Reactions theory for studying rare isotopes: the missing piece of the puzzle

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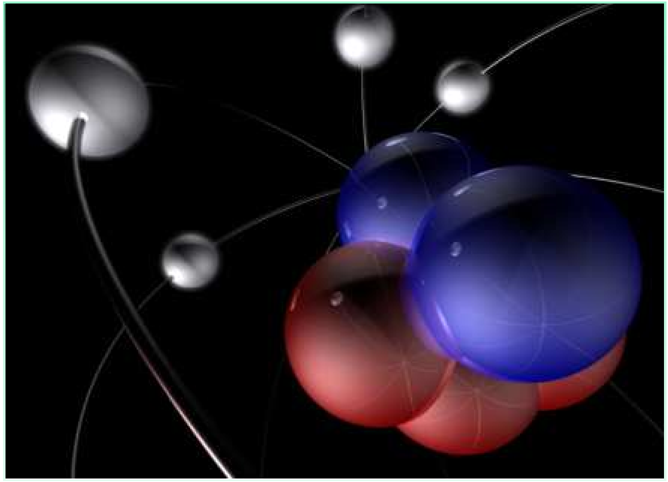
Notre Dame, Oct 2010

reaction theory for studying rare isotopes

- introduction: what are we after?
- motivation: why do we need reactions?
- reaction theory: overall perspective and some connections
- specific examples:
    - breakup reaction for astrophysics
    - combined method for transfer reactions
    - transfer and breakup: CDCC versus Faddeev
    - transfer versus knockout (the Ar isotopes)
- summary and outlook

what are we after?

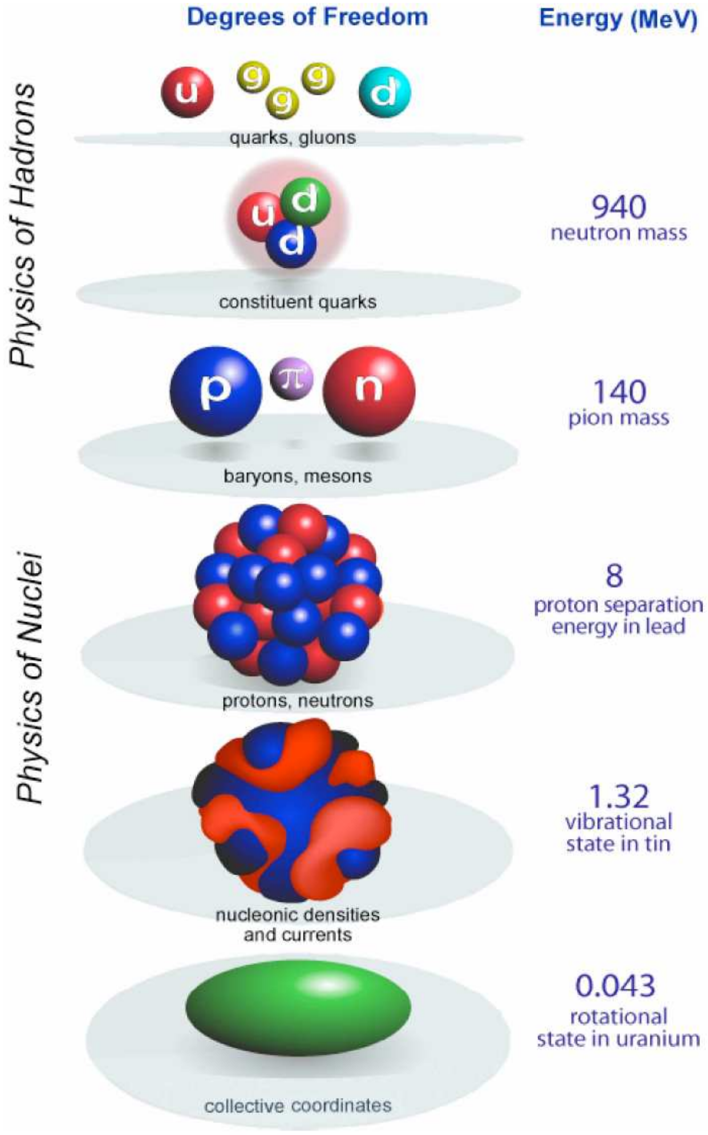
Unified description of nuclei and their reactions



Need a good model!

- describes data
- predictable outside known regions

relevant scales for nuclei



many body problem

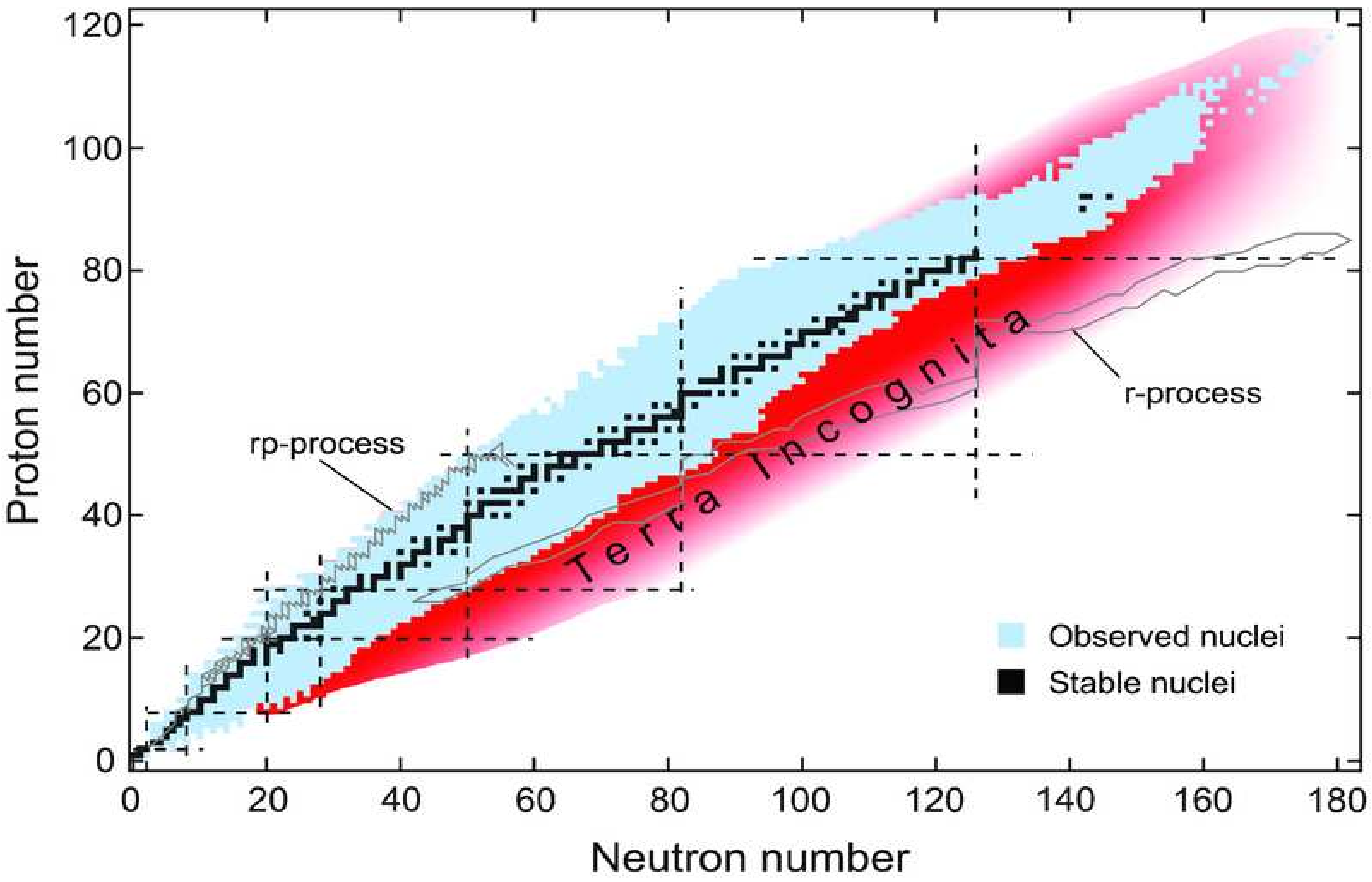
$$H_A = -\sum_{i=1}^{A} \frac{\hbar^2}{2m_i}\nabla^2_{\mathbf{r}_i} + \frac{\hbar^2}{2M}\nabla^2_{\mathbf{S}} + \sum_{i>j}^{A} V^{(2)}(\mathbf{r}_i - \mathbf{r}_j)$$

$$\boxed{H_A \Phi_{I\mu}(\boldsymbol{\rho}_1, \ldots, \boldsymbol{\rho}_{A-1}) = E_I \Phi_{I\mu}}$$

$$\lim_{\boldsymbol{\rho}_i \to \infty} \Phi_{I\mu}(\ldots, \boldsymbol{\rho}_i, \ldots) = 0$$

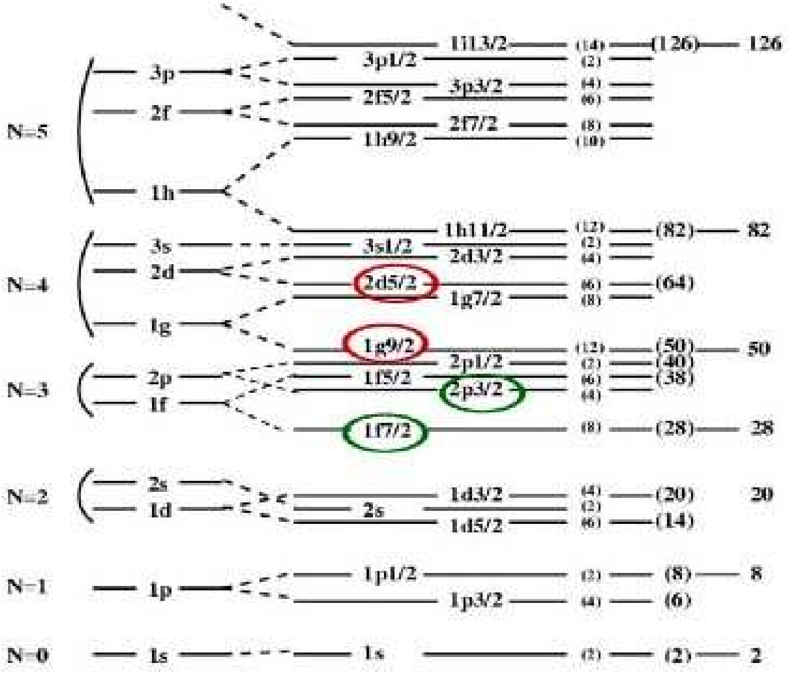
$$\int \mathrm{d}\boldsymbol{\rho}_1 \ldots \int \mathrm{d}\boldsymbol{\rho}_{A-1} |\Phi_{I\mu}(\boldsymbol{\rho}_1, \ldots, \boldsymbol{\rho}_{A-1})|^2 = 1$$

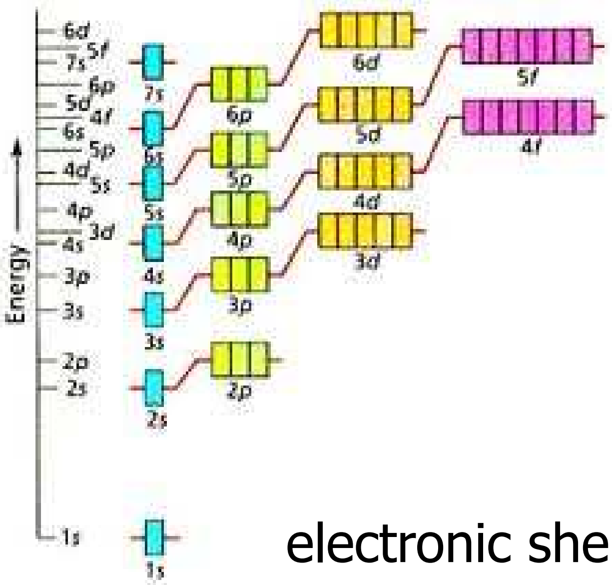
have a good model? go for it!



understanding nuclei

nuclear shell model

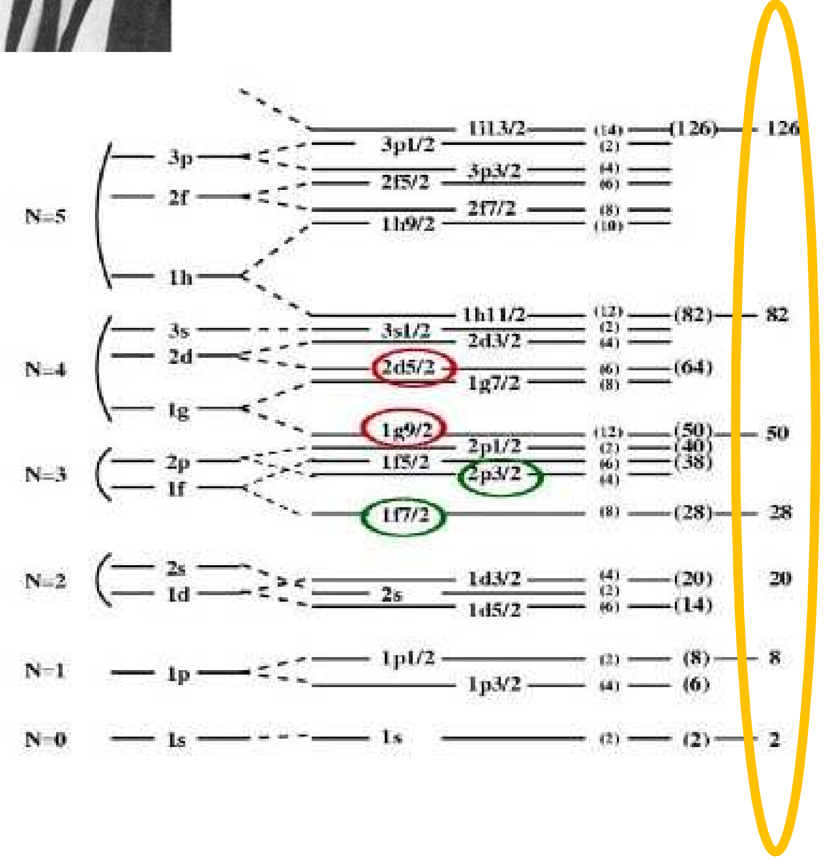




electronic shells

understanding nuclei

nuclear shell model



magic numbers

^{208}Pb

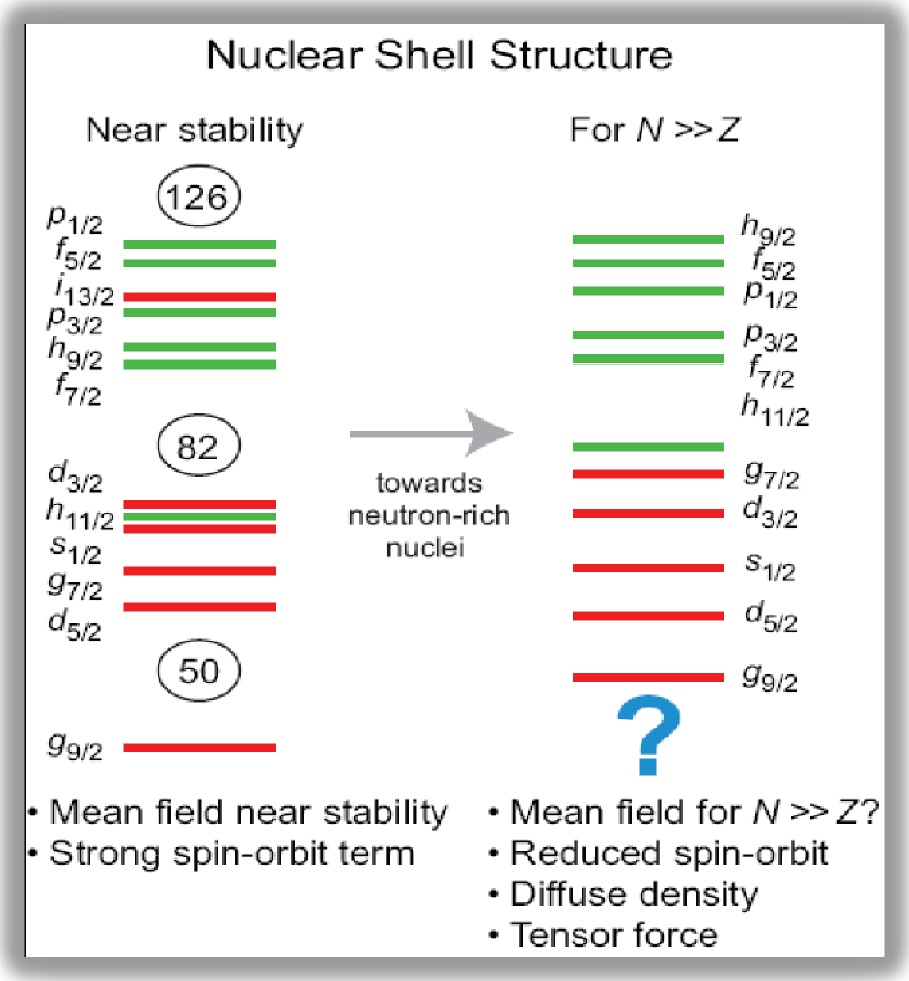
^{100}Sn, ^{132}Sn

^{56}Ni, ^{78}Ni

^{40}Ca, ^{48}Ca

^{16}O

shell structure away from stability?

