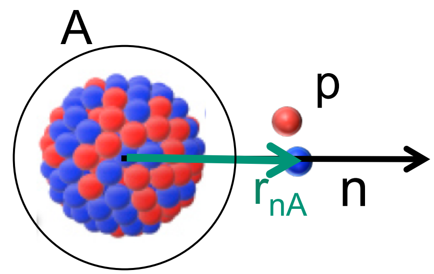
$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = < \varphi_A \mid \varphi_F >$ one-body overlap

Exploring R-matrix ideas for (d,p) one-nucleon transfers II

3-body system



Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(post)} = < \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} >$$

<u>DWBA</u>

$$< \varphi_F \, \chi_{pF}^{(-)} \mid \Delta V_{pF} \mid \varphi_d \varphi_A \, \chi_{dA}^{(+)} >$$

$$< I_A^F \, \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_d \, \chi_{dA}^{(+)} >$$

<u>3-body</u>

$$< \varphi_F \, \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_A \, \Psi_i^{3B(+)} >$$

$$< I_A^F \, \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{3B(+)} >$$

<u>CDCC</u>

$$< I_A^F \, \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{CDCC(+)} >$$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \, \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = < \varphi_A \mid \varphi_F >$ one-body overlap

$$M^{(prior)} = < \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} >$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Generalized R-matrix formalism for (d,p) reactions I

Splitting the transition matrix element M:

- Interior and exterior with respect to r_{nA}

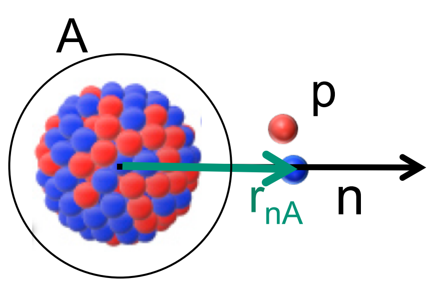
$$M^{(post)} = \langle \Phi_f^{(-)} \,|\, \Delta V_{pF} \,|\, \Psi_i^{(+)} \rangle$$

DWBA

$$\langle \varphi_F \chi_{pF}^{(-)} \,|\, \Delta V_{pF} \,|\, \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \varphi_d \chi_{dA}^{(+)} \rangle$$

3-body system



Interior + exterior

$$M^{(post)} = M^{(post)}(0,a) + M^{(post)}(a,\infty)$$

$$I_A^F = \langle \varphi_A \,|\, \varphi_F \rangle = I_A^F(r_{nA})$$

Mukhamedzhanov

$$M^{(post)}(a,\infty) = M_{surf}(a) + M^{(prior)}(a,\infty)$$

$$M_{surf}(a) = \langle I_A^F \chi_{pF}^{(-)} \,|\, [\overleftarrow{T} - \overrightarrow{T}] \,|\, \varphi_d \chi_{dA}^{(+)} \rangle_{ext}$$

$$\int_{r \leqslant R} \mathrm{d}\mathbf{r}\, f(\mathbf{r})[\overleftarrow{T} - \overrightarrow{T}]g(\mathbf{r})$$
$$= -\frac{1}{2\mu} \oint_{r=R} \mathrm{d}\mathbf{S}[g(\mathbf{r})\nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r})\nabla_{\mathbf{r}} g(\mathbf{r})]$$
$$= -\frac{1}{2\mu} R^2 \int \mathrm{d}\Omega_{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right]_{r=R}$$

