Interior and Exterior Contributions to Transfer Cross Sections

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The Debate: Surface or Volume?

- Do transfer or knockout experiments measure surface properties (ANC, reduced width), or volume properties (norm of overlap function) ?
- Theory: only 'asymptotic properties' are observable: invariant under off-shell (interior) unitary transformations.
- Reply: We have relied on local potentials for interior forms
- Conclusion: We must:
  - pay attention to invariance if we derive effective potentials (which may be local or non-local)
  - separate the contributions from interior and exterior
  - see if/how these contributions depend on higher-order couplings.

Transfers to Resonances?

Not clear what do we measure when we compare

(a) experimental magnitude to theory magnitude?

(b) experimental width to theory width?

Need a new general theory for resonant transfers!

- Preferably one easy to calculate!
- At present, to get convergence at large radii: we use bins, or complex contour, or damping
- Should calculate actual shape of resonance peak
  - Include wide / overlapping / multichannel resonances
  - Ideally should fit using R-matrix resonance parameters

New work

A.M. Mukhamedzhanov, PRC 84, 044616 (2011)

- Look at dependence of transfer rate on r_{nA}
  = radius of neutron wave function $\phi_n(r_{nA})$ being probed
  - Remember that $\phi_n(r_{nA})$ for $r_{nA} > r_s$ (surface radius) depends on the reduced width: γ^2 or the ANC: C

$$\gamma^2 = \frac{\hbar^2}{2\mu a}\phi_n(a)^2 \qquad \phi_n(r_{nA}) \rightarrow_{r_{nA}\rightarrow\infty} C\ W(kr_{nA})$$

$$(\text{when} \quad \int_0^a \phi_n(r)^2 dr = 1)$$

- Look at how post and prior transfers depend on maximum value of r_{nA} (cut wfn to zero outside).
- Later, try to express as much of the transfer as possible in terms of the γ^2.
- This will help calculation of transfers to resonances
  - Needed e.g. for Trojan Horse methods, and many expts.