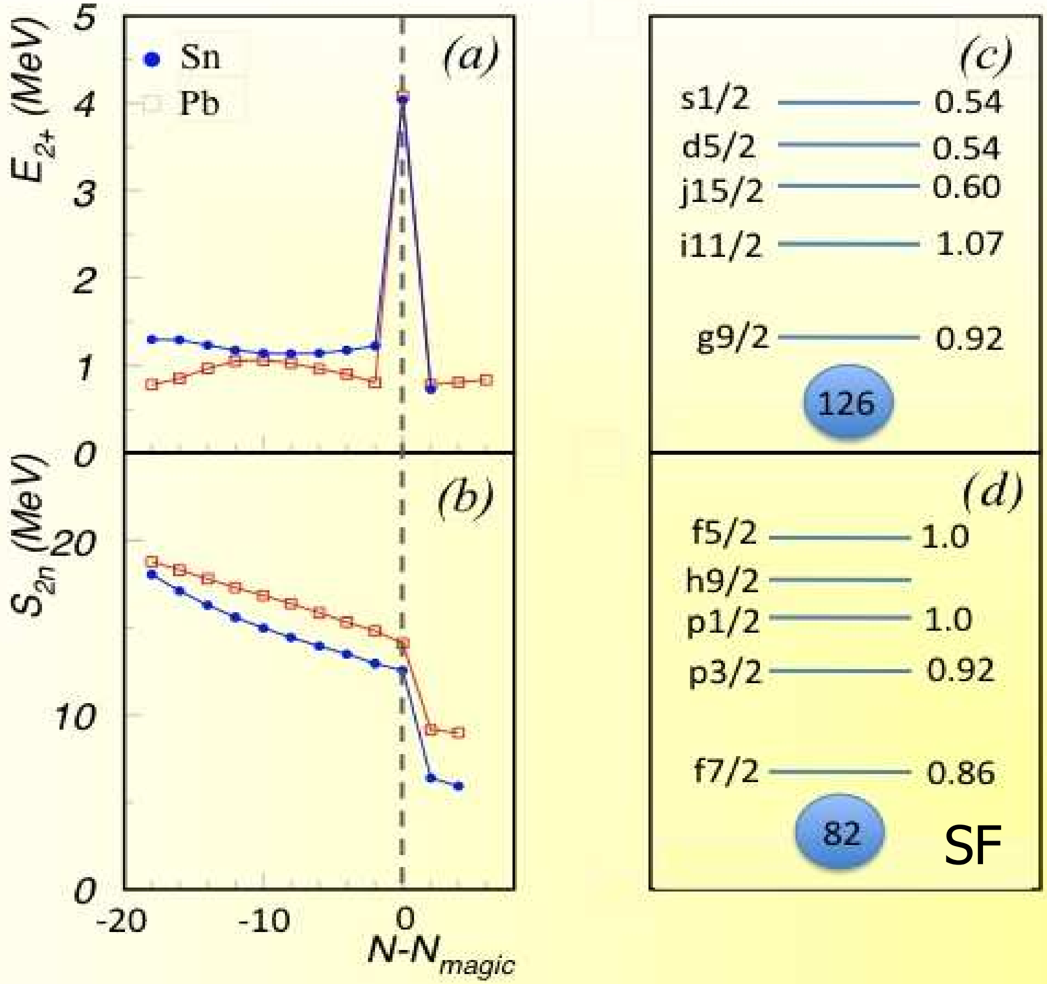


- Mean field near stability
- Strong spin-orbit term

- Mean field for $N >> Z$?
- Reduced spin-orbit
- Diffuse density
- Tensor force

magic numbers

Doubly magic nuclei

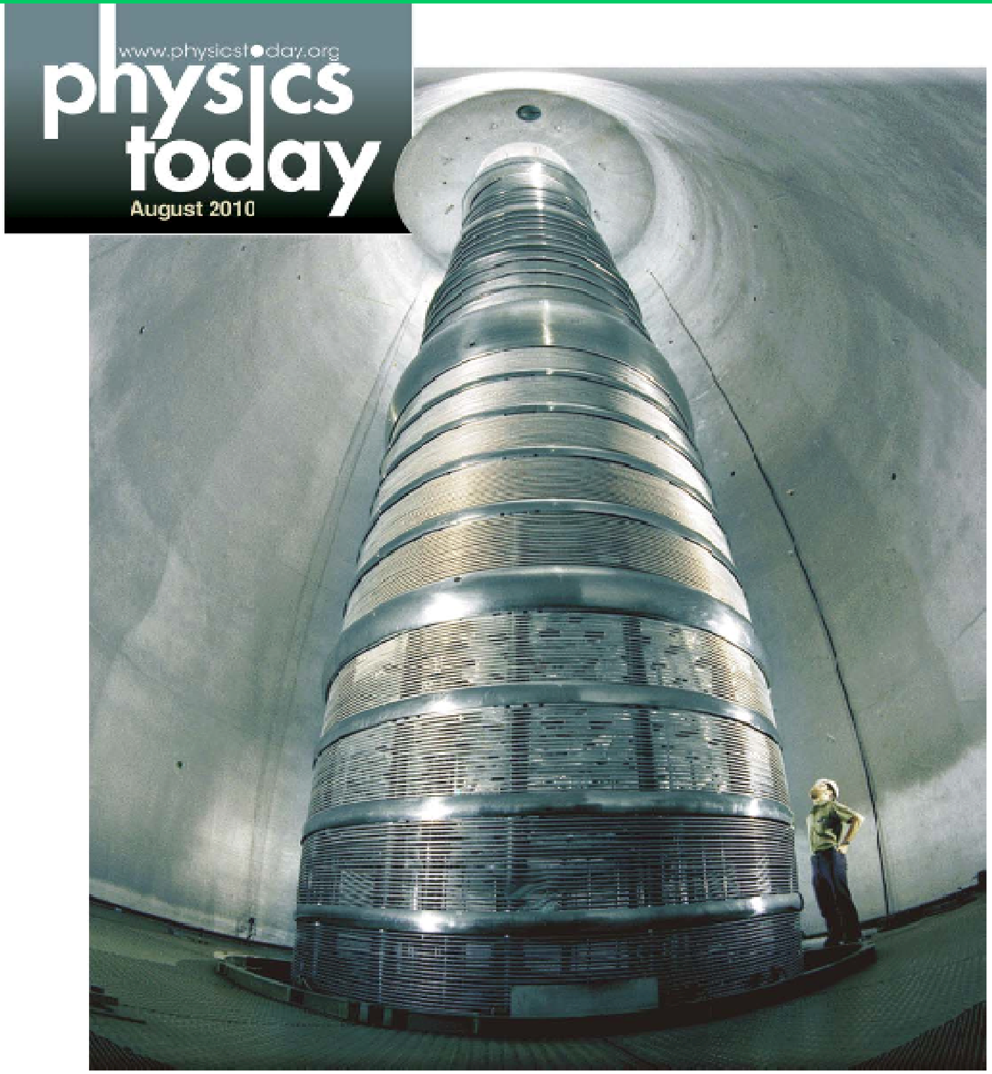


^{208}Pb(d,p)^{209}Pb

^{132}Sn(d,p)^{133}Sn

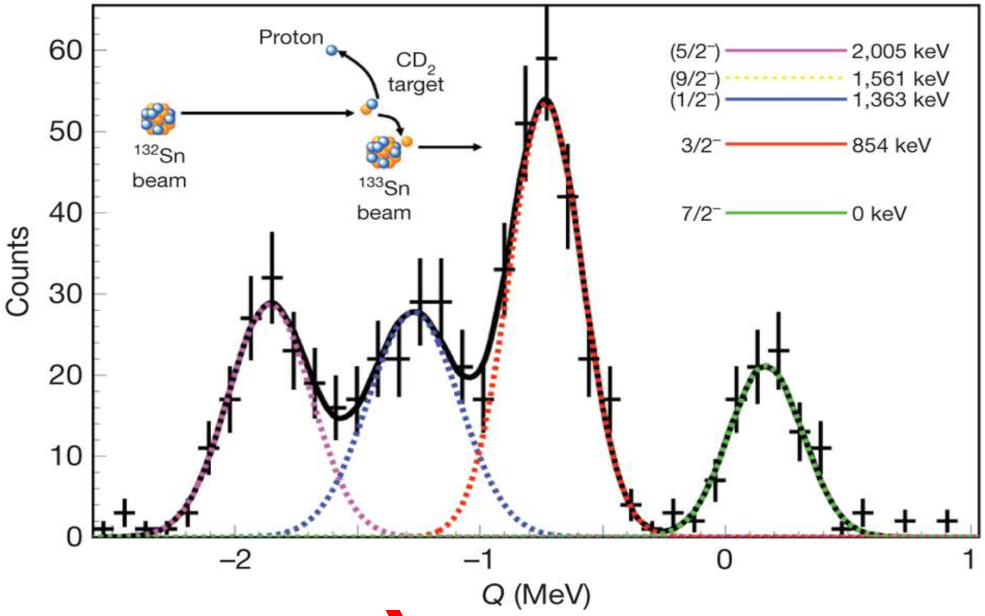
[K. Jones et al, Nature 465 (2010) 454]

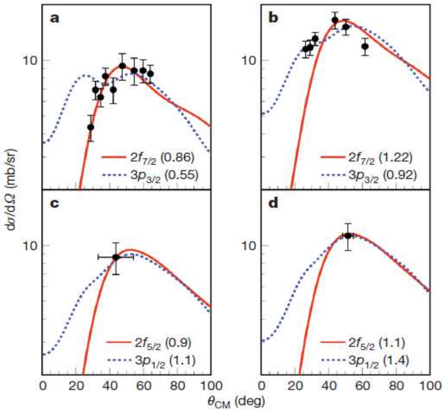
studying double magic nuclei away from stability



Doubly magic shell game

d(^{132}Sn,^{133}Sn)p@5 MeV/u



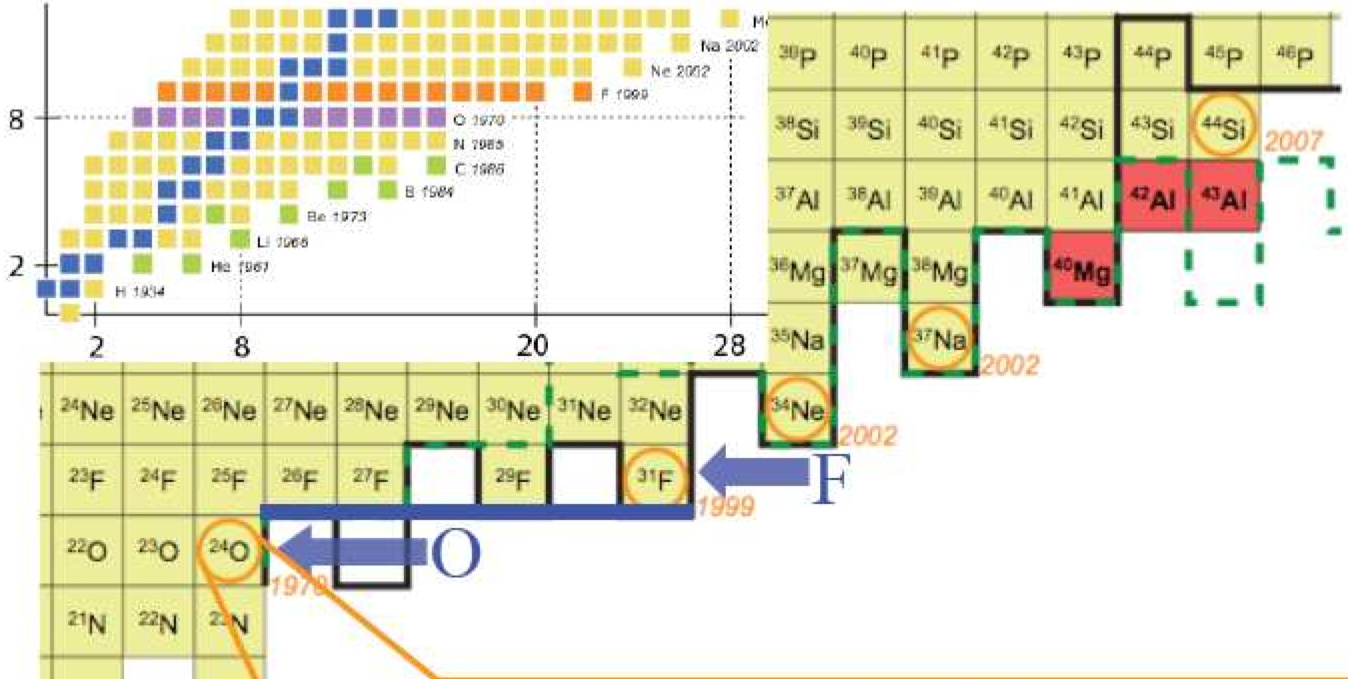


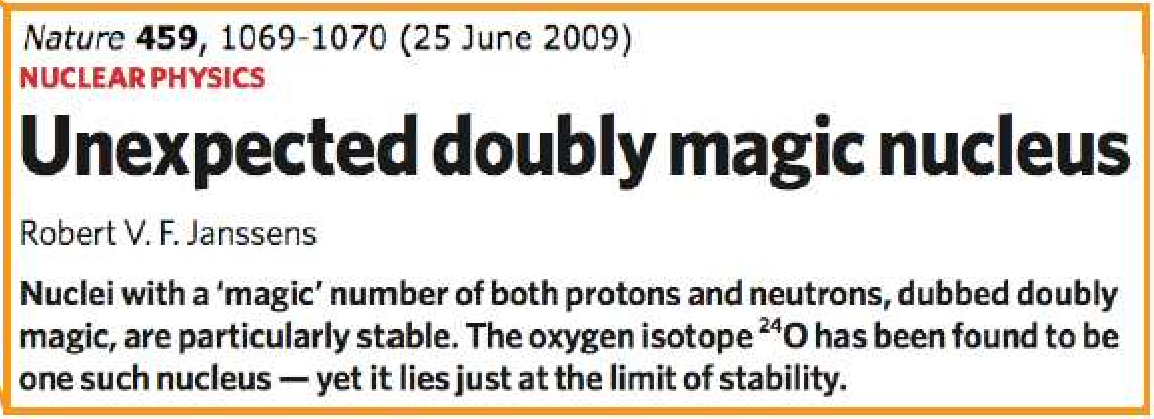
connection
to r-process

^{132}Sn(n,γ)^{133}Sn

[K. Jones et al, Nature 465 (2010) 454]

where is the oxygen dripline?