Surface term

$M_{surf}(a) = f(a, C_A^F, B_{nA})$

B_{nA} = log derivative of I_A^F at surface radius a

ANC: C_A^F defined through: $I_A^F(r_{nA}) \rightarrow C_A^F\, W(kr_{nA})$

related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

Generalized R-matrix formalism for (d,p) reactions II

DWBA matrix element

$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$

$$M_{(surf)}(a) = \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \sum_{j_{nA} m_{j_{nA}} m_{l_{nA}} M_n} \langle J_A M_A j_{nA} m_{j_{nA}} | J_F M_F \rangle \langle J_n M_n l_{nA} m_{l_{nA}} | j_{nA} m_{j_{nA}} \rangle \langle J_p M_p J_n M_n | J_d M_d \rangle \gamma_{nA j_{nA} l_{nA}}$$
$$\times \int d\mathbf{r}_{pF}\, \chi^{(+)}_{-\mathbf{k}_{pF}}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{nA}}\, Y^{*}_{l_{nA} m_{l_{nA}}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA}) (B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi^{(+)}_{\mathbf{k}_{dA}}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA}=R_{nA}}$$

Assessing the approach:

- Separation into internal and external regions sensible?
- Is the surface term dominant and what is the size of the corrections?
- Study cross sections arising from different terms
- Start with DWBA and bound states
- Investigate resonances

Cases considered so far:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs, 1st excited state, $2f_{7/2}$ resonance
- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs, 1st excited state
- ^{12}C(d,p) for E_d=30 MeV
- ^{40}Ca(d,p) for E_d=34.4 MeV
- ^{209}Pb(d,p) for E_d=52 MeV
- Planned: ^{48}Ca(d,p) for E_d=19.3 and 56 MeV

Assessing the R-matrix ideas Ia

1. Interior vs exterior contributions

$$M = M(0,a) + M(a,\infty)$$

This case:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs (5/2+) ←
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance

Observations

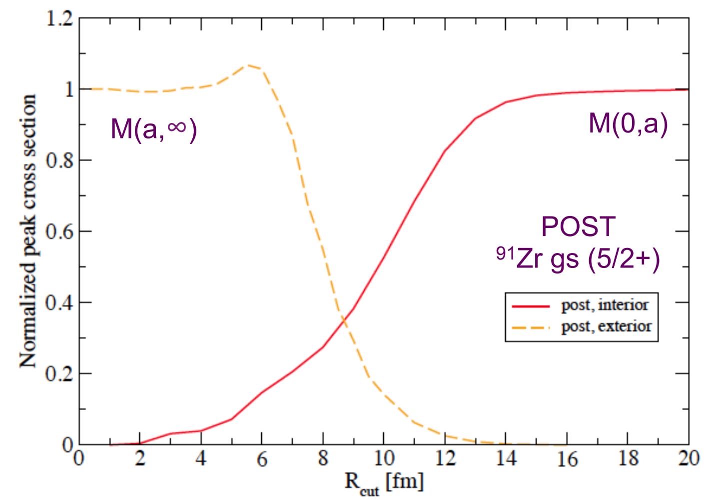
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

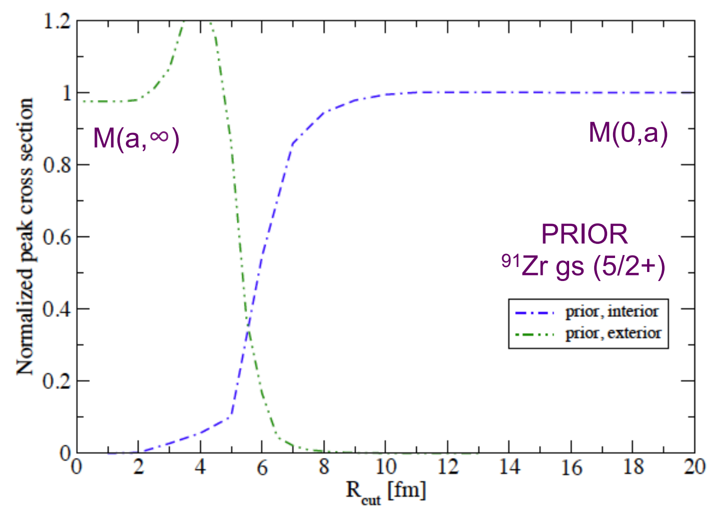
$$M^{(post)} = < \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} >$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = < \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} >$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