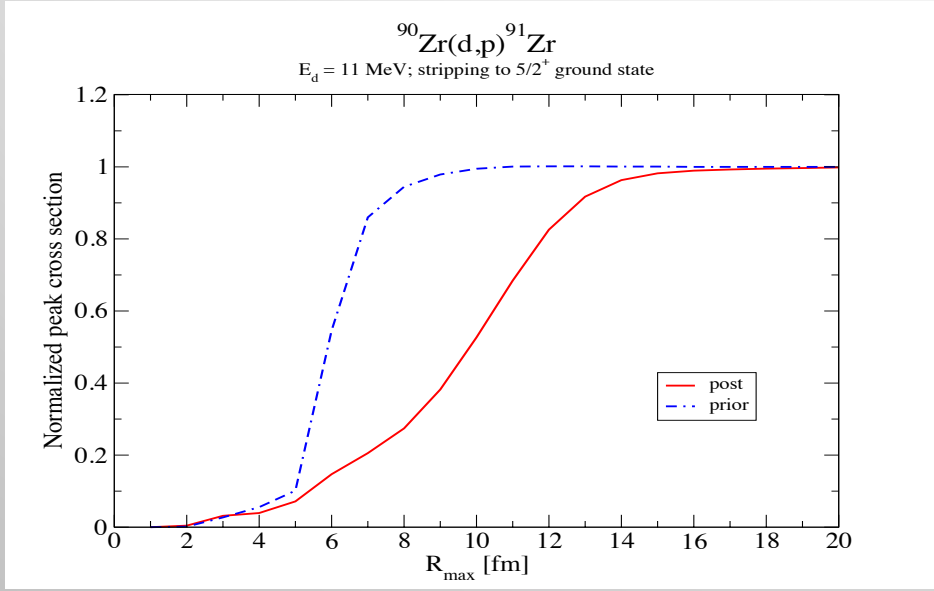
Post and Prior DWBA Transfer Couplings

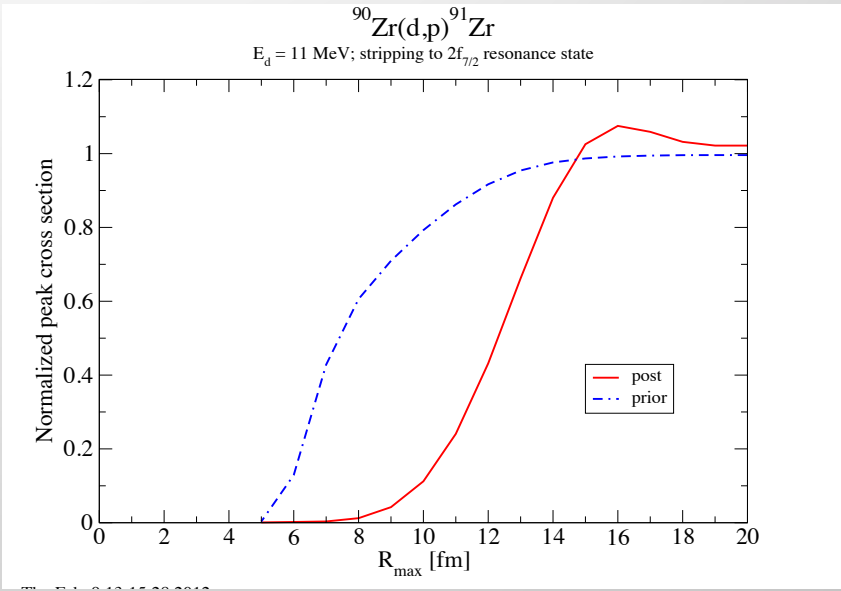
- Consider a deuteron d=n+p incident on target A, and the A(d,p)B reaction, with B=A+n.
- Binding potentials V_{np} for $\phi_d(r)$, V_{nA} for $\phi_n(r_{nA})$
  - Entrance & exit optical potentials $U_{dA}(R)$, $U_{pB}(R)$
  - Also need 'core-core' potential U_{pA}

<u>Look at DWBA as first approximation:</u>

- $T_{post} = \langle f_p^{(-)} \phi_n \mid V_{np} + U_{pA} - U_{pB}(R) \mid \phi_d f_d^{(+)} \rangle$ (has ZR limit)
  - As long-ranged in r_{nA} as ϕ_n, as V_{np} acts at all distances from target
- $T_{prior} = \langle f_p^{(-)} \phi_n \mid V_{nA} + U_{pA} - U_{dA}(R) \mid \phi_d f_d^{(+)} \rangle$
  - Short-ranged in r_{nA} than ϕ_n, as V_{nA} , U_{pA} , U_{dA} all cut off away from target

Effects of limiting max r_{nA}





Bound state

Resonance bin at 1 MeV

Peak cross sections, calculated in the post and prior formalisms, are shown as a function of the cutoff radius, (beyond which contributions from the neutron wave function are set to zero)
The cross sections are normalized relative to the peak cross sections obtained in the full calculation.

See that Post contributions are from large neutron radii.
Convergence to resonances is slow (especially for post form)
Very small post contributions from the interior

‘Surface Amplitude’ component

- Define $T_{post}(a,b)$ & $T_{prior}(a,b)$ with $a < r_{nA} < b$ limits

Mukhamedzhanov (PRC **84**, 044616, 2011) showed recently:

$$T = T_{post}(0,a) + T_{surf}(a) + T_{prior}(a,\infty)$$

where $T_{surf}(a) = \langle f_p^{(-)} \phi_n \left| [\overleftarrow{T} - \overrightarrow{T}] \right| \phi_d f_d^{(+)} \rangle_{(in)}$

- Evaluate:

$$\int_{r\leqslant R} \mathrm{d}\mathbf{r}\, f(\mathbf{r})[\overleftarrow{T} - \overrightarrow{T}]g(\mathbf{r})$$

$$= -\frac{1}{2\mu} \oint_{r=R} \mathrm{d}\mathbf{S}[g(\mathbf{r})\nabla_{\mathbf{r}} f(\mathbf{r}) - f(\mathbf{r})\nabla_{\mathbf{r}} g(\mathbf{r})]$$

$$= -\frac{1}{2\mu} R^2 \int \mathrm{d}\Omega_{\mathbf{r}} \left[g(\mathbf{r}) \frac{\partial f(\mathbf{r})}{\partial r} - f(\mathbf{r}) \frac{\partial g(\mathbf{r})}{\partial r} \right]_{r=R}$$