Nature **459**, 1069-1070 (25 June 2009)

NUCLEAR PHYSICS

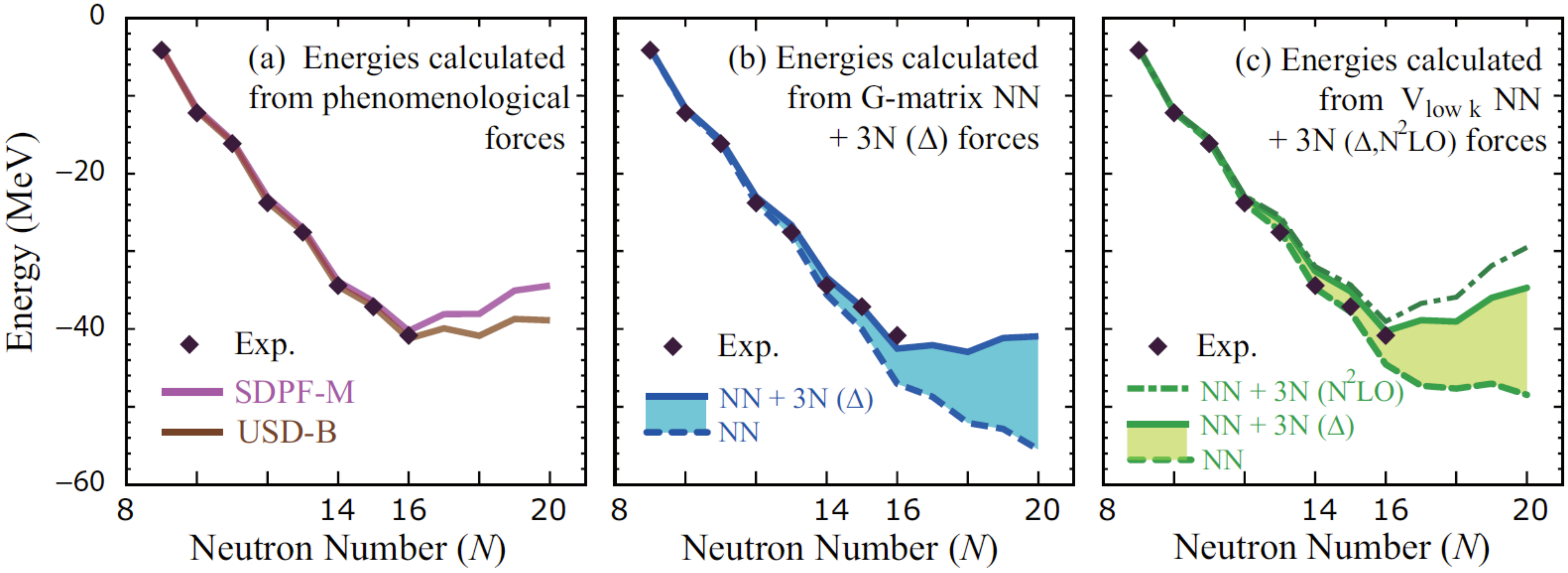
Unexpected doubly magic nucleus

Robert V. F. Janssens

Nuclei with a 'magic' number of both protons and neutrons, dubbed doubly magic, are particularly stable. The oxygen isotope ^{24}O has been found to be one such nucleus — yet it lies just at the limit of stability.

Slide from INT talk, A. Schwenk

three-body force for Oxygen isotopes



Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2009)

reaction theory for studying rare isotopes

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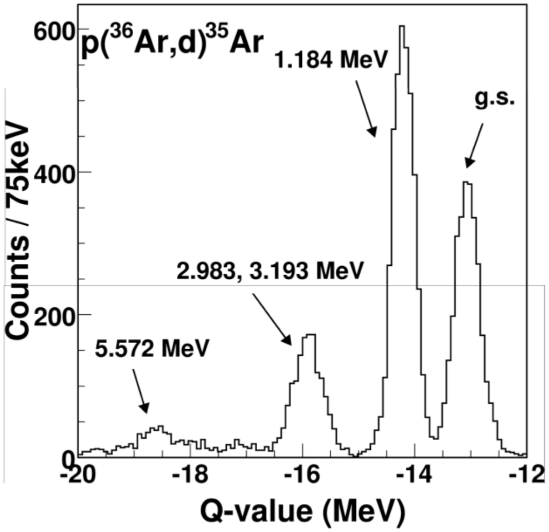
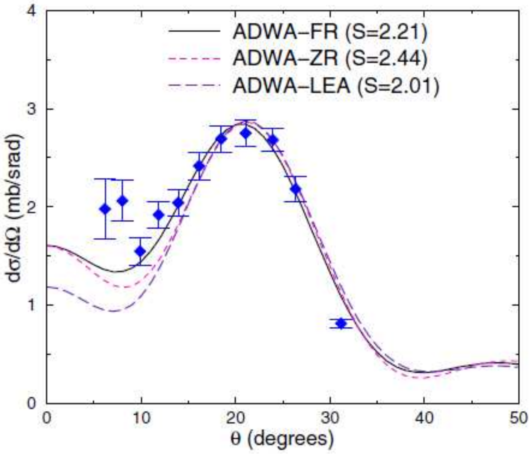
why bother with reactions?

a) nuclei of interest are beams

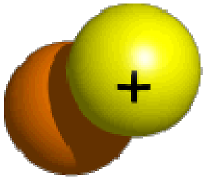


b) offers much more than energy levels

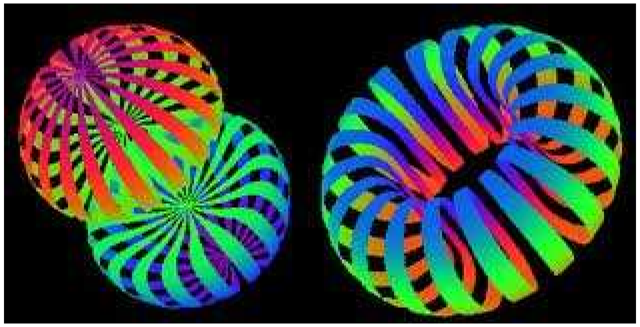


example of deuteron and V_{np}

any simple central interaction can give correct binding



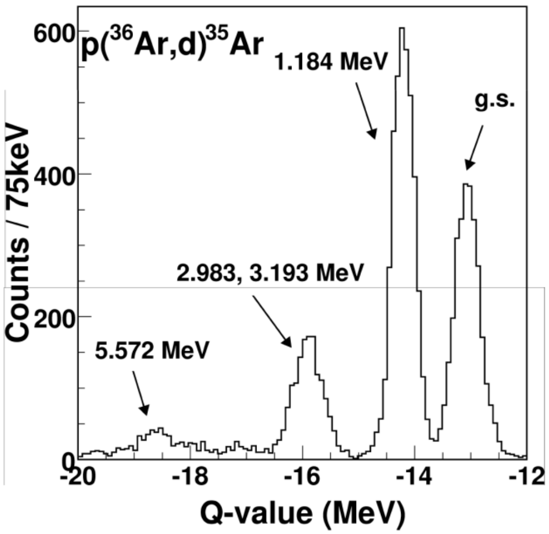
only the large body of reaction analysis could provide the detailed structure of the deuteron and the tensor interaction



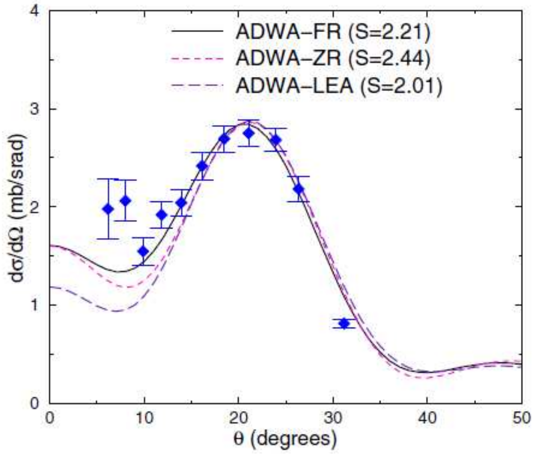
Pieper and Wiringa, ANL

why bother with reactions?

a) nuclei of interest are beams

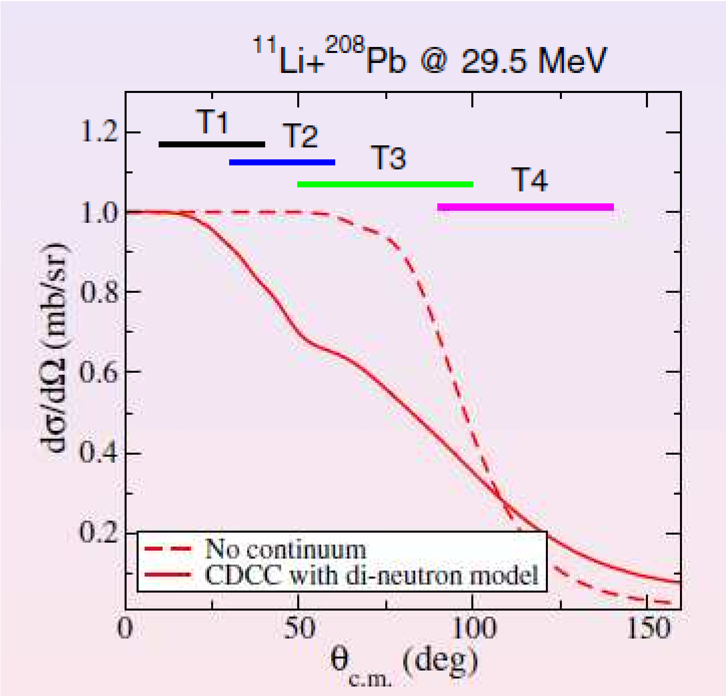


b) offers much more than energy levels

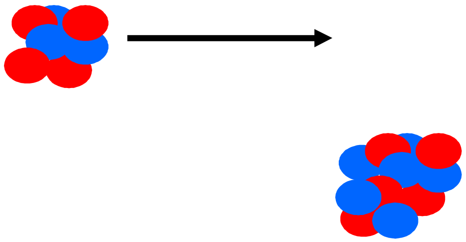


why do reactions? elastic

traditionally used to extract optical potentials, rms radii, density distributions.

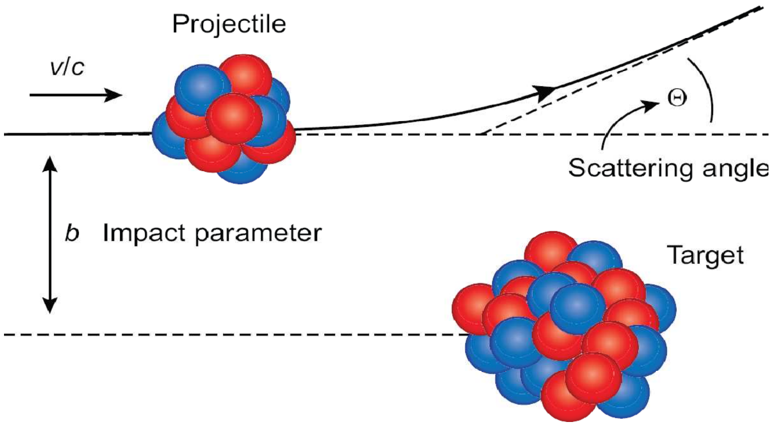


[Moro, talk at ECT* April 2010]



why do reactions? inelastic

X(^{11}Be, ^{11}Be*)



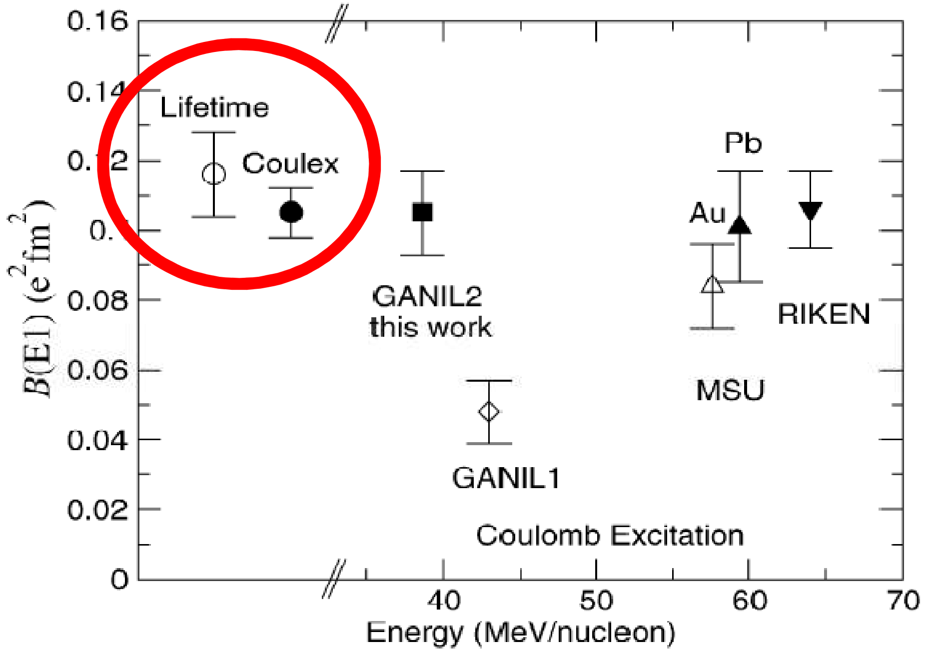


Fig. 2. Comparison of B(E1) values obtained from lifetime and Coulomb excitation measurements. The weighted average of lifetime measurements [3] (open circle) is plotted on the left along with the weighted average (solid circle) of three Coulomb excitation measurements (solid symbols). The individual Coulomb excitation measurements, GANIL (this work, square), MSU (up triangle) [6], RIKEN (down triangle) [7], and a previous GANIL experiment (diamond) [4], are plotted versus the beam energy.

traditionally used to extract electromagnetic transitions or nuclear deformations

[Summers, FN, et al, PLB 650 (2007) 124]

why do reactions? transfer

traditionally used to extract spin,parity and spectroscopic factors

example:
^{132}Sn(d,p)^{133}Sn

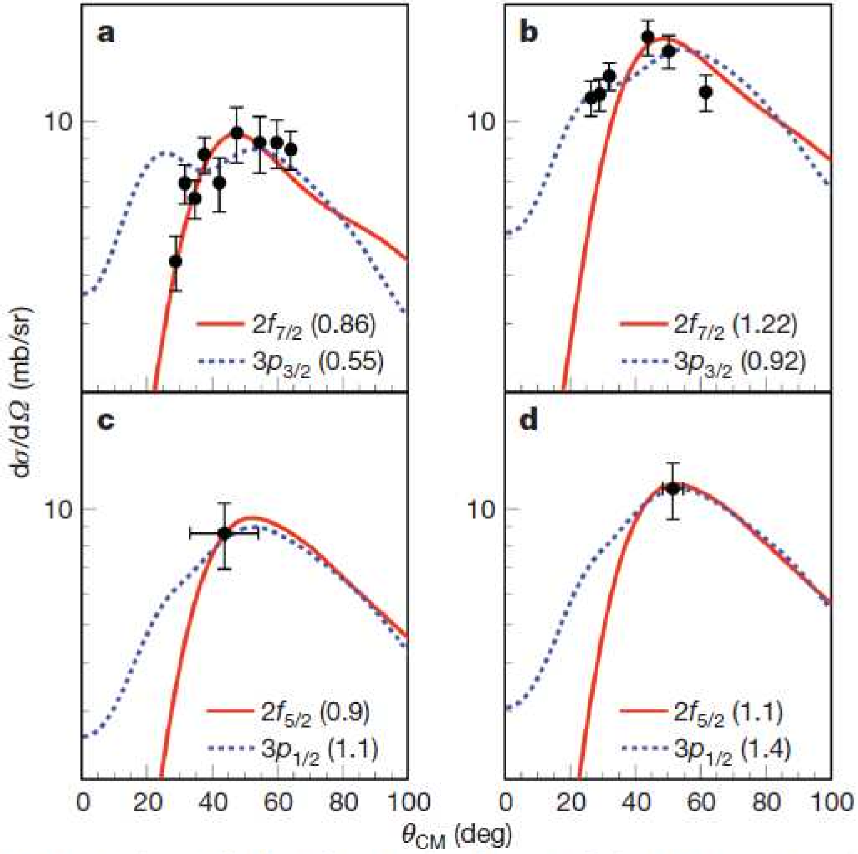


Table 1 | Properties of the four single-particle states populated by the ^{132}Sn(d,p)^{133}Sn reaction

