Using R-matrix ideas to describe one-nucleon transfers to resonance states

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Present theories provide valuable information on angular momenta …but have serious limitations in resonance cases

- Problem: present theories rely heavily on one-body overlap function of A and A+1 systems

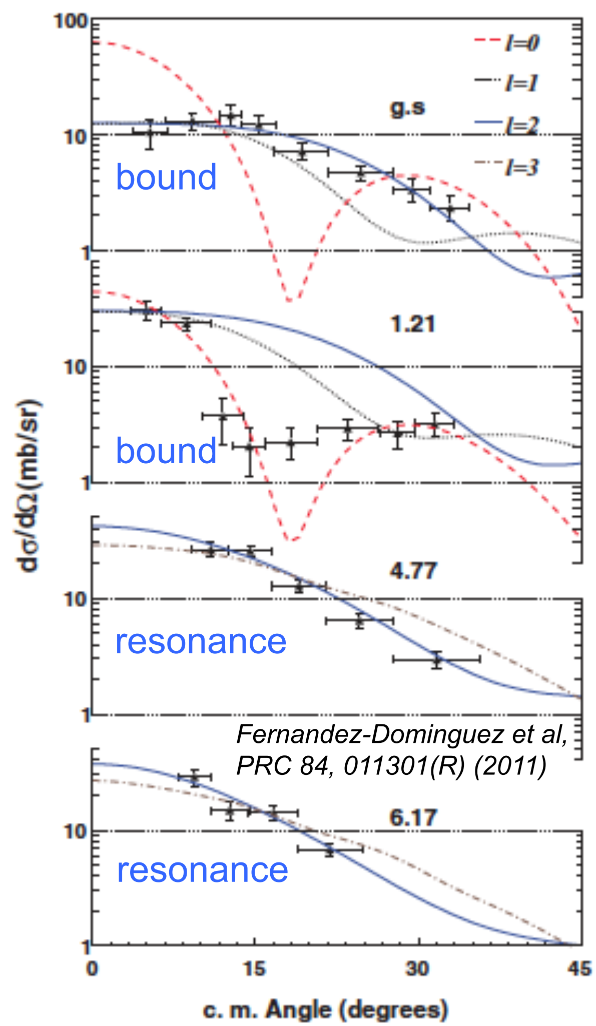
$$I_A^F(r) = \langle \varphi_A | \varphi_F \rangle$$

  - carries structure information
  - not well-known in nuclear interior
  - typically approximated by single-particle function

- Calculations converge very slowly
- Not appropriate for describing reactions involving wide resonances
- Desired resonance properties (energies and widths) cannot be reliably obtained

Suggestion from PRC 84, 044616 (2014): Extend R-matrix description to transfer reactions ➔ `Surface Formalism'

Example: $^{20}O(d,p)^{21}O$ inverse-kinematics experiment - Intepreted the traditional way



Transition matrix element – DWBA and CDCC approximations

Transition matrix element M:

- Connects initial to final wave function
- Cross section $\sigma \sim M^2$

$$M^{(post)} = \langle \Phi_f^{(-)} \mid \Delta V_{pF} \mid \Psi_i^{(+)} \rangle$$

DWBA

$$\langle \varphi_F \chi_{pF}^{(-)} \mid \Delta V_{pF} \mid \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_d \chi_{dA}^{(+)} \rangle$$

3-body

$$\langle \varphi_F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \varphi_A \Psi_i^{3B(+)} \rangle$$

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{3B(+)} \rangle$$

CDCC

$$\langle I_A^F \chi_{pF}^{(-)} \mid \Delta \underline{V}_{pF} \mid \Psi_i^{CDCC(+)} \rangle$$

$\Psi_i^{(+)}$: exact d+A scattering function

$\Phi_f^{(-)} = \varphi_F \chi_{pF}^{(-)}$ exit channel function

$\Delta V_{pF} = V_{pA} + V_{pn} - U_{pF}$

$I_A^F = \langle \varphi_A \mid \varphi_F \rangle$ one-body overlap

$$M^{(prior)} = \langle \Psi_f^{(-)} \mid \Delta V_{dA} \mid \Phi_i^{(+)} \rangle$$

$$\Delta V_{dA} = V_{pA} + V_{nA} - U_{dA}$$

Surface formalism for DWBA

$$M^{(post)} = \langle \Phi_f^{(-)} \,|\, \Delta V_{pF} \,|\, \Psi_i^{(+)} \rangle$$

Transition matrix element M:

- Cross section $\sigma \sim M^2$
- Is split into interior and exterior parts

DWBA

$$\langle I_A^F \chi_{pF}^{(-)} \,|\, \Delta \underline{V}_{pF} \,|\, \varphi_d \chi_{dA}^{(+)} \rangle$$

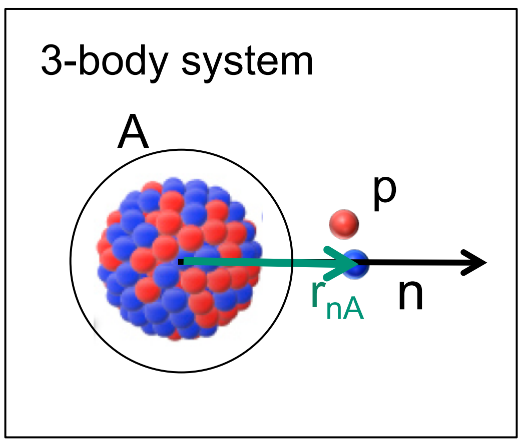
$$I_A^F = \langle \varphi_A \,|\, \varphi_F \rangle = I_A^F(r_{nA})$$

One-body overlap function of A and A+1 systems

Mukhamed-Zhanov, PRC 2011

Interior + exterior

$$M^{(post)} = M^{(post)}(0,a) + \underbrace{M^{(post)}(a,\infty)}_{M_{surf}(a) + M^{(prior)}(a,\infty)}$$