$T = T_{post}(0,a) + T_{surf}(a) + T_{prior}(a,\infty)$

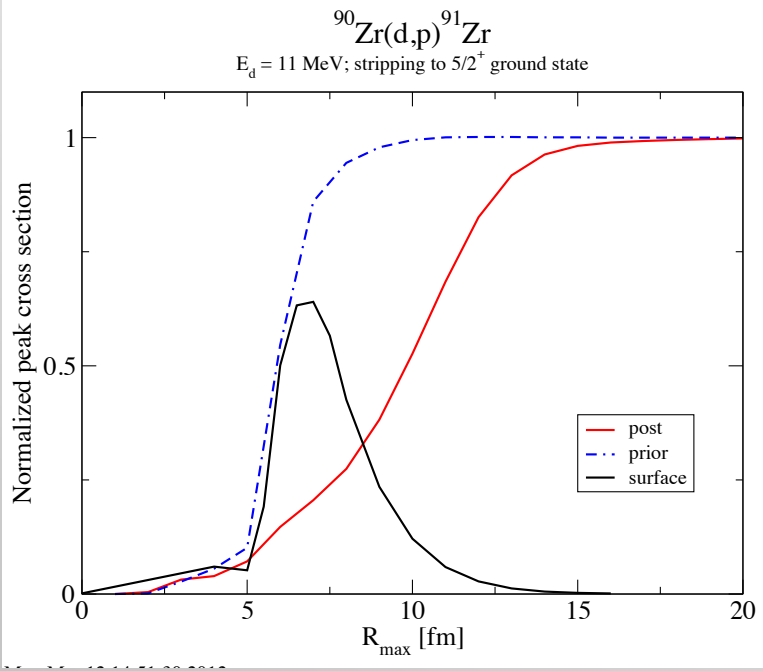
Prove post-prior equivalence in DWBA:

- If a=0, then, since $T_{surf}(0) = 0$, find $T = T_{prior}(0,\infty)$
- If a=∞, then, since $T_{surf}(\infty) = 0$, find $T = T_{post}(0,\infty)$

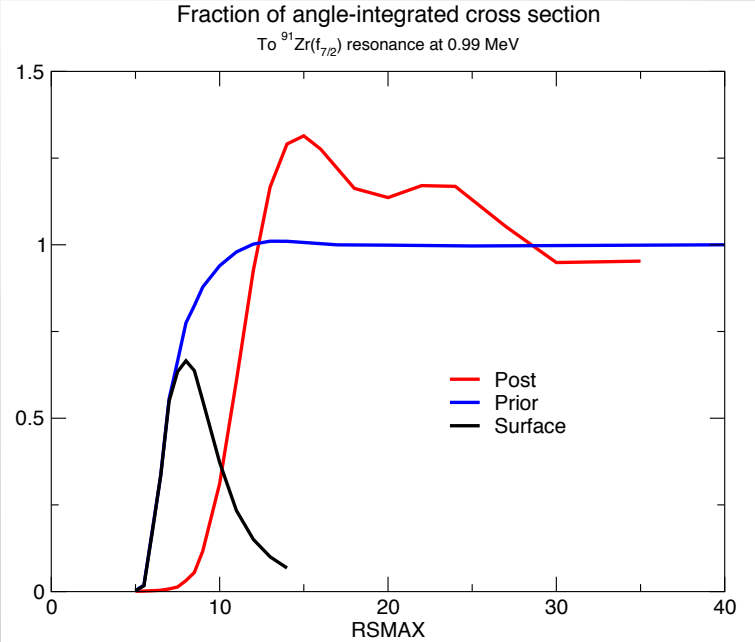
Dependence on reduced width γ^2 of neutron wf:

- If a is outside radius of the potential, then $T_{surf}(a) + T_{prior}(a,\infty)$ depend on wfn only by γ^2
- Only dependence on interior is by (small) $T_{post}(0,a)$

Size of the Surface Term



Bound state



Resonance

$$T_{surf}(a) = T_{prior}(0,a) - T_{post}(0,a)$$

Now we see the surface term peaked at the surface (as expected).