E_x (keV)	J^π	Configuration	S	C^2 (fm^{-1})
0	$7/2^-$	$^{132}\mathrm{Sn}_{gs} \otimes \nu_{f7/2}$	0.86 ± 0.16	0.64 ± 0.10
854	$3/2^-$	$^{132}\mathrm{Sn}_{gs} \otimes \nu_{p3/2}$	0.92 ± 0.18	5.61 ± 0.86
1,363 ± 31	$(1/2^-)$	$^{132}\mathrm{Sn}_{gs} \otimes \nu_{p1/2}$	1.1 ± 0.3	2.63 ± 0.43
2,005	$(5/2^-)$	$^{132}\mathrm{Sn}_{gs} \otimes \nu_{f5/2}$	1.1 ± 0.2	$(9 \pm 2) \times 10^{-4}$

[K. Jones et al, Nature 2010]

why do reactions? transfer

^{11}Li(p,t)^{9}Li@ 3 A MeV

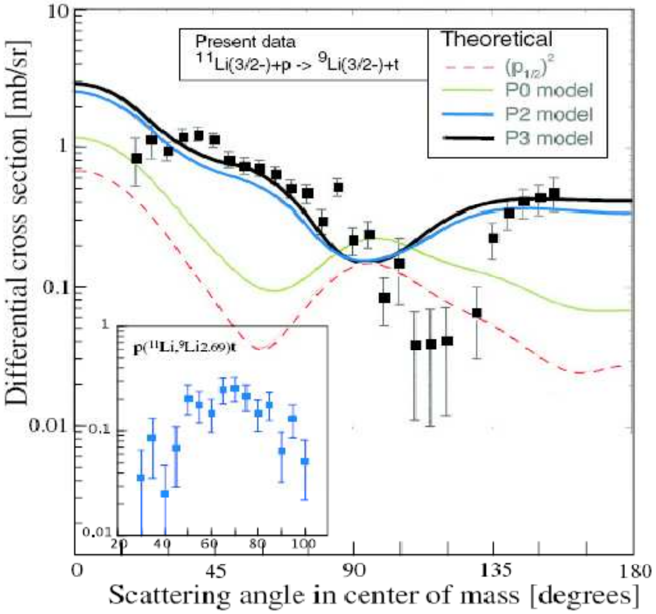


FIG. 3 (color online). Differential cross sections of the (p, t) reaction to the ground state of ^{9}Li and to the first excited state (insert). Theoretical predictions using four different wave functions were shown by curves. See the text for the difference of the wave functions.

measured both ground state and excited state ^{9}Li

[Tanihata et al, PRL 100, 192502 (2008)]

traditionally used to study two nucleon correlations and pairing

why do reactions? breakup

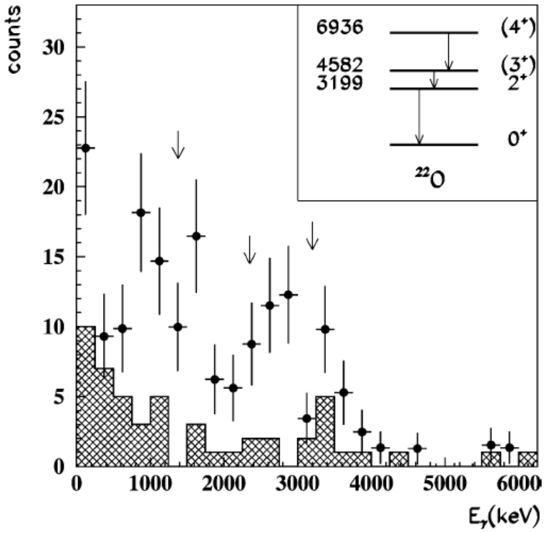
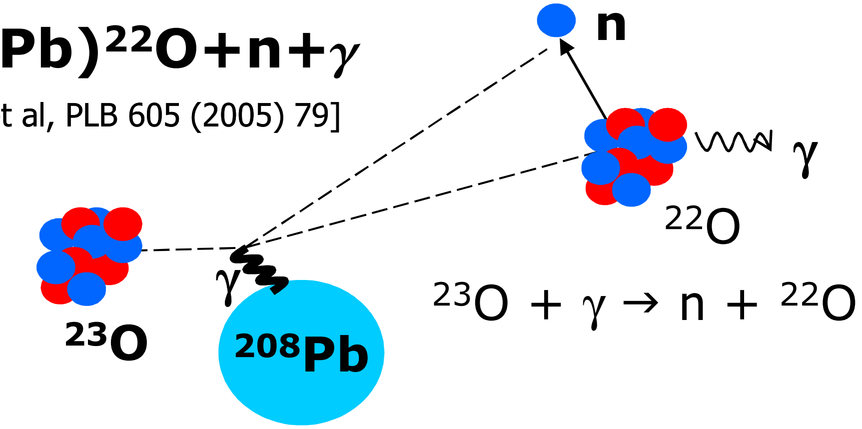


Fig. 1. Doppler corrected γ-ray spectra measured in coincidence with an ^{22}O fragment and one neutron for Pb (symbols) and C (shaded area) targets. Arrows indicate the strongest γ transitions as expected from the ^{22}O level scheme of Ref. [10] (partial level scheme shown as inset; level energies are in keV).

^{23}O(Pb,Pb)^{22}O+n+γ

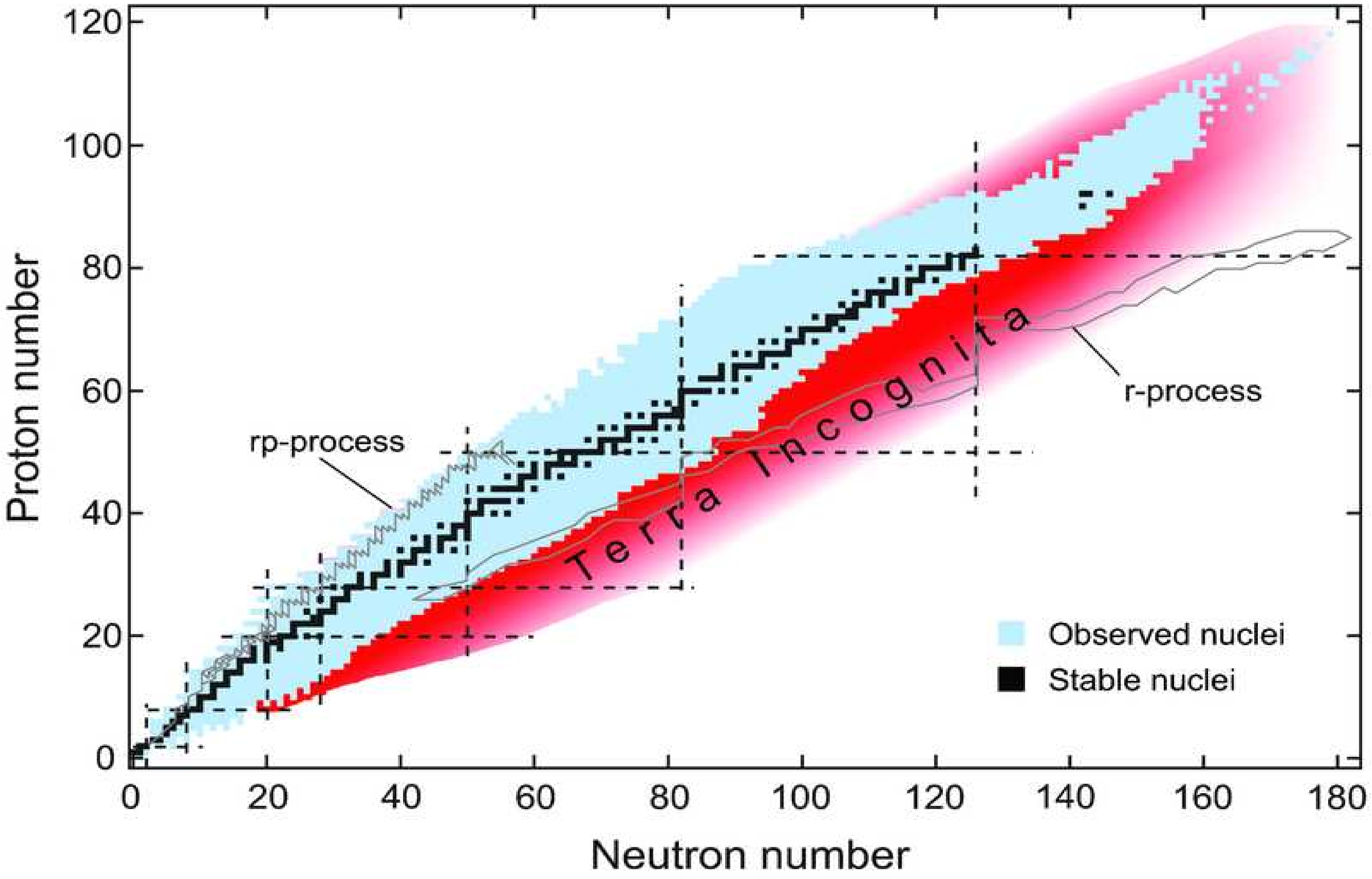
[Nociforo et al, PLB 605 (2005) 79]



^{23}O + γ → n + ^{22}O

and then there is the whole universe…

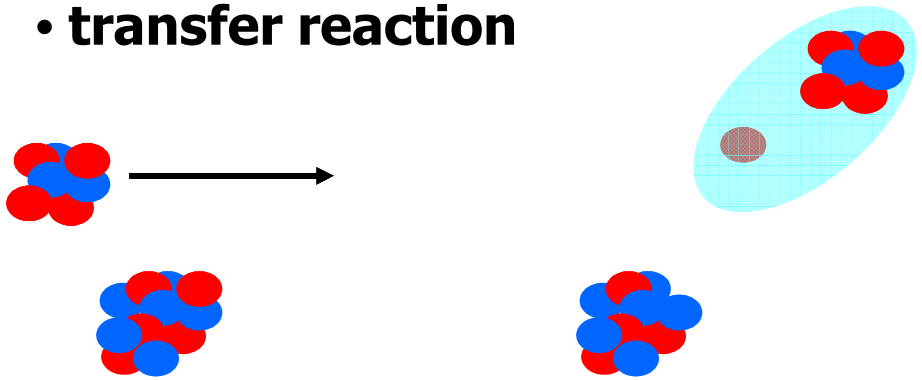
nucleosynthesis in the nuclear chart



indirect methods: nuclear reactions

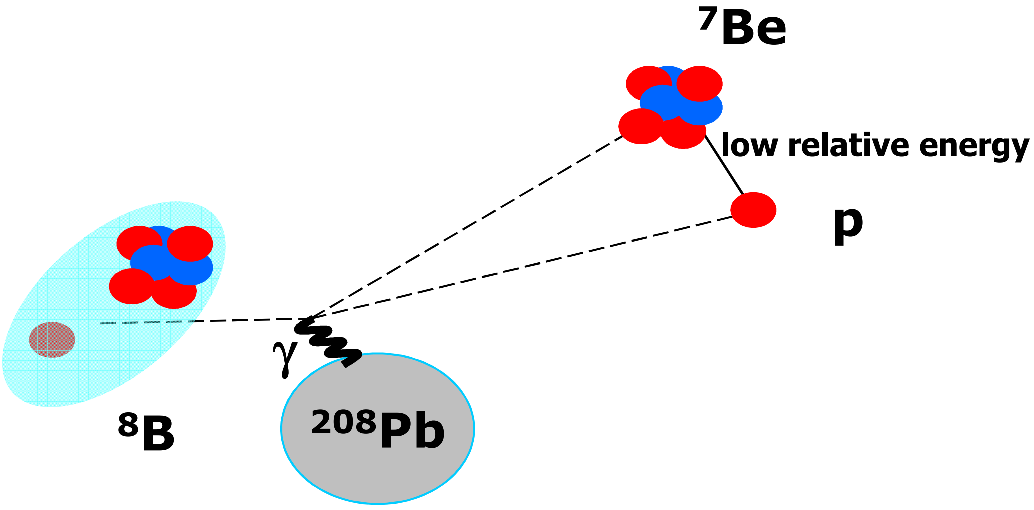
- **direct measurement** $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$