Surface formulation

$$\mathbf{M = \underbrace{M^{(post)}(0,a)}_{\textit{model dependence}} + \underbrace{M_{surf}(a) + M^{(prior)}(a,\infty)}_{\textit{asymptotic quantities}}}$$

$M_{surf}(a) = f(a, C_A^F, B_{nA})$

B_{nA} = log derivative of I_A^F at surface radius a

ANC: C_A^F defined through: $I_A^F(r_{nA}) \rightarrow C_A^F\, W(kr_{nA})$

related to reduced width amplitude $C_A^F \sim \gamma_{nA}$

Internal, surface, external contributions – $^{90}Zr(d,p)$ at E_d=11 MeV

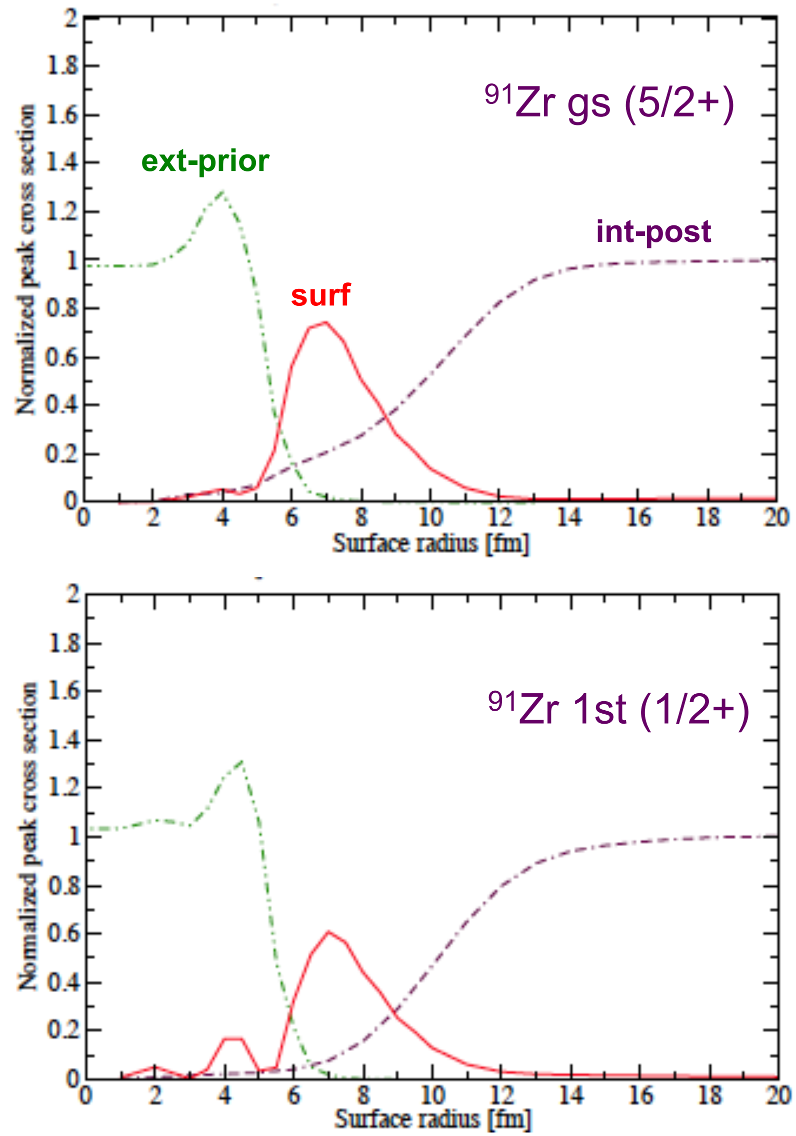
$$M = \underbrace{M^{(post)}(0,a)}_{\textit{model dependence}} + \underbrace{M_{(surf)}(a) + M^{(prior)}(a,\infty)}_{\textit{asymptotic quantities}}$$

Observations

- Surface term dominant at 6-8 fm
- Small interior contributions
- Small exterior contributions
- Surface term does not produce the whole cross section

The surface term is dominant, but contributions from the interior and exterior terms remain.

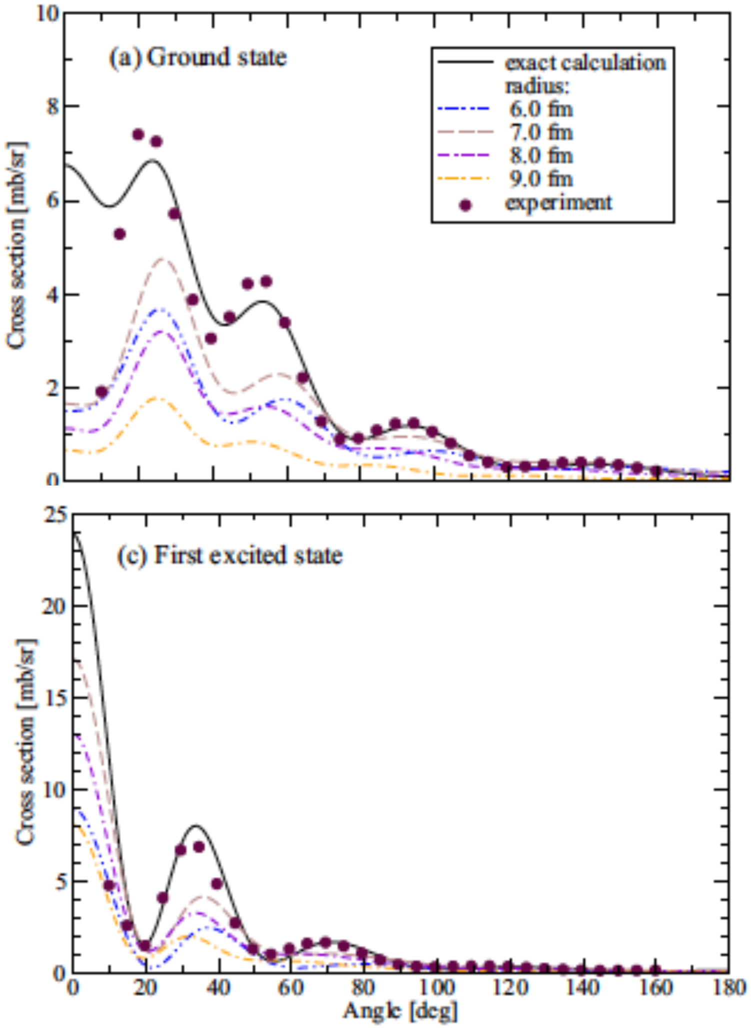
Peak cross section relative to full calculation