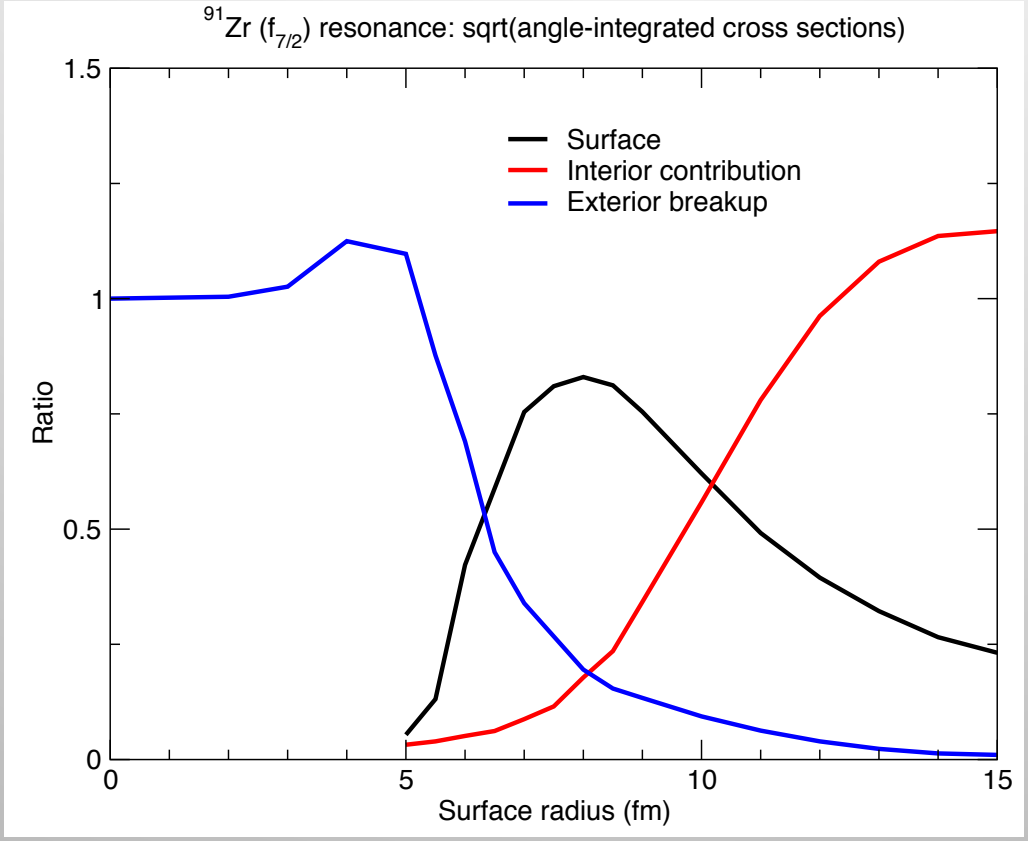
But it does not produce all the cross section peak, or all the integral

CDCC Breakup exterior calculations

- The potentials in the prior matrix element
  $$V_n + U_{pA} - U_d(R)$$
  are very similar to the
  $$U_{nA} + U_{pA} - U_d(R)$$
  used in CDCC breakup calculations.
  Difference is that V_n = binding potl and U_{nA} = optical potl.
- If we can ignore this difference, and calculate Ψ^{CDCC}, then the 'exterior prior' term disappears:
  $$T = T^{CDCC}_{post}(0,a) + T^{CDCC}_{surf}(a)$$
- For now:
  regard the 'exterior prior' as indicator of <u>breakup</u>.

Interior / Surface / Breakup

Breakup outside radius on x-axis



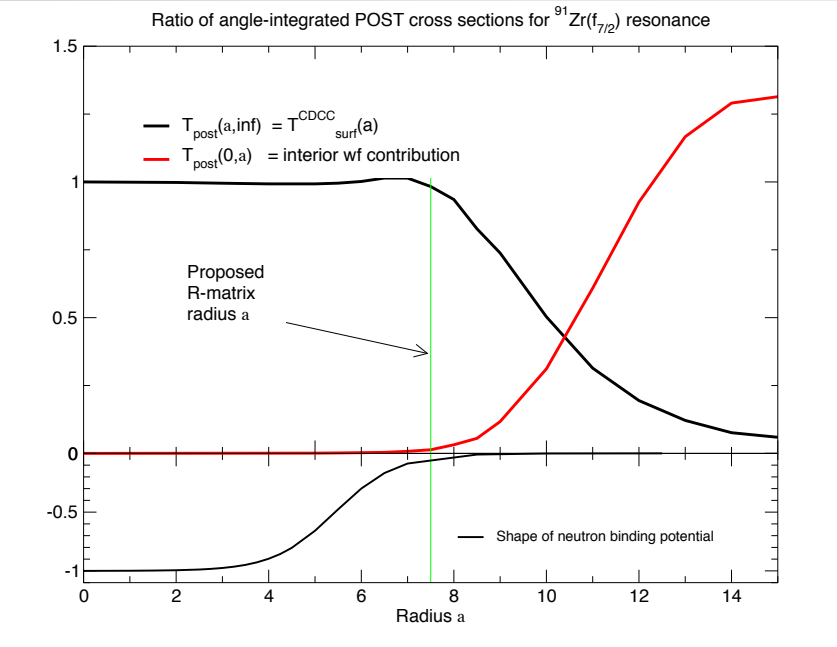
Interior wfn contribution inside x-axis radius