- **transfer reaction**

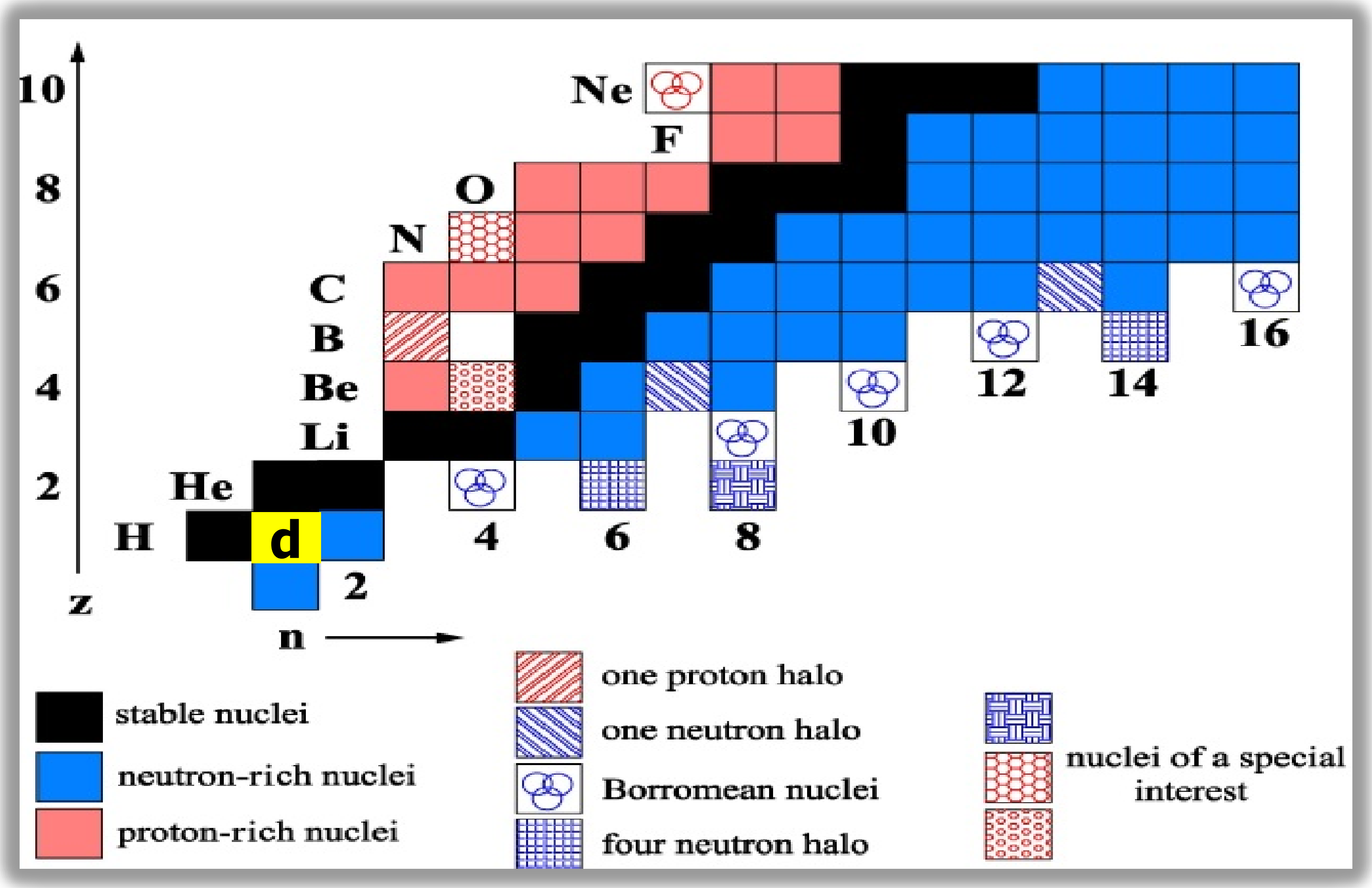


$^{14}\mathrm{N}(^{7}\mathrm{Be},{}^{8}\mathrm{B})^{13}\mathrm{C}$

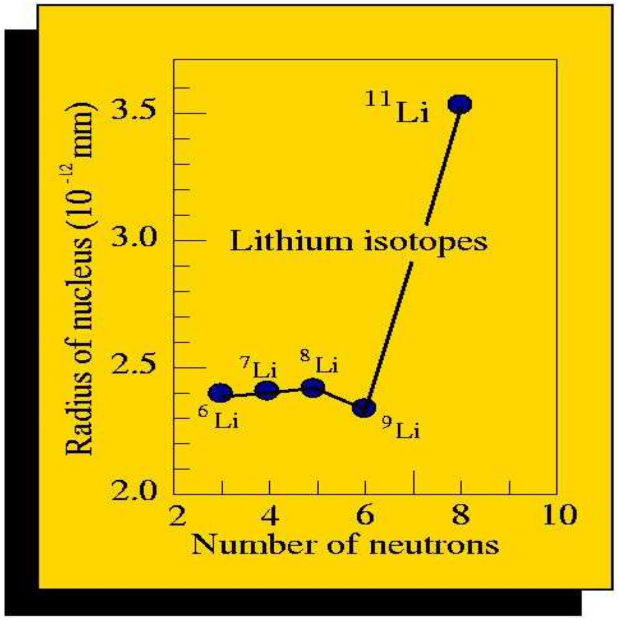
- **Coulomb dissociation**

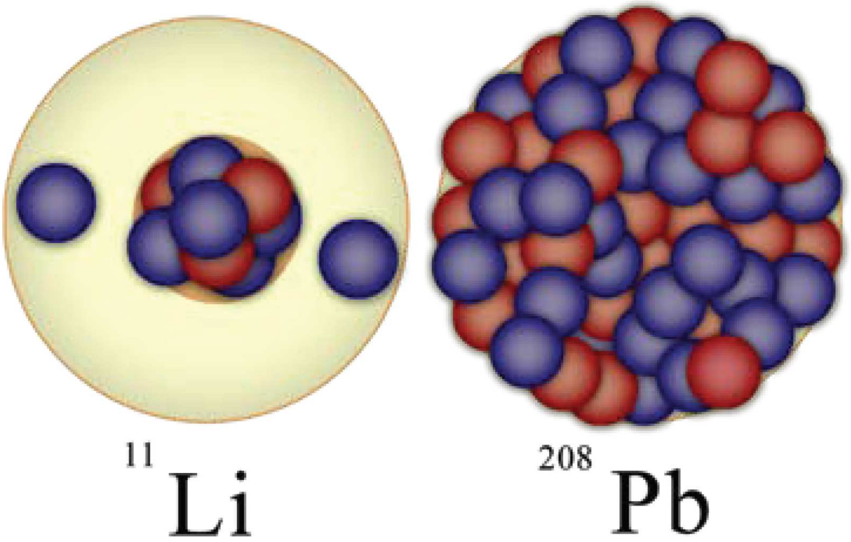


new phenomena with rare isotopes



weakly bound systems: halo nuclei





Very large spatial extension:

correct asymptotic behaviour needed

finite range effects crucial

the latest on halos

^{22}C

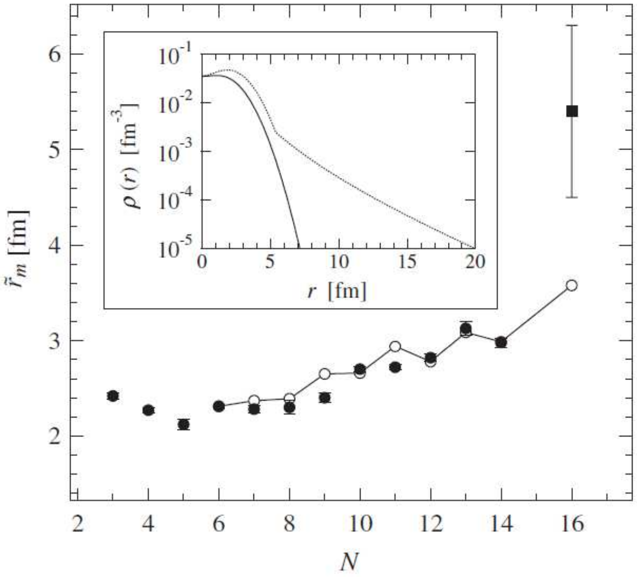


FIG. 2. The $\tilde{r}_m$ as a function of the neutron number of C isotopes. The filled square and circles show the present result and those determined at GSI [14], respectively, while open symbols are the result of the calculation [22]. The lines connect the open circles. The inset shows $\rho_p(r)$ (solid line) and $\rho_n(r)$ (dotted line) of ^{22}C for the determined parameter. See text.

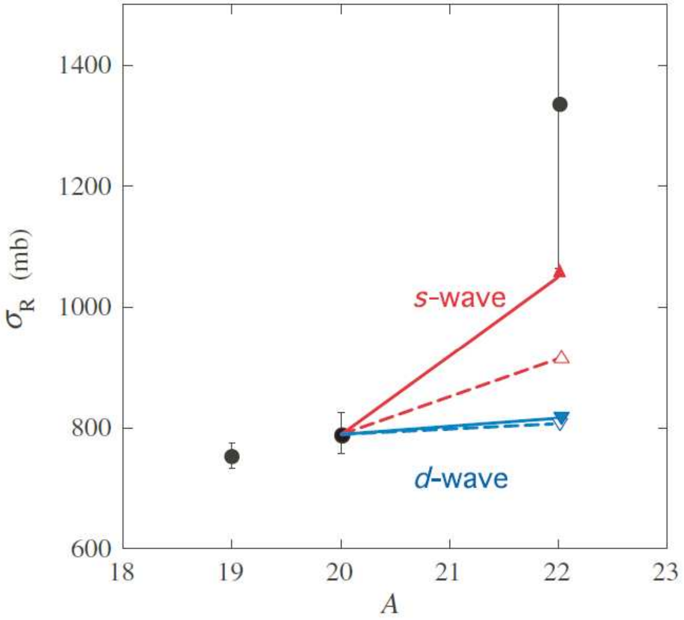
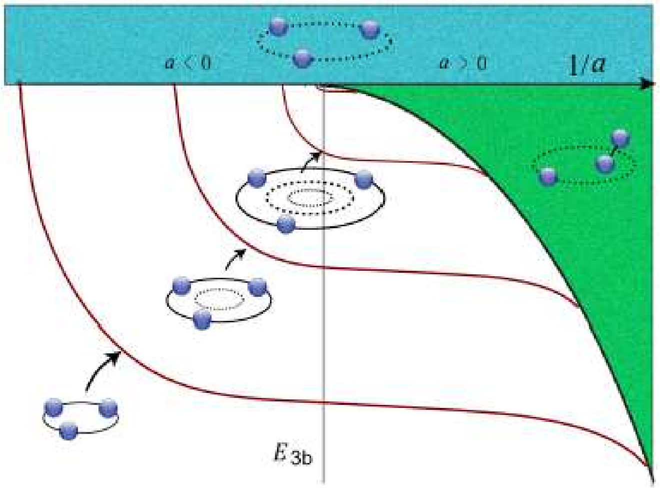


FIG. 3 (color). The σ_R for $f = 1.0$ (red triangles) and that for $f = 0.0$ (blue triangles), with $S_{2n} = 420$ keV (open symbols) and $S_{2n} = 10$ keV (closed symbols), respectively. The lines are to guide the eye. The experimental data (solid circles) as a function of the mass number of C isotopes are also plotted.

PRL **104,** 062701 (2010)

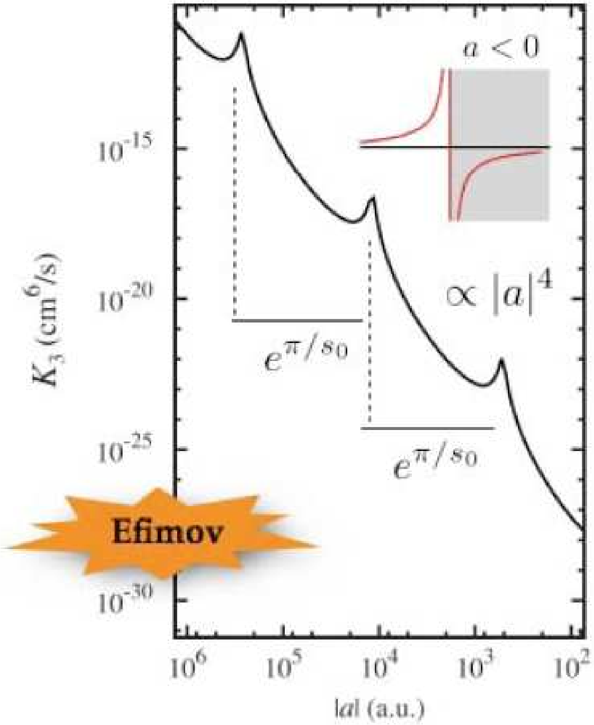
efimov states in nuclear systems?



Efimov spectrum

An infinite series of three-body bound states with $E_n = E_0 e^{-2n\pi/s_0}$ when two-body scattering length $a \rightarrow \infty$ ($s_0 \approx 1.00624$)

Wang@Weakly bound systems, INT 2010



in nuclei how can we vary the scattering length!?

D'Incao@Weakly bound systems, INT 2010

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putting in perspective reaction theory

theory = structure x reaction

Compare theory to data: structure=data/reaction

putting in perspective reaction theory

theory = restruciontaucre

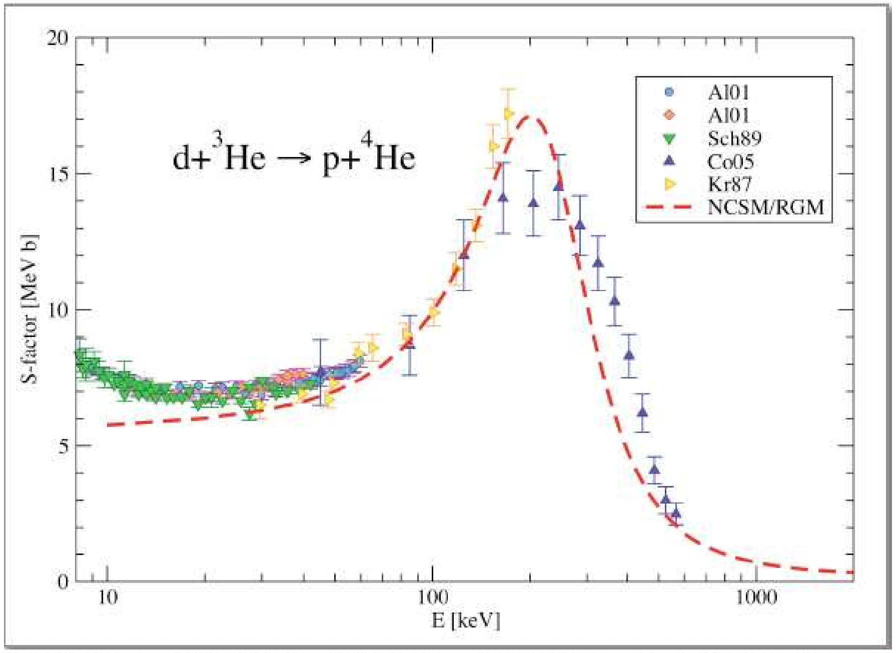
Compare theory to data:
cross section(theory)=cross section(exp) ?

If yes: structure assumptions correct
If no: try again!

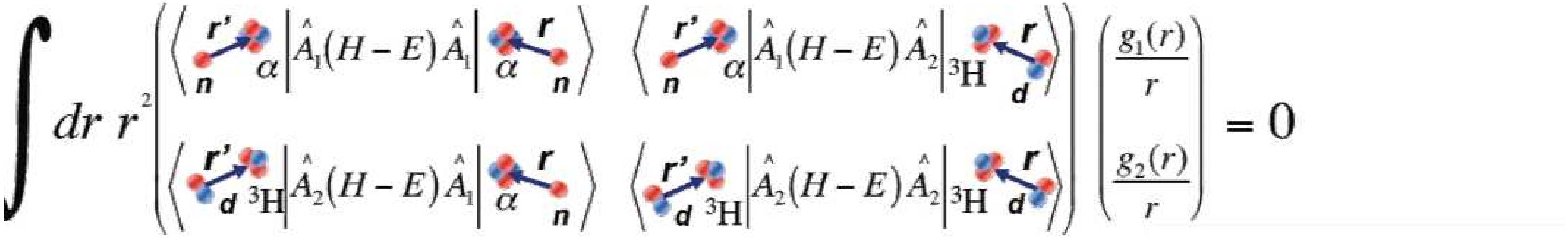
need absolute confidence in reaction model

putting in perspective reaction theory

Unified structure and reactions



[S. Quaglioni and P. Navrátil, PRL101, 092501 (2008); PRC79, 044606 (2009)]