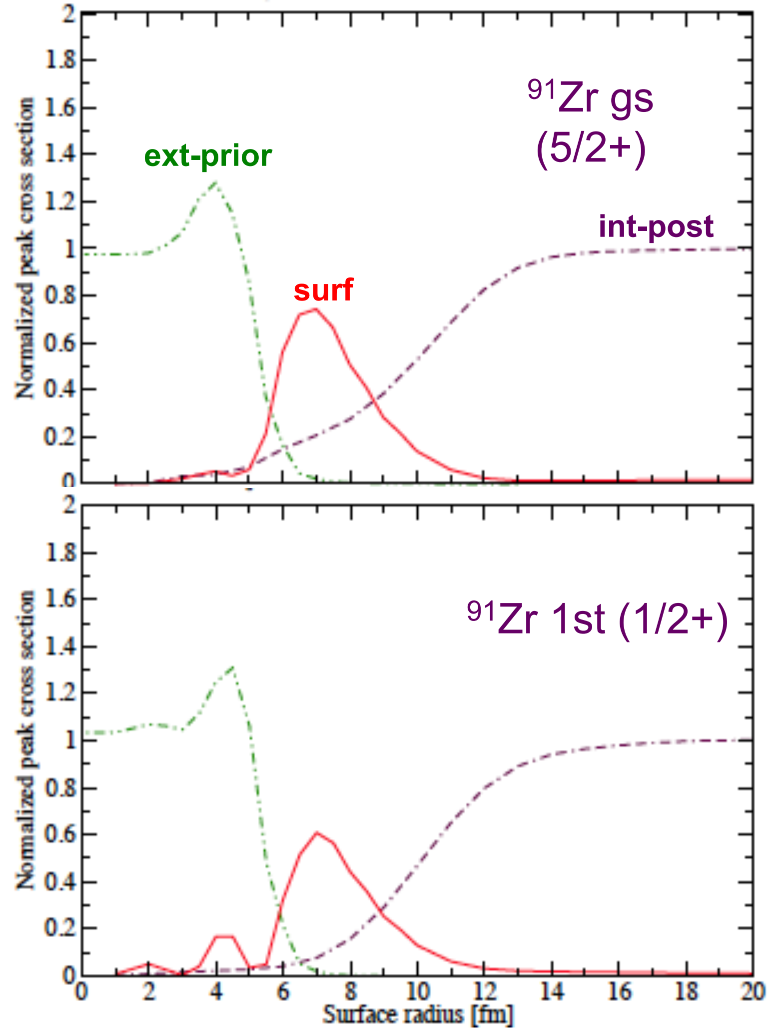
Escher, Thompson, Mukhamedzhanov, JPCS (2012).

The surface contribution – $^{90}Zr(d,p)$ at E_d=11 MeV

Angular cross section – Surface term only



Peak cross section relative to full calculation

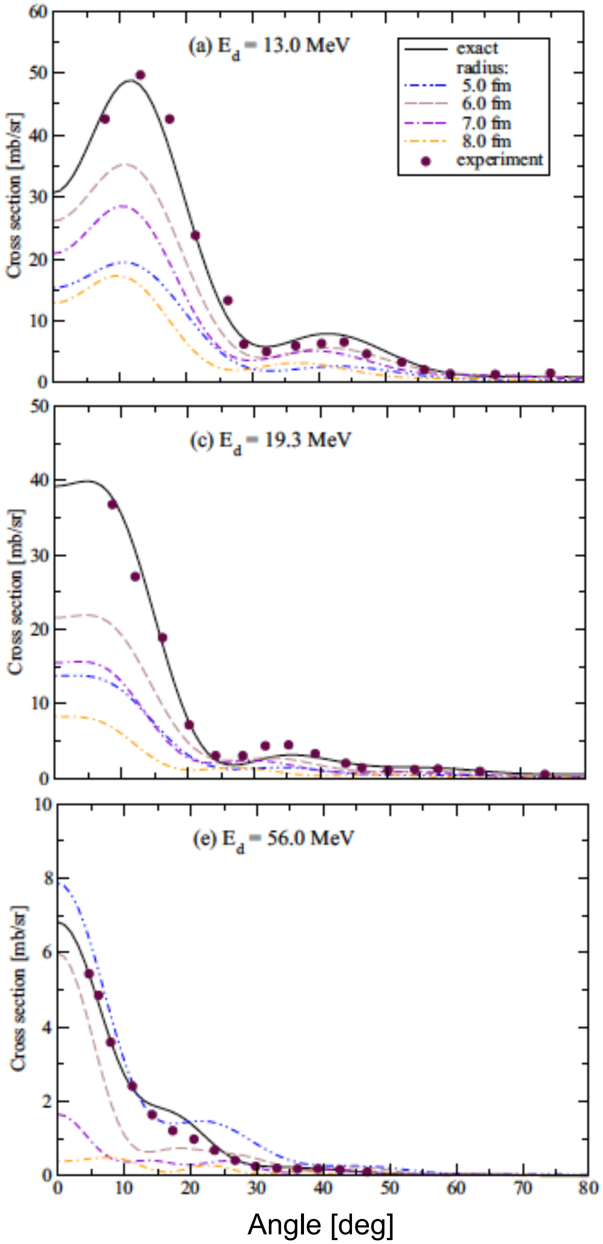


Escher et al, PRC 89, 054605 (2014)