- Exterior contributions: Post requires contributions further out; also an issue for resonance calculations

Peak cross section relative to full calculation





Assessing the R-matrix ideas Ib

1. Interior vs exterior contributions

$M = M(0,a) + M(a,\infty)$

This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1^{st} excited state (1/2+) ←
  - $2f_{7/2}$ resonance

Observations

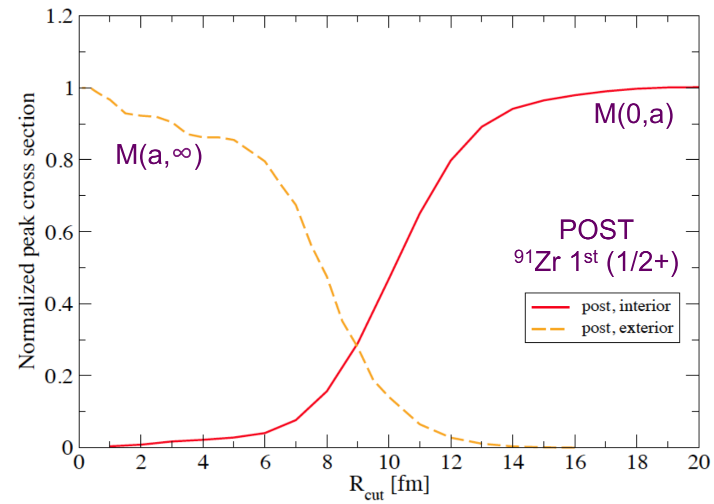
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

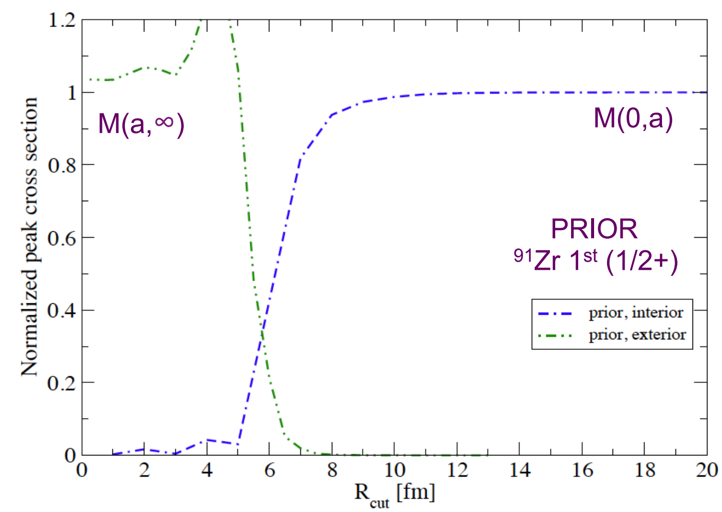
$$M^{(post)} = < \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} >$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = < \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} >$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

- Exterior contributions: Post requires contributions further out; also an issue for resonance calculations

Peak cross section relative to full calculation





Assessing the R-matrix ideas Ic

1. Interior vs exterior contributions

$M = M(0,a) + M(a,\infty)$

This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance ←

Observations

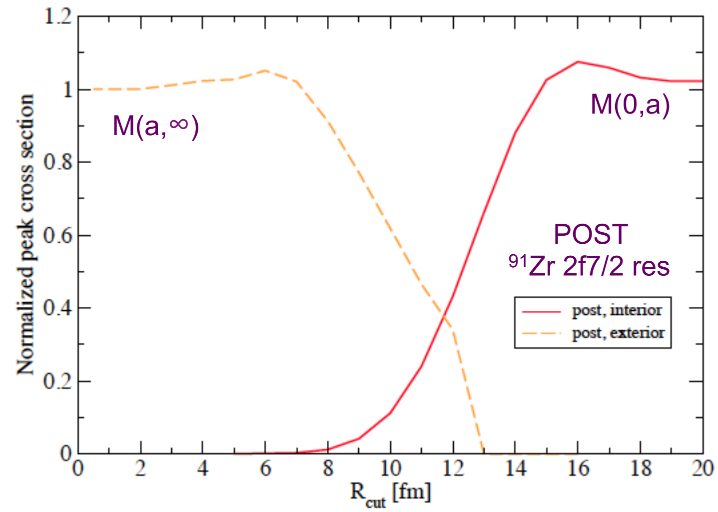
- 'action is in the nuclear surface'
- Post formalism more sensitive to larger radii than prior:

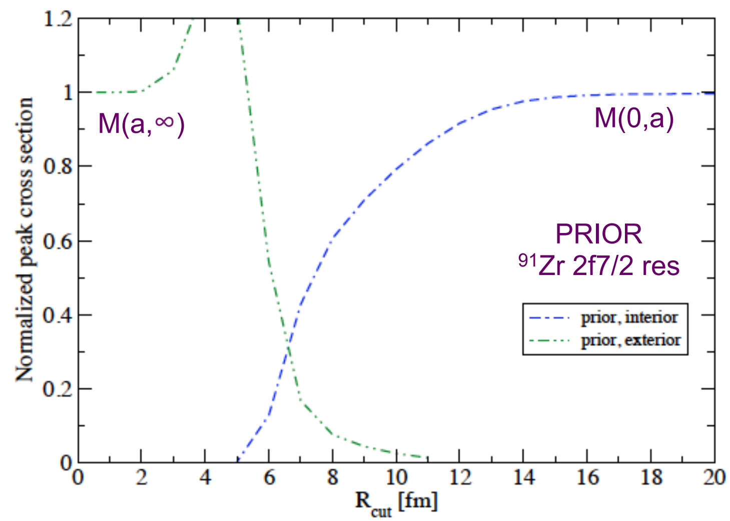
$$M^{(post)} = \langle \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} \rangle$$
$$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$$

$$M^{(prior)} = \langle \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} \rangle$$
$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

- Exterior contributions: Post requires contributions further out; also an issue for resonance calculations

Peak cross section relative to full calculation





Assessing the R-matrix ideas IIa

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

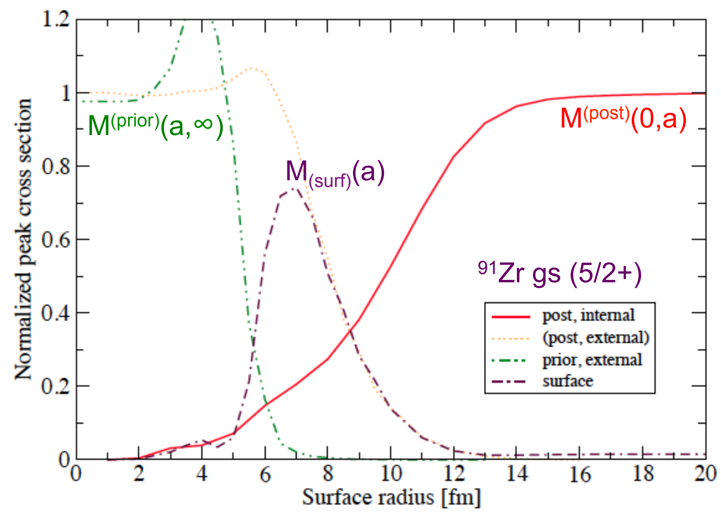
This case:

- ^{90}Zr(d,p) for E_d=11 MeV
  - ^{91}Zr gs (5/2+) ←
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance

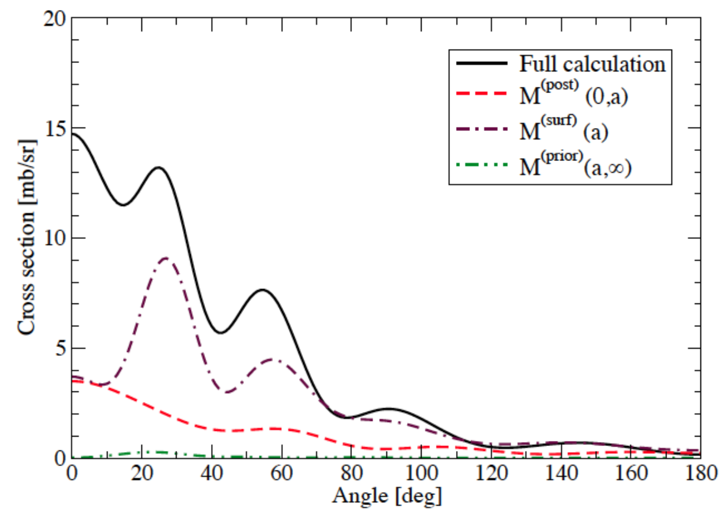
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation



Angular differential cross sections



Assessing the R-matrix ideas IIb

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

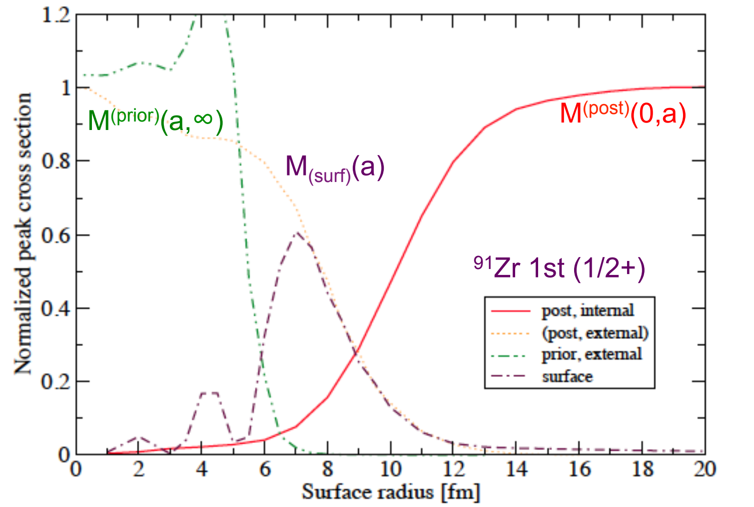
This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+) ←
  - $2f_{7/2}$ resonance

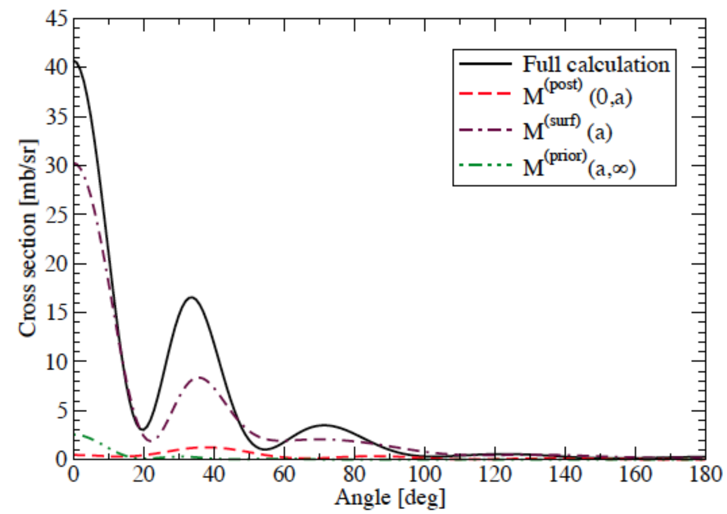
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation