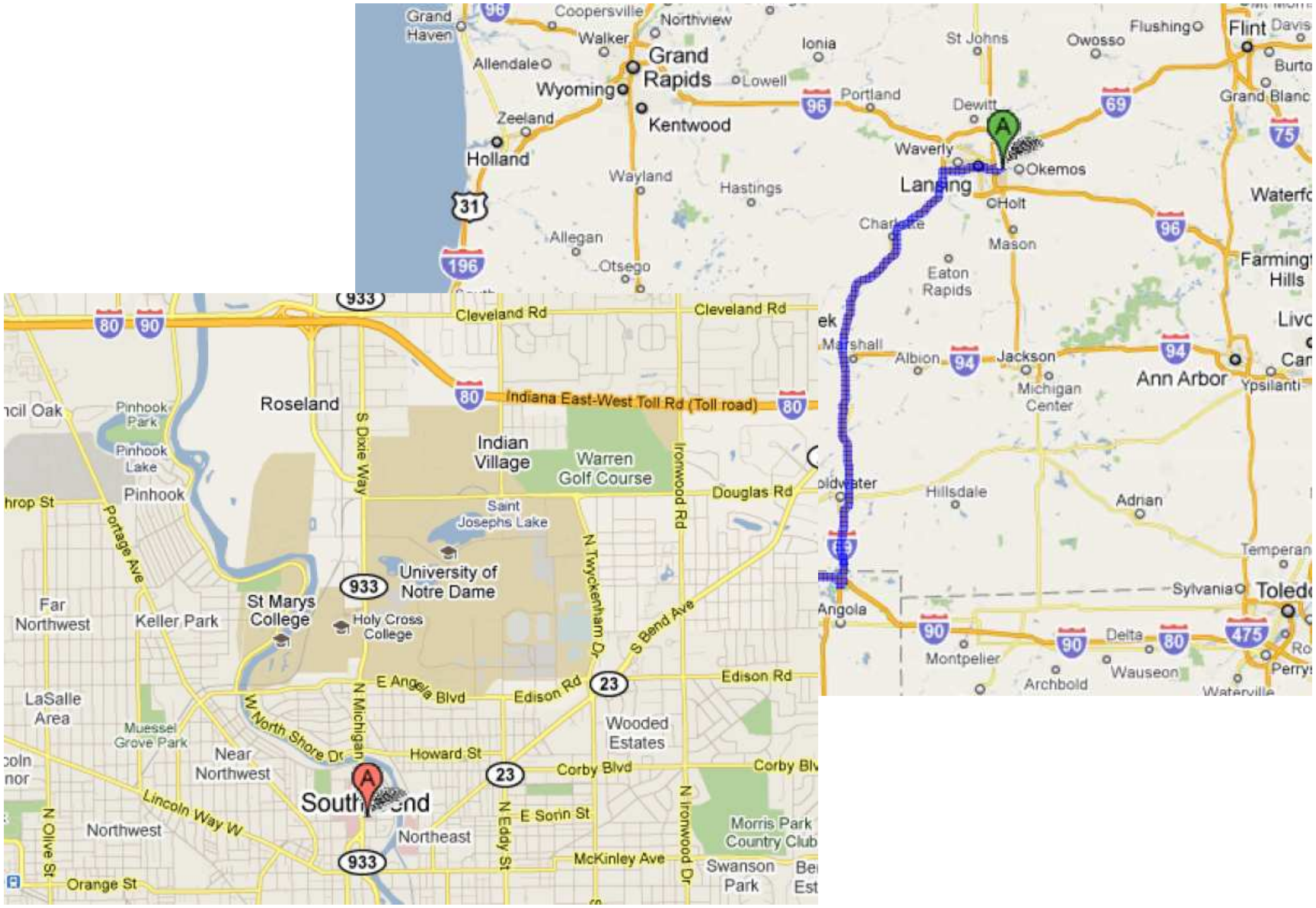


$$\int dr\, r^2 \begin{pmatrix} \left\langle {}^{r'}_{n}\ \alpha \middle| \hat{A}_1 (H-E) \hat{A}_1 \middle| \alpha\ {}^{r}_{n} \right\rangle & \left\langle {}^{r'}_{n}\ \alpha \middle| \hat{A}_1 (H-E) \hat{A}_2 \middle| {}^3\mathrm{H}\ {}^{r}_{d} \right\rangle \\ \left\langle {}^{r'}_{d}\ {}^3\mathrm{H} \middle| \hat{A}_2 (H-E) \hat{A}_1 \middle| \alpha\ {}^{r}_{n} \right\rangle & \left\langle {}^{r'}_{d}\ {}^3\mathrm{H} \middle| \hat{A}_2 (H-E) \hat{A}_2 \middle| {}^3\mathrm{H}\ {}^{r}_{d} \right\rangle \end{pmatrix} \begin{pmatrix} \frac{g_1(r)}{r} \\ \frac{g_2(r)}{r} \end{pmatrix} = 0$$

multiple channel reaction theory

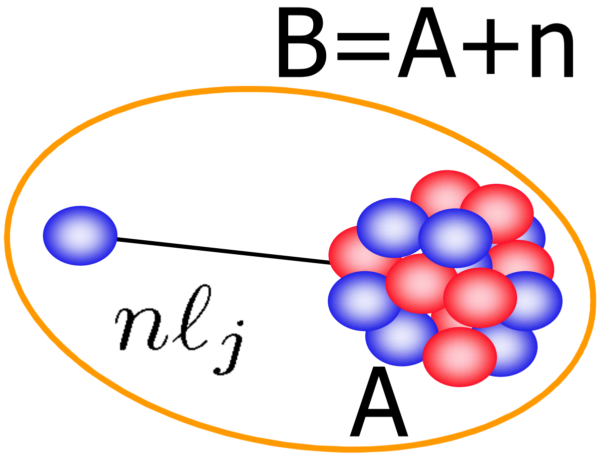
Reaction theories need to keep track of multiple channels

from the many body problem to few body reactions



Reaction theories need to map onto the many-body problem!

meeting point between reactions and structure



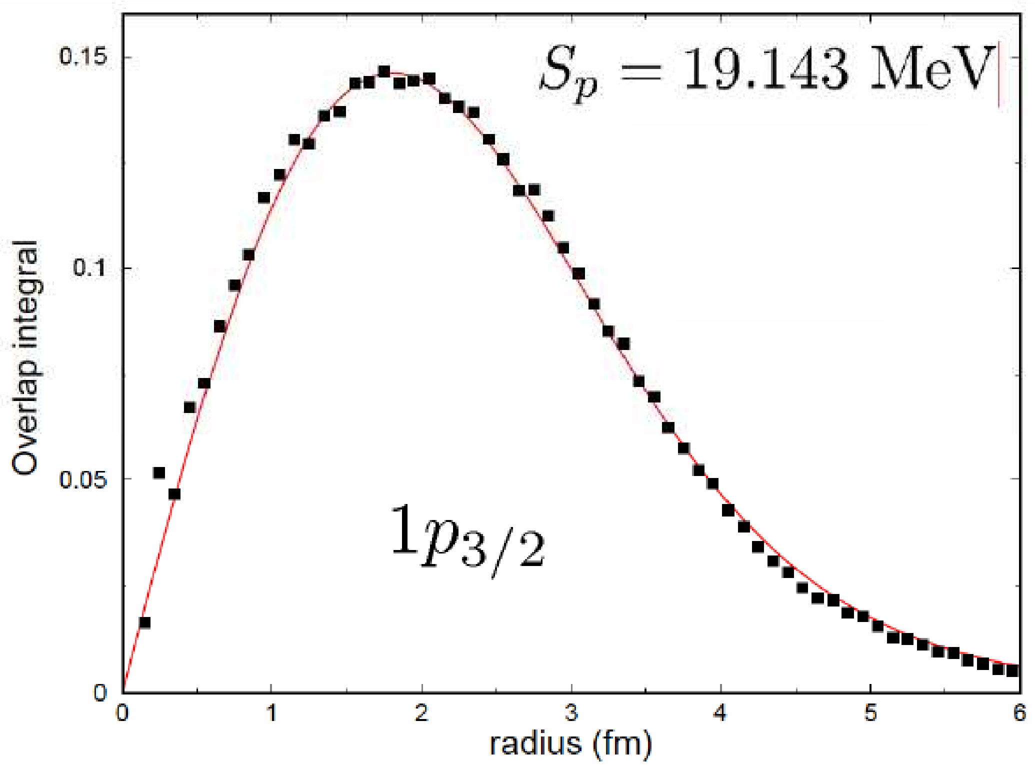
overlap function

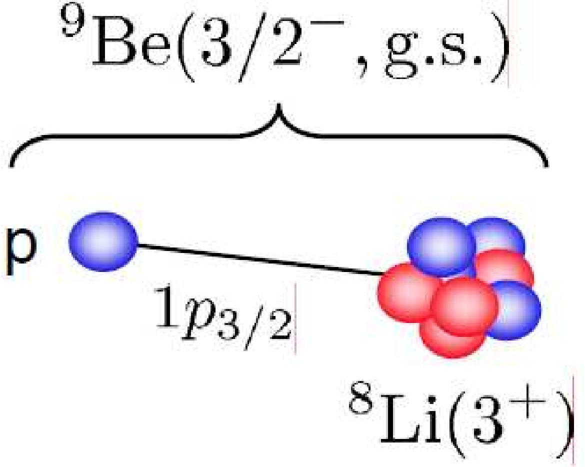
$$I_{I_A:I_B}(\mathbf{r}) = \langle \Phi^A_{I_A}(\xi_A) | \Phi^B_{I_B}(\xi_A, \mathbf{r}) \rangle$$

spectroscopic factor (S_{nlj}):
norm of overlap function

microscopic one nucleon overlap functions

$$\langle \vec{r}, {}^{8}\mathrm{Li}(3^{+}) | {}^{9}\mathrm{Be}(3/2^{-}, \mathrm{g.s.}) \rangle$$





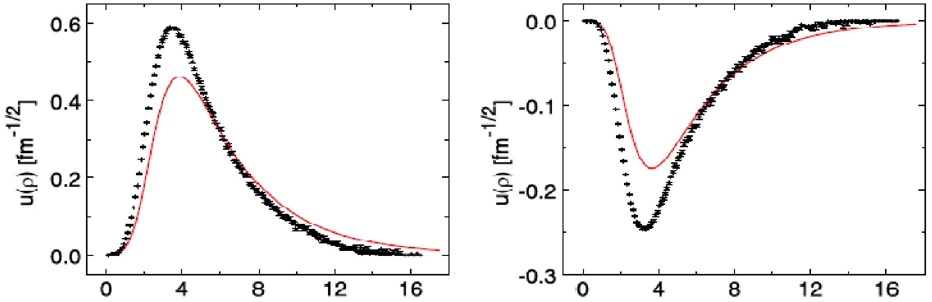
- Microscopic overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a few cases

— Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ Spectroscopic factor

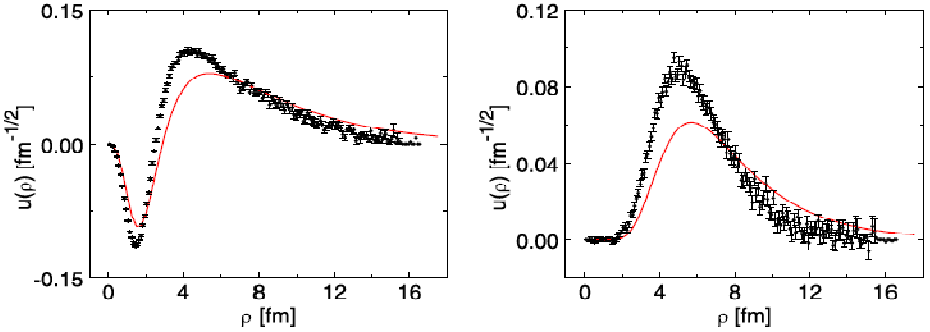
$$r_V = r_{so} = \text{fitted},\ a_V = a_{so} = \text{fitted},\ V_{so} = 6.0$$

[Wiringa et al., ANL 2007]

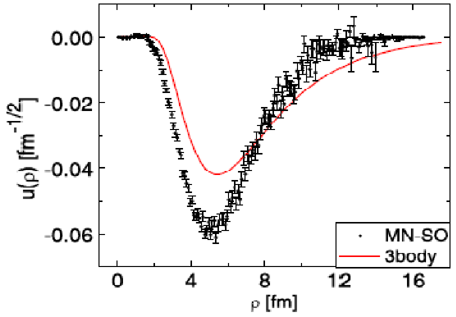
microscopic 2n overlap functions



(a) $K = 2$ s-waves (b) $K = 2$ p-waves



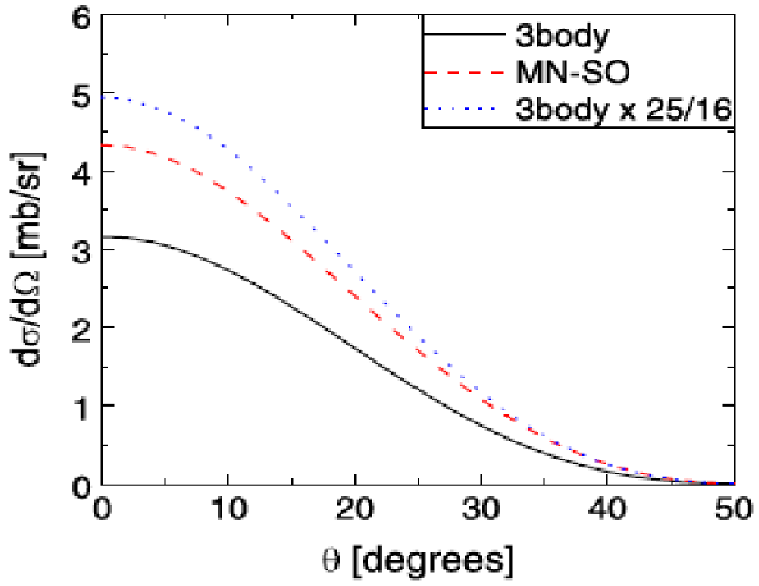
(c) $K = 0$ s-waves (d) $K = 6$ d-waves



(e) $K = 6$ f-waves

^{6}He 2n overlap functions

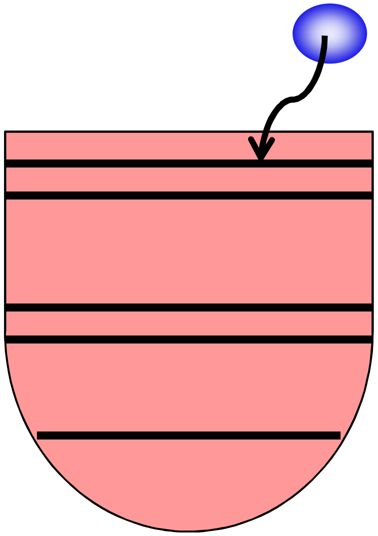
^{6}He(p,t)^{4}He @ 25 MeV



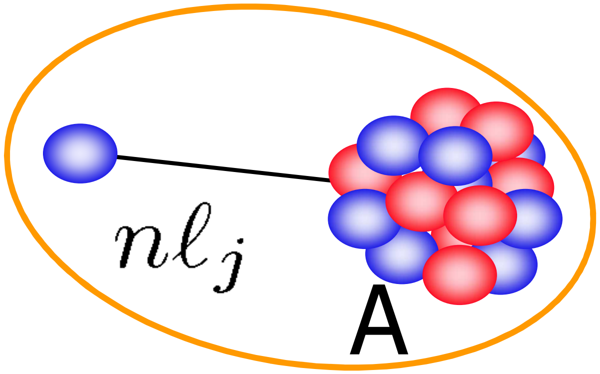
(b) Linear scale for small angles.