- Cross sections depend on surface radius
- The surface term is dominant, but corrections remain

Numerical tests of the formalism (DWBA) – ^{48}Ca(d,p) at E_d=13, 19.3, 56 MeV

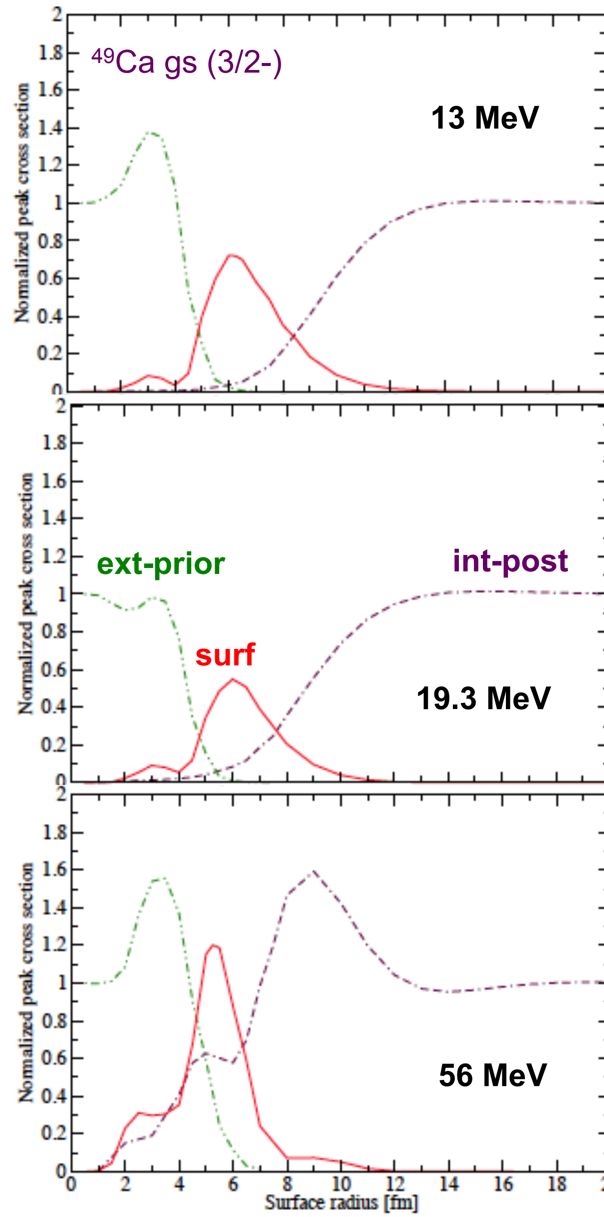
Angular cross section – Surface term only



Surface term approximation improves with decreasing energy

Calculations for ^{49}Ca 1st (1/2-) give similar results

Peak cross section relative to full calculation



Escher et al, PRC (2014)

Surface formalism for DWBA – resonance states

d+A → p + (n+A)

↓

b+B

$$M^{\mathrm{DW(post)}}(P,\mathbf{k}_{dA}) = \langle \chi_{pF}^{(-)} \Psi_{bB}^{(\mathrm{int})(-)} | \Delta\overline{V}_{pF} | \varphi_d \varphi_A \chi_{dA}^{(+)} \rangle, \qquad (94)$$

<u>Surface formulation</u>

M = $M^{(post)}(0,a)$

+ $M_{surf}(a)$

+ $M^{(prior)}(a,\infty)$

→ $f(\Gamma^{1/2}, [A^{-1}], I_A^F)$: contribution hopefully small

→ b + B = n + A; b + B ≠ n + A

→ b + B = n + A; b + B ≠ n + A

Total post matrix element for b + B ≠ n + A example:

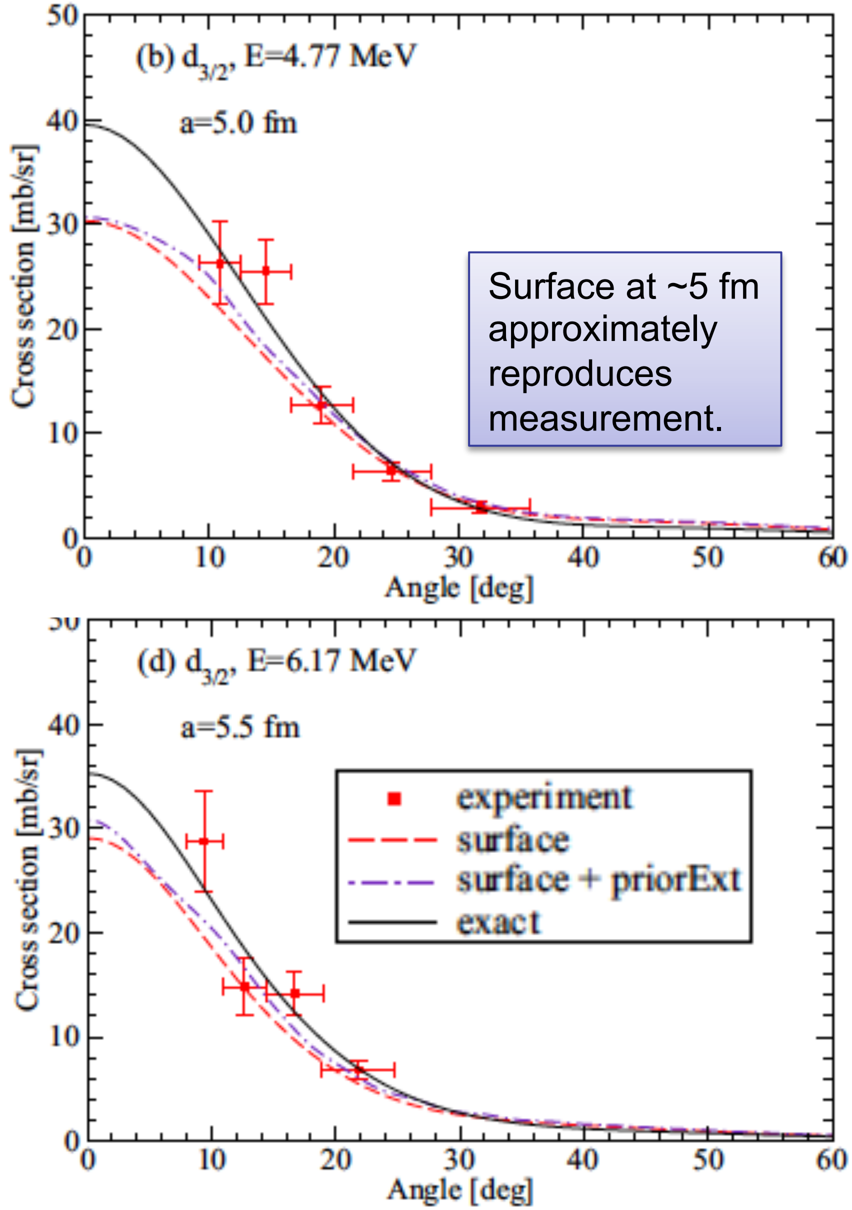
$$\begin{aligned}
M^{\mathrm{DW(post)}}(P,\mathbf{k}_{dA}) = {} & 2\pi\sqrt{\frac{1}{\mu_{bB}k_{bB}}} \sum_{J_F M_F s' l l' m_{s'} m_l m_{l'} M_n} i^l \langle s m_s l m_l | J_F M_F\rangle \langle s' m_{s'} l' m_{l'} | J_F M_F\rangle \langle J_n M_n J_A M_A | s' m_{s'}\rangle \langle J_n M_n J_p M_p | J_d M_d\rangle \\
& \times e^{-i\delta_{bBl}^{hs}} Y_{l m_l}^{*}(-\hat{\mathbf{k}}_{bB}) \sum_{\nu,\tau=1}^{N} [\Gamma_{\nu bBslJ_F}(E_{bB})]^{1/2} [\mathbf{A}^{-1}]_{\nu\tau} \Bigg\{ \langle \chi_{pF}^{(-)} I_{As'l'J_F}^{F} | \Delta\overline{V}_{pF} | \varphi_d \chi_{dA}^{(+)} \rangle \Big|_{r_{nA} \leqslant R_{nA}} \\
& + \sqrt{\frac{2\mu_{nA}}{R_{nA}}} \gamma_{\tau nAs'l'J_F} \left\langle \chi_{pF}^{(-)} \frac{O_{l'}^{*}(k_{nA}, r_{nA})}{r_{nA}} \frac{R_{nA}}{O_{l'}^{*}(k_{nA}, R_{nA})} Y_{l' m_{l'}}^{*}(\hat{\mathbf{r}}_{nA}) \right| \Delta\overline{V}_{dA} \left| \varphi_d \chi_{dA}^{(+)} \right\rangle \Bigg|_{r_{nA} > R_{nA}} + \sqrt{\frac{R_{nA}}{2\mu_{nA}}} \gamma_{\tau nAs'l'J_F} \\
& \times \int d\mathbf{r}_{pF} \chi_{-\mathbf{k}_{pF}}^{(+)}(\mathbf{r}_{pF}) \int d\Omega_{\mathbf{r}_{nA}} Y_{l' m_{l'}}(\hat{\mathbf{r}}_{nA}) \left[\varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})(B_{nA} - 1) - R_{nA} \frac{\partial \varphi_d(\mathbf{r}_{pn}) \chi_{\mathbf{k}_{dA}}^{(+)}(\mathbf{r}_{dA})}{\partial r_{nA}} \right] \Bigg|_{r_{nA} = R_{nA}} \Bigg\}.
\end{aligned}$$