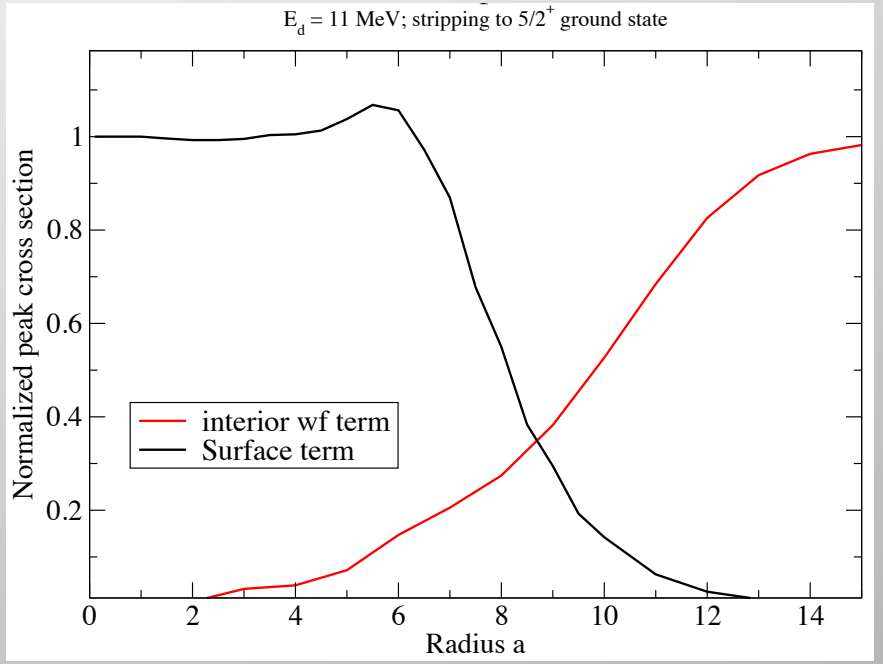
Surface term at x-axis radius

Plotting sqrt(cross-section) – to estimate amplitudes

Predictions for future CDCC-surface calculations

$T^{CDCC}_{surf}(a) = T - T^{CDCC}_{post}(0,a)$
$\approx T_{post}(a,\infty)$ -- use this to estimate:





Black curve ratio of post cross sections $\sigma_{post}(a,\infty)/\sigma = |T_{post}(a,\infty)/T|^2$:

Try to choose radius around 7.5 – 8 fm outside potential, where the CDCC-surface contribution is complete

Ratio of cross section peaks

Larger interior contribution for this bound-state transfer

Conclusions

- See development of a model that separates
  1. Interior contributions from shape of wave function
  2. Breakup contributions from exterior tails
  3. Dominant 'surface contribution' from exterior tails.
- 'Surface Approximation': if neglect other terms
- Good prospects for
  - a new model of transfer reactions to resonances, that
  - uses small-radius calculations (convergent!),
  - to map R-matrix parameters onto resonance shapes.
- We are now developing the CDCC approach
- In future: fit neutron R-matrix parameters from expt.

Some history from 1960: Transfers measure reduced widths

REVIEWS OF MODERN PHYSICS VOLUME 32, NUMBER 3 JULY, 1960

Stripping Reactions and the Structure of Light and Intermediate Nuclei*

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I. INTRODUCTION

A REDUCED width for the emission of a single nucleon in a transition between two specific nuclear states can be regarded as a product of two factors. Of these factors, the first is a measure of the probability that, in the initial nuclear state, all but one of the nucleons will find themselves in an arrangement corresponding to the final state; the second factor measures the probability that, when this happens, the two components will actually separate. The factorization is formally expressed by

$$\theta^2 = S\theta_0^2. \qquad \text{(I.1)}$$

It is clear that the "spectroscopic factor" S, which depends only on the wave functions of the nuclear states involved, provides a useful basis for comparison between experiment and the predictions of current nuclear models.

Define 'spectroscopic factor' S
= ratio of observed reduced width
to that of single-particle state

Maybe something for us to learn here?

TORUS
Theory of Reactions of Unstable Isotopes

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Aim: develop new methods that will advance nuclear reaction theory for unstable isotopes by using three-body techniques to improve direct-reaction calculations

Year 2 out of 5.

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