[Brida, PhD thesis, MSU 2009]
[Brida and Nunes, NPA]

single particle approximation



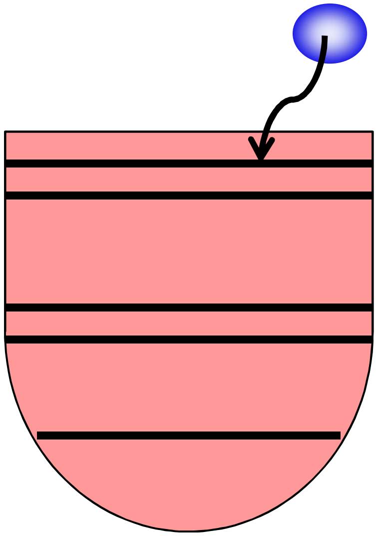
$$(T_r + V_{nA} + \varepsilon)\varphi_{nlj}(r) = 0$$



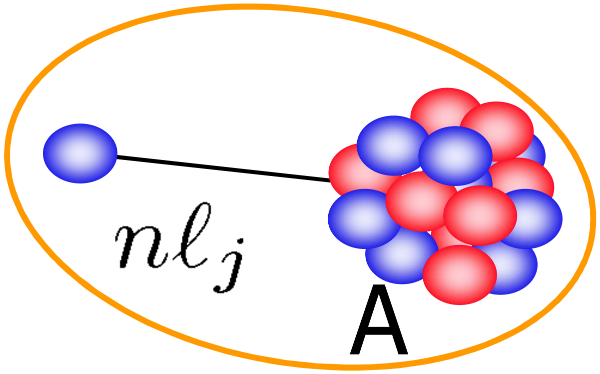
nucleon feels a mean field generated by core nucleons V_{nA}

- specific n,l,j and separation energy
- assumptions about single particle parameters V_{nA}

single particle approximation



$$\boxed{S^{B}_{nlj} = A^{2}_{nlj} = \frac{C^{2}_{lj}}{b^{2}_{nlj}}}$$



Same radial dependence at large distances:

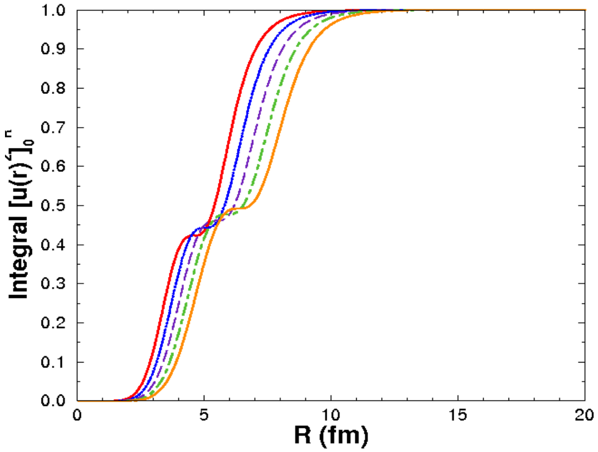
$$I_{AB}(r) \xrightarrow{r>R_N} C_{lj}\, i\kappa \quad h_l(i\kappa r) \qquad \varphi_{nlj}(r) \xrightarrow{r>R_N} b_{nlj}\, i\kappa \quad h_l(i\kappa r)$$

Extend that assumption within the range of the interaction:

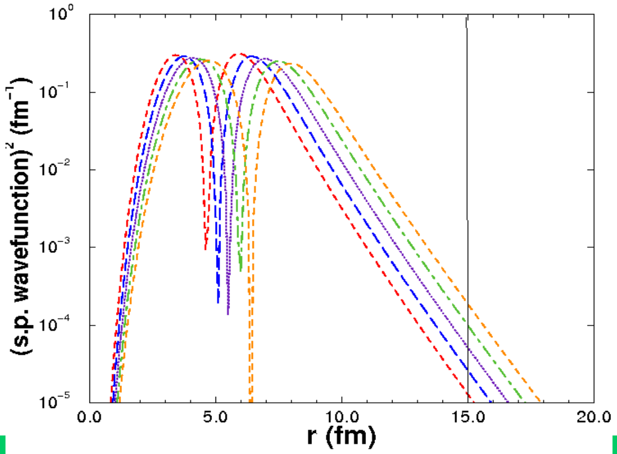
$$\boxed{I_{AB}(r) = A_{nlj}\,\varphi_{nlj}(r)}$$

Overlap function, SF and ANC

S (spectroscopic factor) – **volume** property



C (asymptotic normalization coefficient) – **asymptotic** property



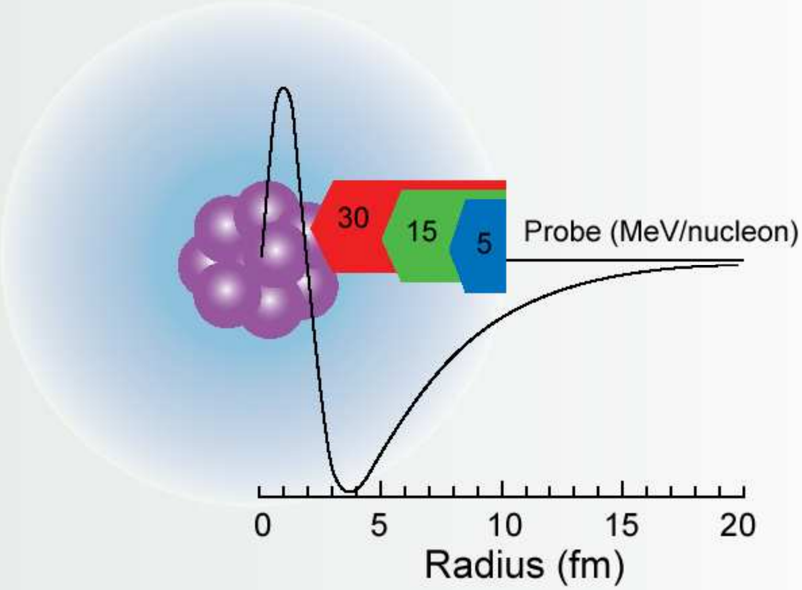
reaction theory for studying rare isotopes

- ☐ introduction: what are we after?
- ☐ motivation: why do we need reactions?
- ☐ reaction theory: overall perspective and some connections
- ☐ specific examples:
  - ☐ combined method for transfer reactions
  - ☐ transfer and breakup: CDCC versus Faddeev
  - ☐ transfer versus knockout (the Ar isotopes)
  - ☐ breakup reaction for astrophysics
- ☐ summary and outlook

(d,p) reactions: combined method

From sub-Coulomb transfer reaction obtain ANC $\sigma^{th}(b) = C^2 \mathrm{M}_{out}^{\;2}$

From higher energy transfer reaction obtain SF consistent with ANC



The overlap function for $^{19}C \rightarrow n + ^{18}C$ in arbitrary units. The radial sensitivity of the $^{18}C(d,p)^{19}C$ cross section is represented by the colored bars for different beam energies.

$$\sigma^{th}(b) = (S\,\mathrm{M}_{in}(b) + C\,\mathrm{M}_{out})^2$$

$$C^2 = Sb^2$$

Combined method provides a handle on single particle parameters!

(d,p) reactions: combined method

Method has been checked for consistency for a number of nuclei

Benchmarked for ^{48}Ca(d,p)^{49}Ca against ^{48}Ca(n,γ)^{49}Ca

Motivated several new experiments at NSCL, ORNL, TexasA&M, etc

Mukhamedzhanov and FN, Phys. Rev. C 72, 017602 (2005)

Pang, Mukhamedzhanov and FN, Phys. Rev. C 75, 024601 (2007)

Mukhamedzhanov, FN and Mohr, Phys. Rev. C 77, 051601R (2008)

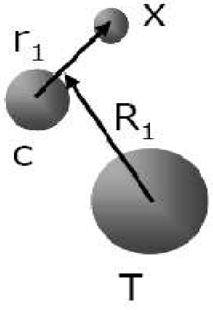
(d,p) reactions: transfer and breakup

**until recently best reaction theories for (d,p)
consider breakup to all orders but transfer to first order.**

**is this a valid assumption?
when is it a valid assumption?**

need full Faddeev calculation

CDCC versus Faddeev formalism

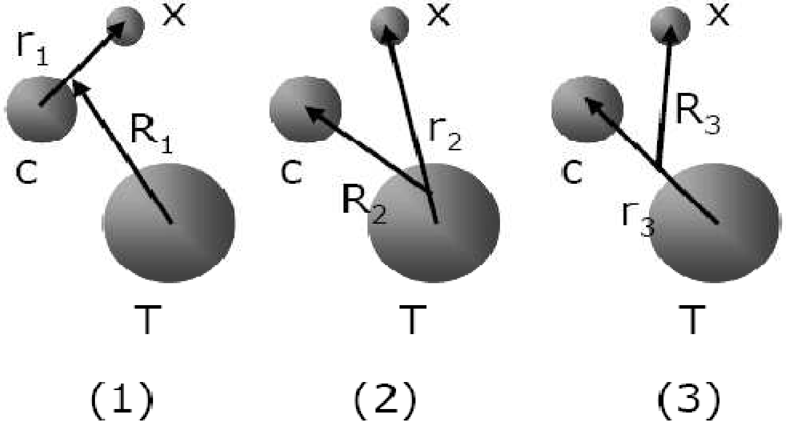


CDCC Formalism

$$\boxed{[H_{3\mathrm{b}} - E]\Psi^{(1)}(\mathbf{r}_1, \mathbf{R}_1) = 0}$$

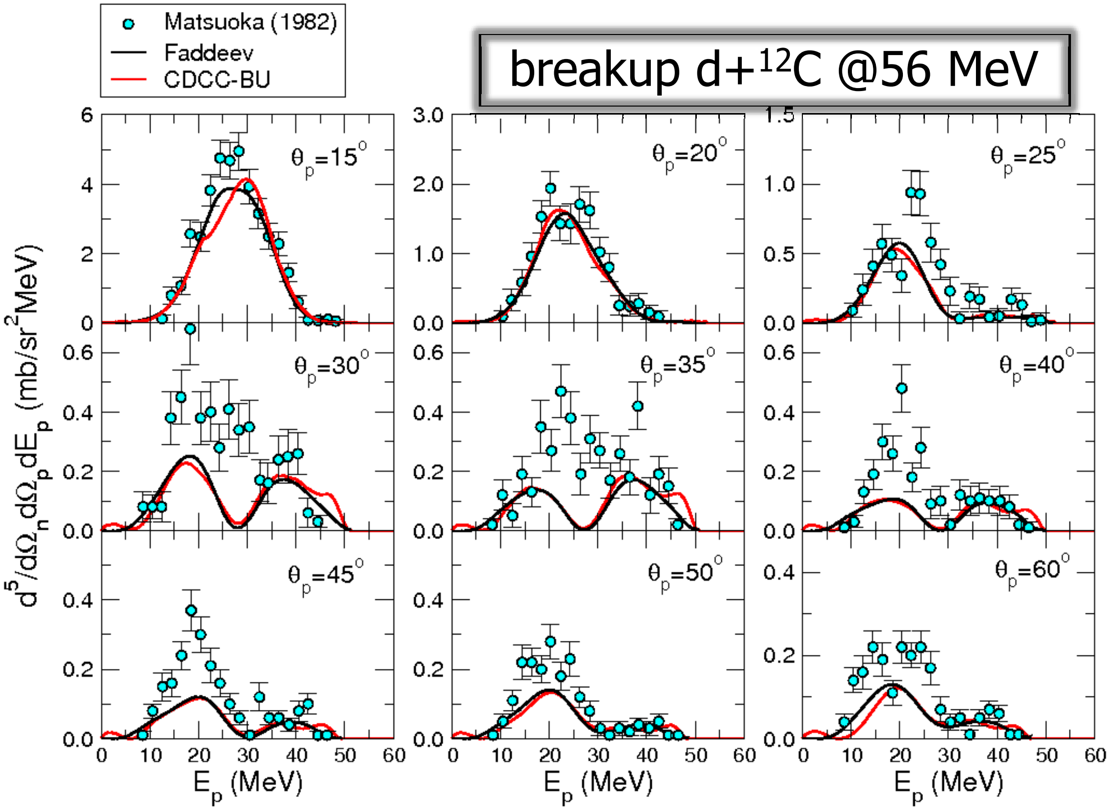
Faddeev Formalism

$$\begin{array}{rcl} (E - T_1 - V_{xc})\Psi^{(1)} & = & V_{xc}(\Psi^{(2)} + \Psi^{(3)}) \\ (E - T_2 - V_{ct})\Psi^{(2)} & = & V_{ct}(\Psi^{(3)} + \Psi^{(1)}) \\ (E - T_3 - V_{tx})\Psi^{(3)} & = & V_{tx}(\Psi^{(1)} + \Psi^{(2)}) \end{array}$$

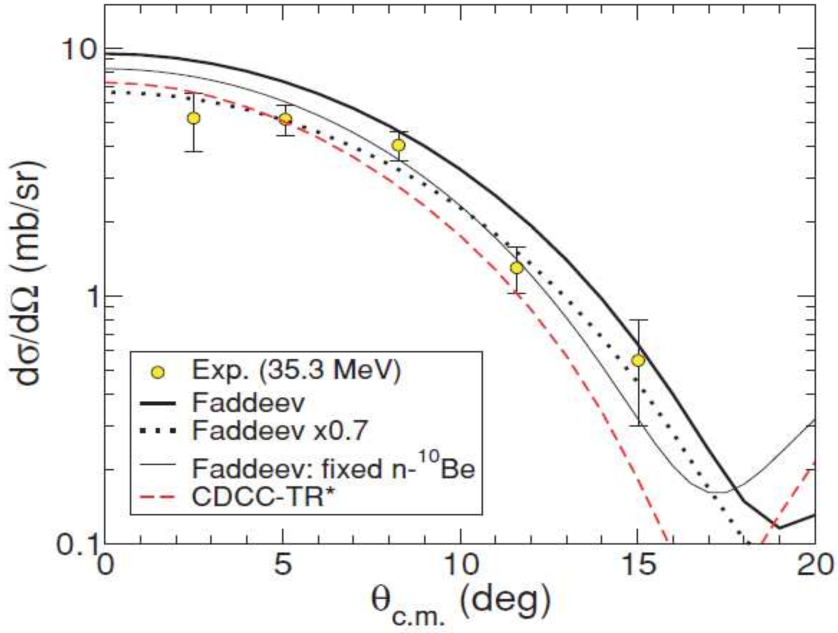


comparing CDCC with Faddeev

breakup d+^{12}C @56 MeV

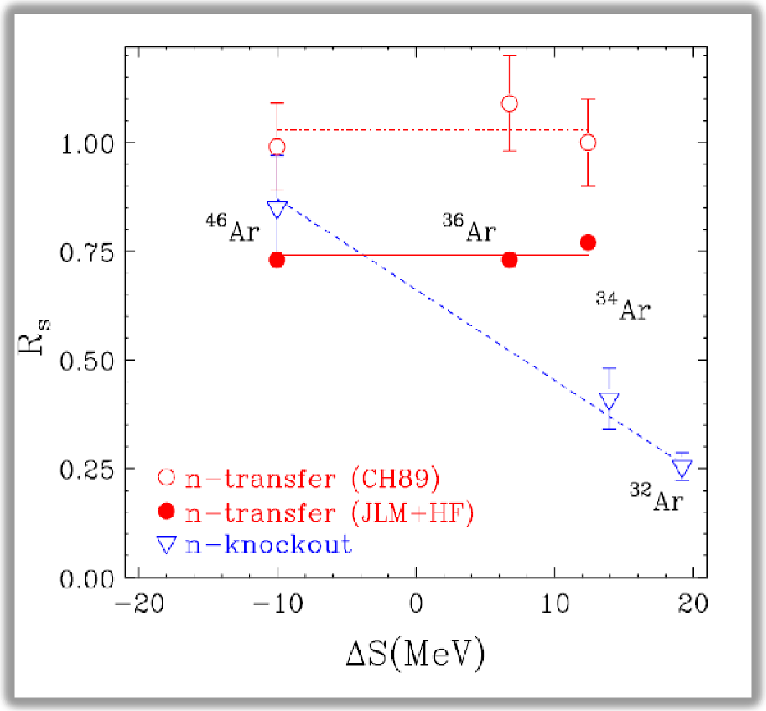


^{11}Be(p,d)^{10}Be at $E_p = 38.4$ MeV



[Deltuva, Moro, Nunes and Fonseca, PRC76, 064602]

transfer versus knockout



[Jenny Lee et al, PRL 2009]

[Gade et al, PRL 93, 042501]