Analogously for CDCC resonance case

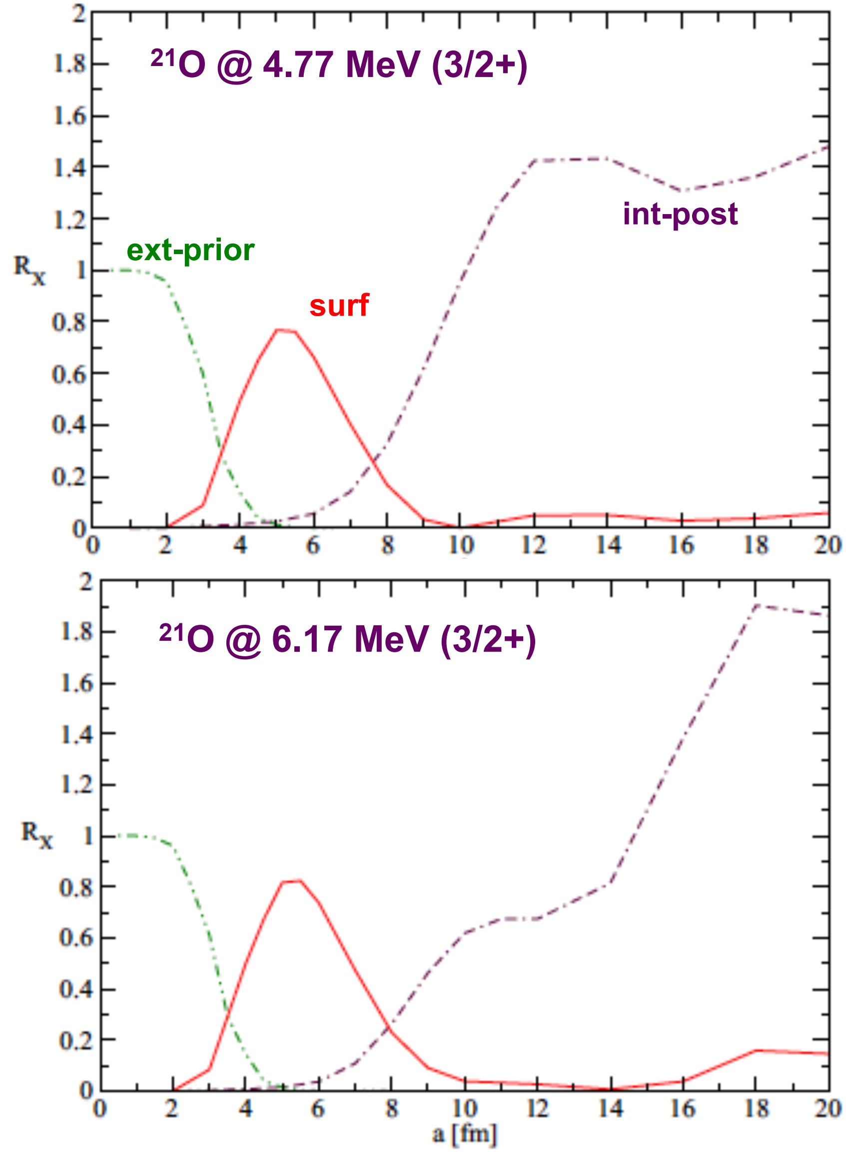
Eq. (117) from Mukhamedzhanov, PRC 2011

The oxygen case - ^{20}O at E_d=21 MeV

Angular cross section – Surface term only



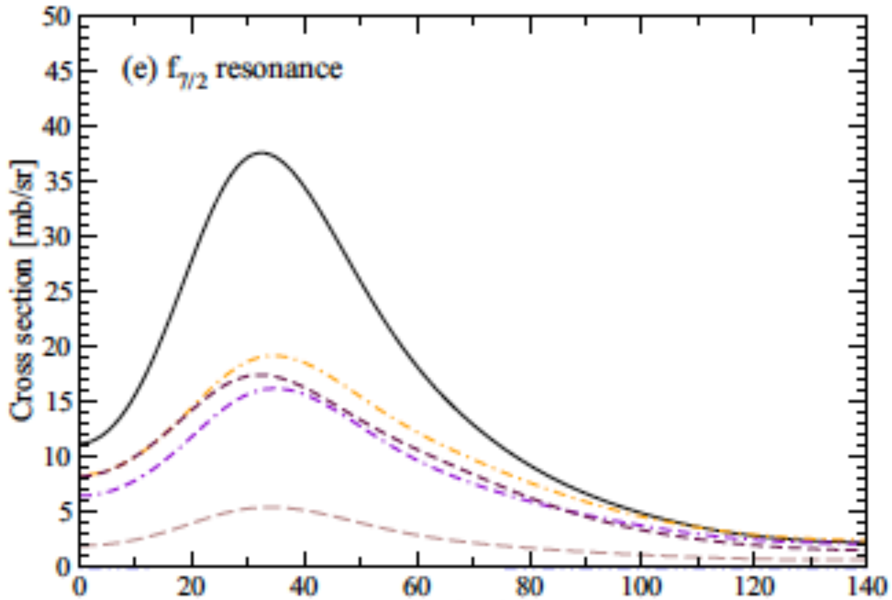
Peak cross section relative to full calculation



Escher et al, PRC 89, 054605 (2014)

Resonances – ^{90}Zr at E_d=11 MeV

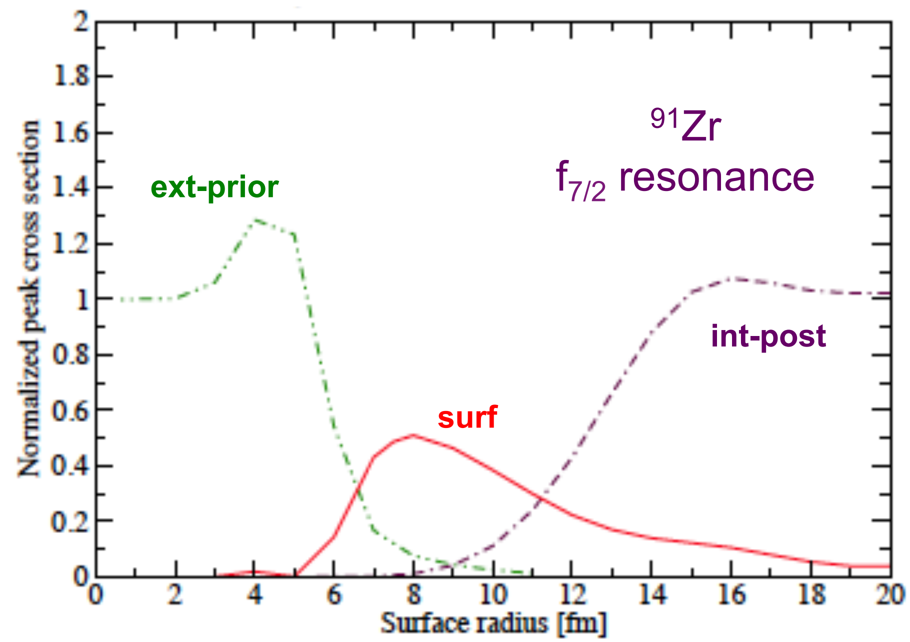
Angular cross section – Surface term only