Angular differential cross sections



Assessing the R-matrix ideas IIc

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

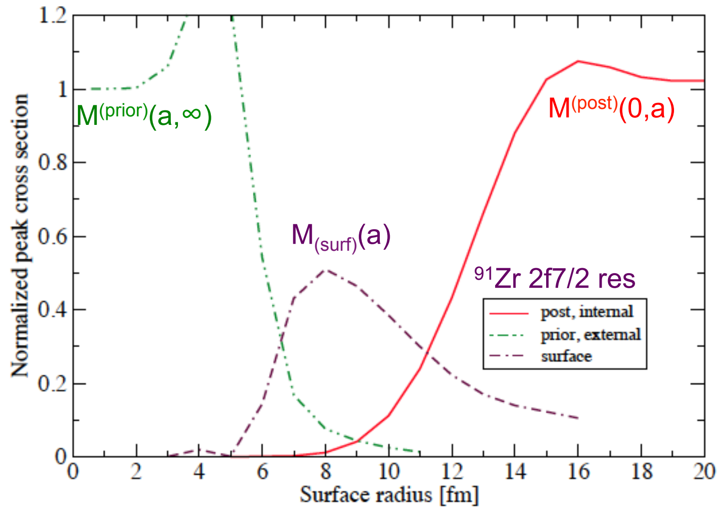
This case:

- $^{90}Zr(d,p)$ for E_d=11 MeV
  - ^{91}Zr gs (5/2+)
  - 1st excited state (1/2+)
  - $2f_{7/2}$ resonance ←

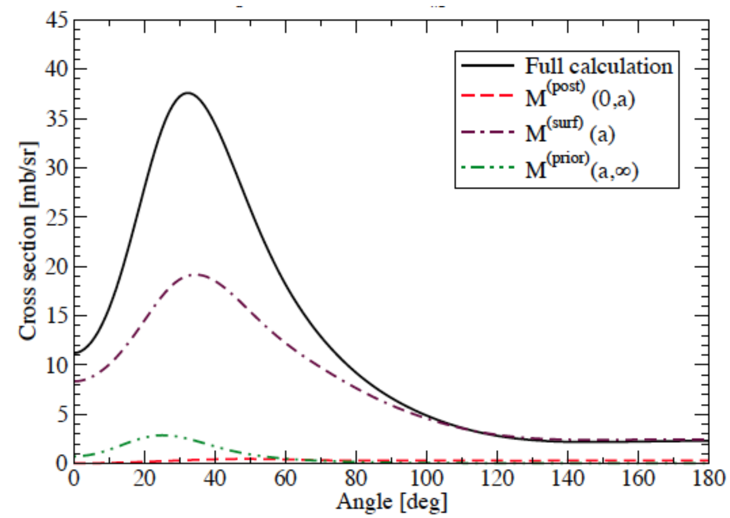
Observations

- Surface term indeed dominant 6-8 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation



Angular differential cross sections



Assessing the R-matrix ideas - ^{48}Ca

2. Surface contribution

$$M = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

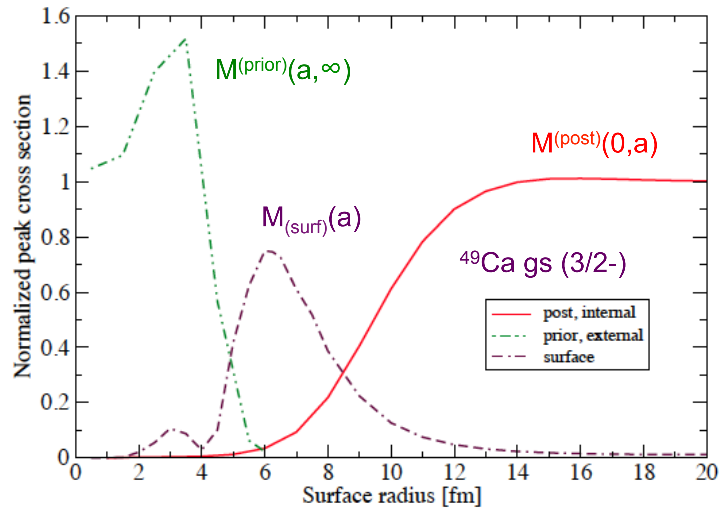
This case:

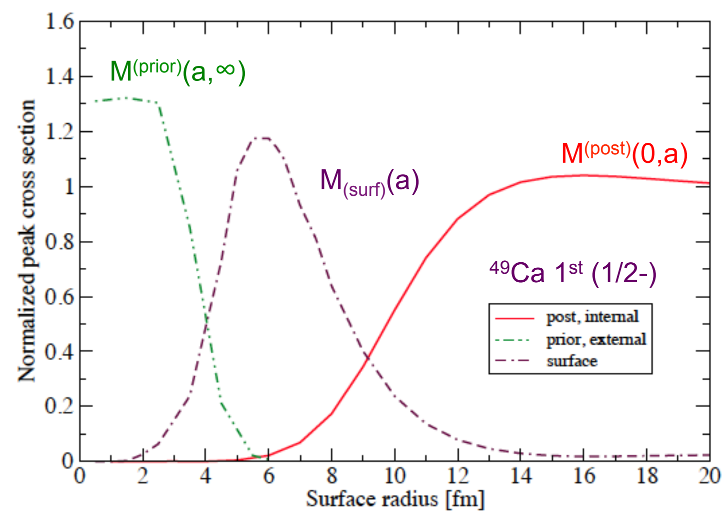
- ^{48}Ca(d,p) for E_d=13 MeV
  - ^{49}Ca gs (3/2-) ←
  - 1st excited state (1/2-) ←

Observations

- Surface term indeed dominant 5-7 fm
- Small interior contributions → little dependence on model for interior
- Small exterior contributions → better convergence for resonance case
- Surface term does not produce the whole cross section, corrections required from internal/external

Peak cross section relative to full calculation





Next: Extension of the formalism to include breakup

DWBA matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a) + M^{(prior)}(a,\infty)$$

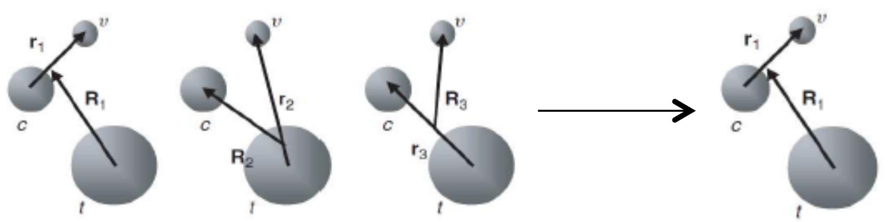
CDCC matrix element

$$M^{(post)} = M^{(post)}(0,a) + M_{(surf)}(a)$$

$M^{(prior)}(a,\infty) = 0$ (is included in breakup)

CDCC (Continum-discretized coupled channels)

- Approximate treatment of 3-body problem
- Describes breakup of deuteron



- Successfully used for describing data
- Currently revisited via comparison with Fadeev
- To be studied in connection with R-matrix approach for (d,p) to resonances

Expectation for CDCC extension

- Simultaneous calculation of breakup and transfer cross sections
- Surface term dominant
- Good convergence

Conclusions

(d,p) reactions:

- Important for nuclear structure studies and astrophysics
- New experimental techniques for radioactive isotopes
- Improved theoretical descriptions required

Studying resonances with (d,p):

- Conceptual and practical problems have to be overcome

New formalism:

- Builds on ideas from successful R-matrix approach
- Separation into interior and exterior regions works formally well, surface term emerges as important contributor, can be expressed in terms of familiar R-matrix parameters
- Test cases show that the surface term is dominant, but other contributions may not be negligible
- Including breakup via CDCC removes exterior prior contribution, thus eliminates convergence problem for resonances

Further studies needed to clarify conditions where the surface formalism will work well.