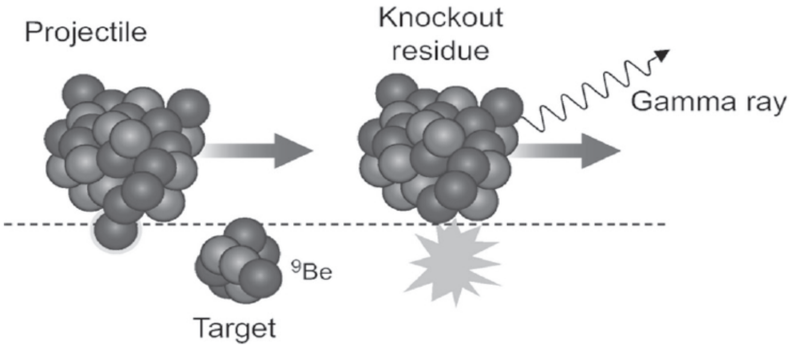


Fig. 14.9. Schematic of a nuclear knockout reaction. Reprinted from [3] with permission.

error from reaction theory

preliminary

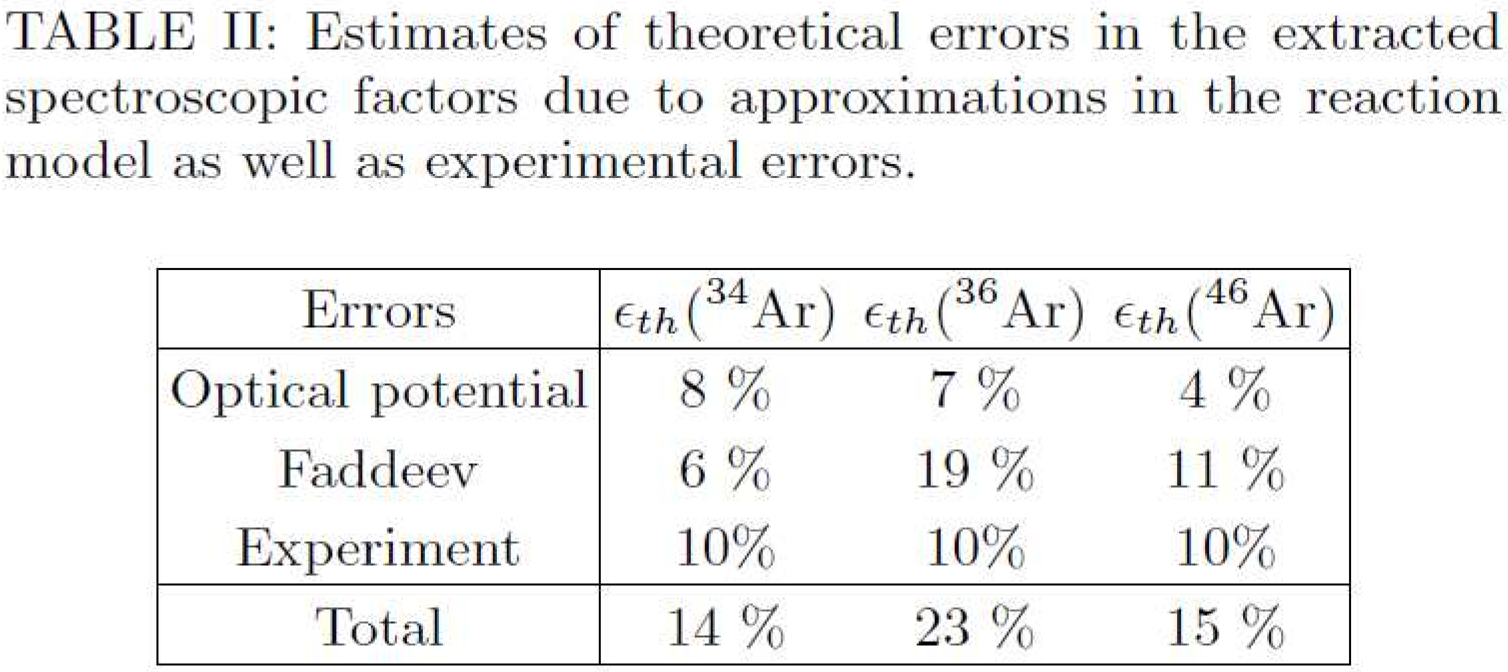
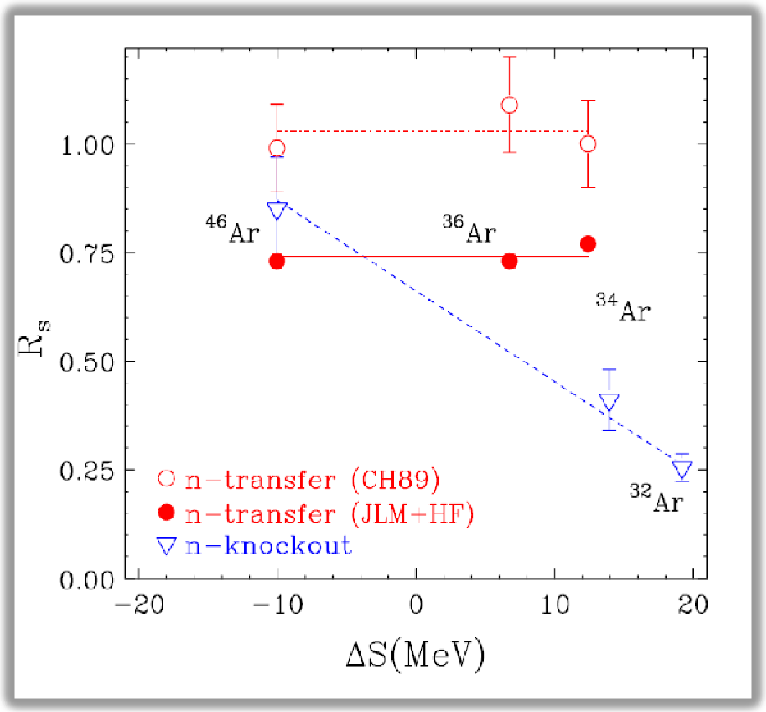


TABLE II: Estimates of theoretical errors in the extracted spectroscopic factors due to approximations in the reaction model as well as experimental errors.

Errors	$\epsilon_{th}(^{34}\mathrm{Ar})$	$\epsilon_{th}(^{36}\mathrm{Ar})$	$\epsilon_{th}(^{46}\mathrm{Ar})$
Optical potential	8 %	7 %	4 %
Faddeev	6 %	19 %	11 %
Experiment	10%	10%	10%
Total	14 %	23 %	15 %

[FN, Deltuva, Hong, 2010]

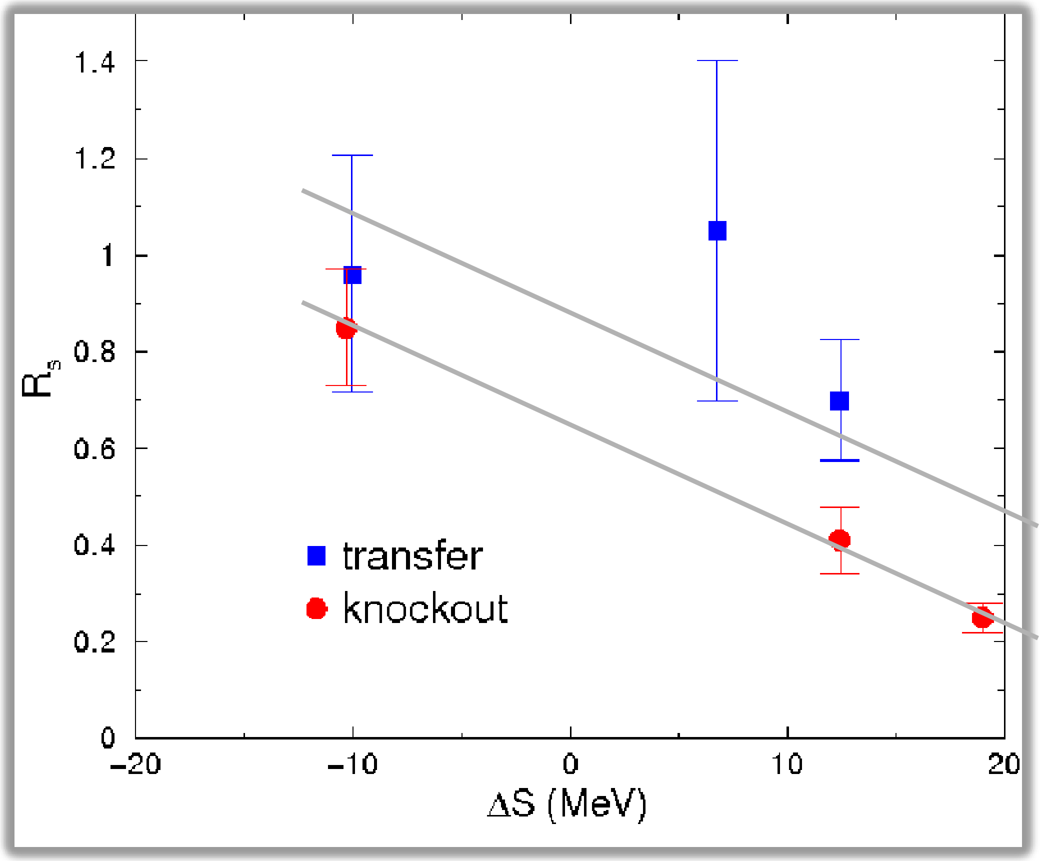
transfer versus knockout



[Jenny Lee et al, PRL 2009]

[Gade et al, Phys. Rev. Lett. 93, 042501]

preliminary



[FN, Deltuva, Hong, 2010]

breakup reactions and (n,γ): methodology

CDCC + set of single particle parameters

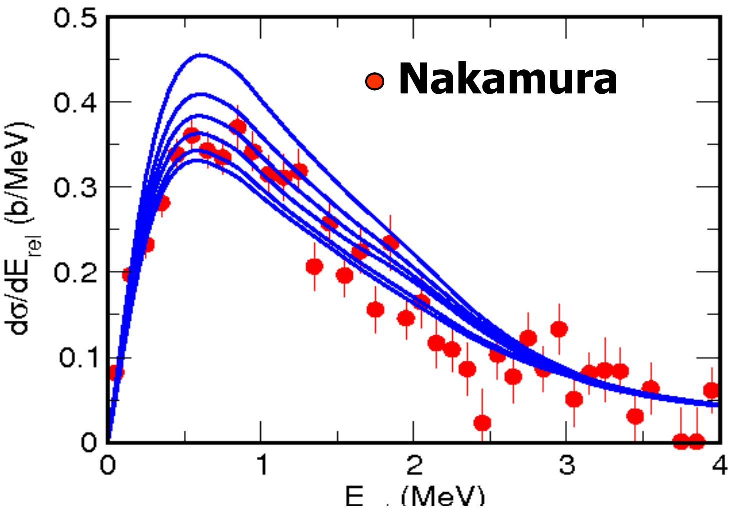
- extract ANC from χ^2 minimum
- error from $\varepsilon=\chi_{min}^2+1$

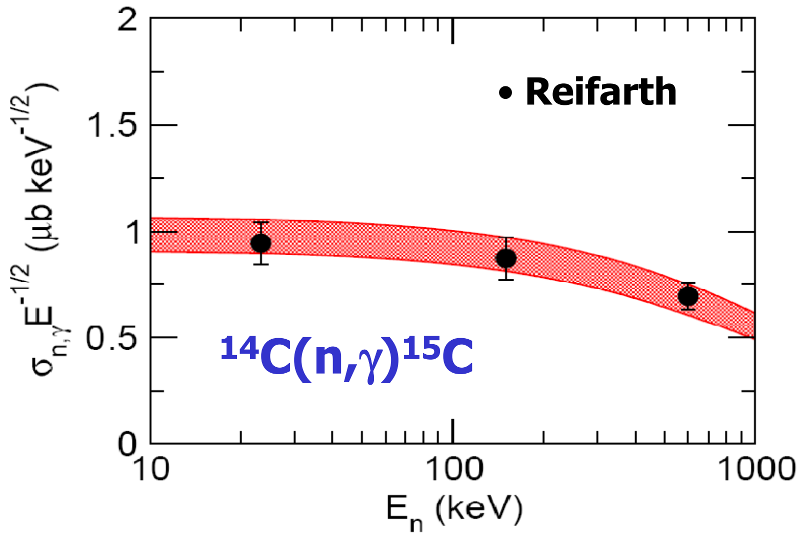
Yao, JPG33 (2006) 1

$$ANC = 1.32 \pm 0.07 \ \mathbf{fm^{-1/2}}$$

Summers and Nunes, PRC78(2009)069908

$^{208}\mathrm{Pb}(^{15}\mathrm{C},^{14}\mathrm{C}+n)^{208}\mathrm{Pb}$@68 MeV/u





Nakamura et al, NPA722(2003)301c
Reifarth et al, PRC77,015804 (2008)

summary of some recent advances

Continuum discretized coupled channels method (CDCC)

- many applications to weakly bound nuclei: good description of data
- extensions to core excitation (also 4-body CDCC)

Coulomb dissociation can be used to extract peripheral (n,γ)

- new methodology based on xs scaling with the ANC
- $^{14}C(n,\gamma)^{15}C$ from Coulomb dissociation consistent with direct capture data

Transfer reactions and combined method

- one benchmark with (n,γ) but many applications with future experiments
- finite range effects can be very important at intermediate energies

Testing CDCC against Faddeev

- disagreement needs to be better understood… new formalism

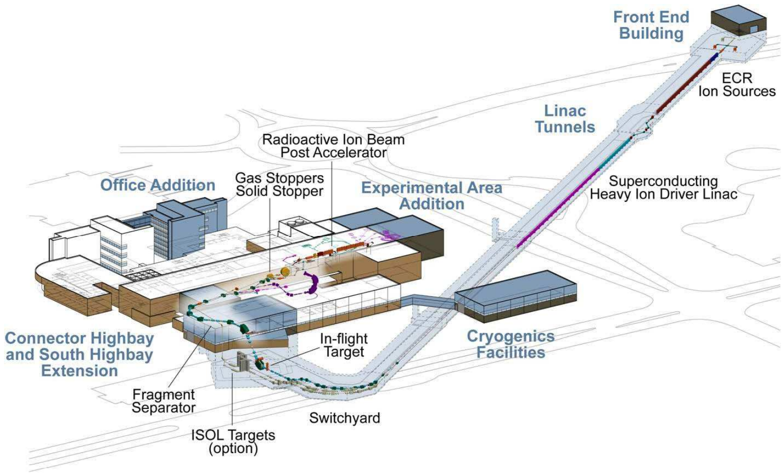
Transfer reactions compared to knockout

- uncertainties in reaction theory have been quantified
- results move toward agreement

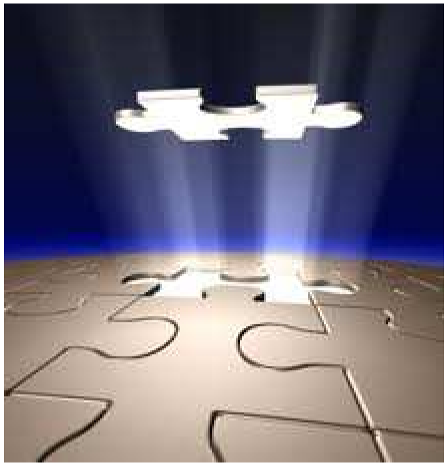
Microscopic overlap functions

- 2n overlap functions show increased spectroscopic strength (compared to 3body)
- significant progress needs to be made in reaction theory before structure models can be tested

the missing piece of the puzzle



- ✓ our hose will increase enormously with FRIB
- ✓ impressive improvements in detector technology
- ✓ last 2 decades incredible advances on nuclear structure theory



reaction theory

main problem

manpower

- less than a handful of researchers at PhD granting institutions

thankyou!

Collaborators: Ngoc Nguyen(MSU), June Hong(MSU), Ivan Brida(ANL),
Pierre Capel, Antonio Moro, Neil Summers, Arnas Deltuva,
Akram Mukhamedzhanov, Peter Mohr, Ron Johnson

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