Angle [deg]

Escher et al, PRC 89, 054605 (2014)

Peak cross section relative to full calculation

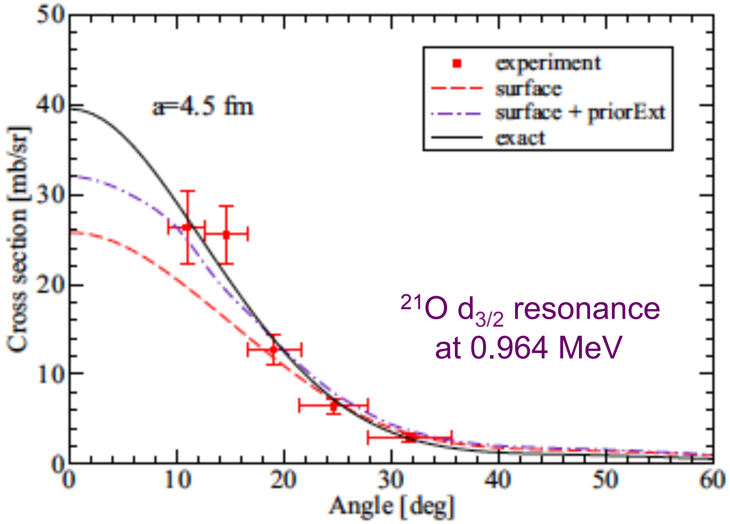


- Results similar to bound-state cases
- Surface term dominant at larger radii
- Interior/exterior terms still contribute

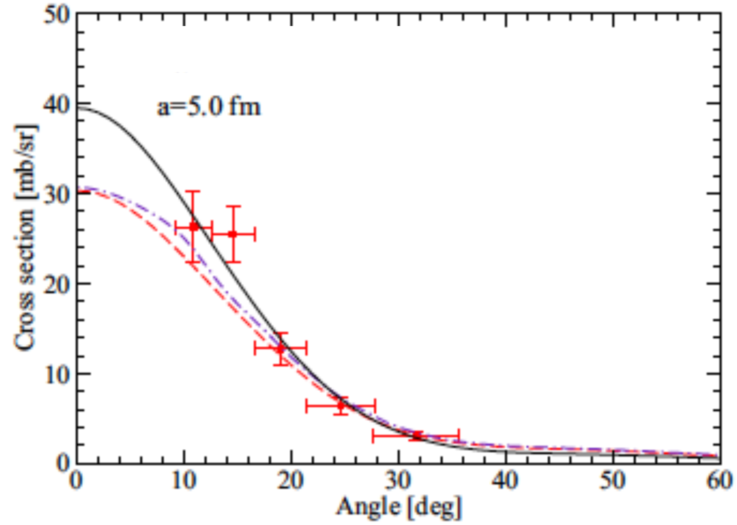
The surface formalism: can we save it?

^{20}O at E_d=21 MeV

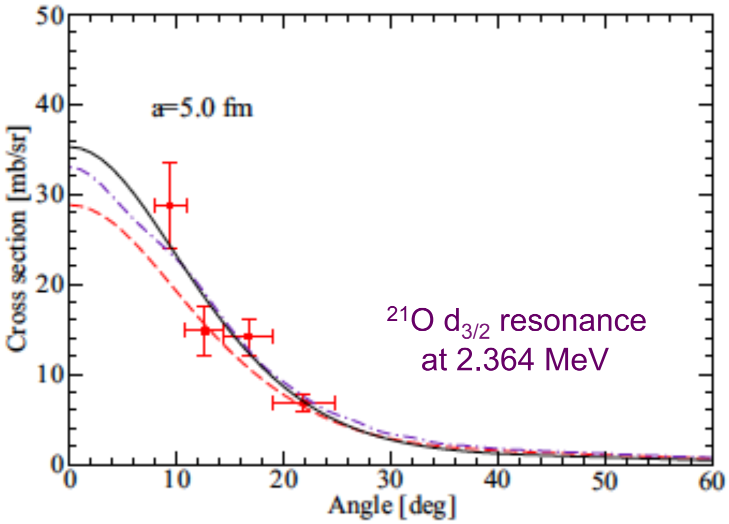
Angular cross section at smaller radius

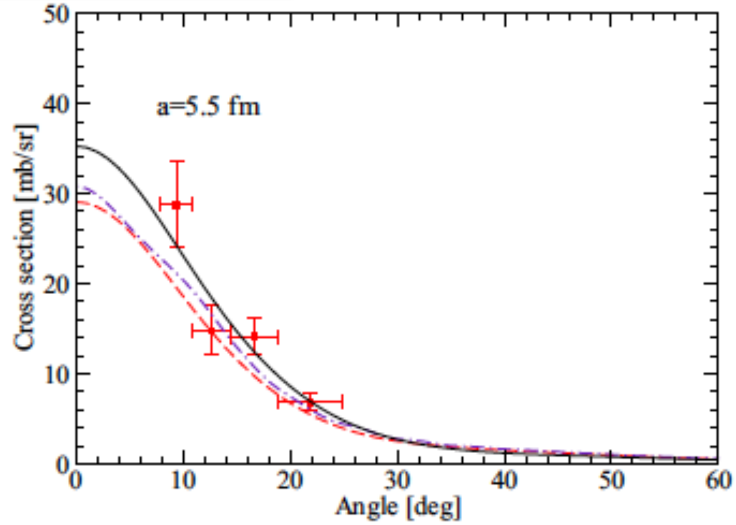


Angular cross section at peak radius



- reducing the surface radius
- adding prior-exterior contribution



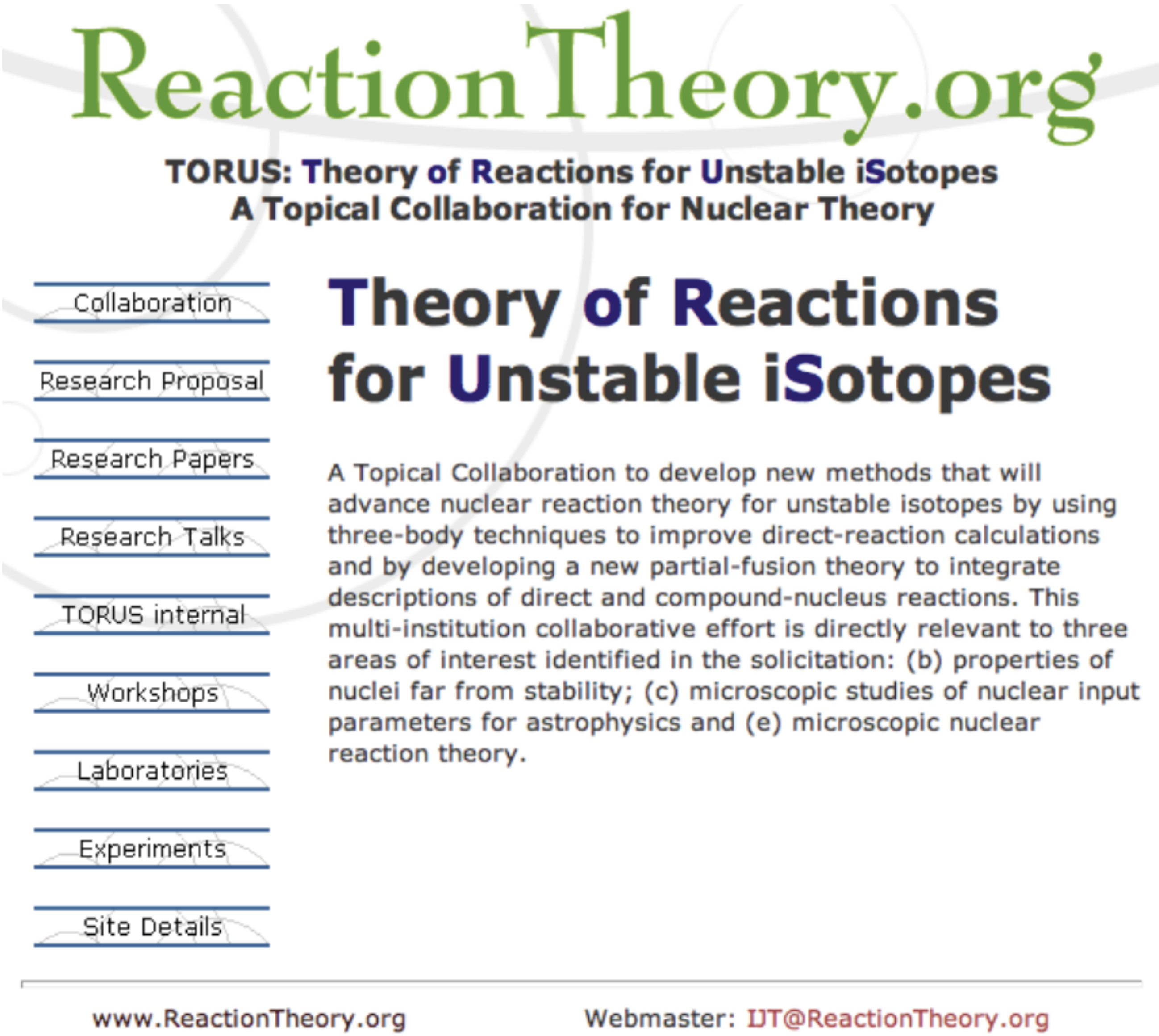


Concluding Notes

Surface formalism for studying resonances with (d,p):

- Uses successful R-matrix ideas to emphasize **asymptotic properties** of the wave function
- Separation into interior and exterior leads to a surface term which can be expressed in terms of familiar R-matrix parameters, thus providing **meaningful spectrosopic information**
- Our studies within a DWBA implementation show that the surface term is dominant; dependence on model for nuclear interior is reduced.
- The surface term alone is **not sufficient** to describe transfer reactions, corrections are required
- **Remains to be seen** whether a CDCC implementation (which includes breakup effects) will give the required improvements.

TORUS Collaboration



ReactionTheory.org

TORUS: Theory of Reactions for Unstable iSotopes
A Topical Collaboration for Nuclear Theory

- Collaboration
- Research Proposal
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A Topical Collaboration to develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations and by developing a new partial-fusion theory to integrate descriptions of direct and compound-nucleus reactions. This multi-institution collaborative effort is directly relevant to three areas of interest identified in the solicitation: (b) properties of nuclei far from stability; (c) microscopic studies of nuclear input parameters for astrophysics and (e) microscopic nuclear reaction theory.

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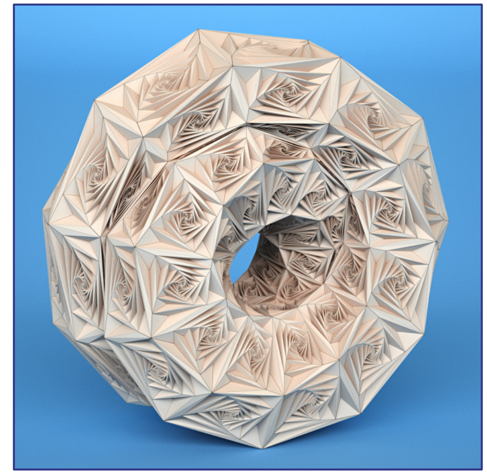

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