Improving the theory for transfer reactions

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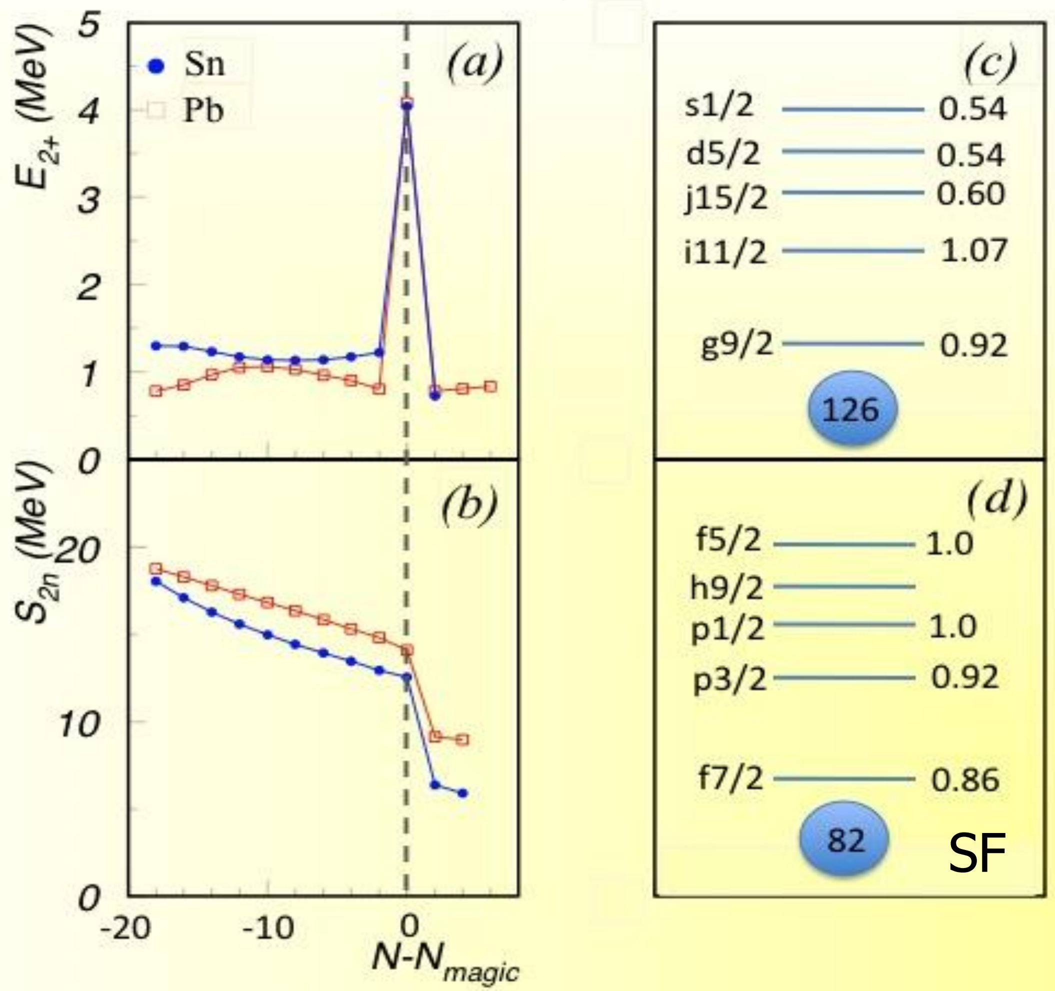
magic numbers

Doubly magic nuclei



^{208}Pb(d,p)^{209}Pb

^{132}Sn(d,p)^{133}Sn

[K. Jones et al, Nature 2010]

spectroscopy of ^{132}Sn

$^{132}Sn(d,p)^{133}Sn$

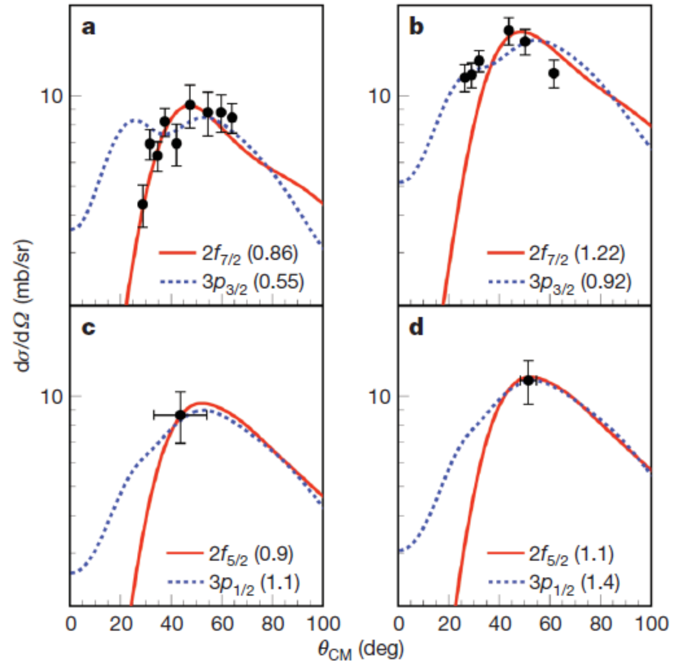


Table 1 | Properties of the four single-particle states populated by the $^{132}Sn(d,p)^{133}Sn$ reaction

E_x (keV)	J^π	Configuration	S	C^2 (fm^{-1})
0	$7/2^-$	$^{132}Sn_{gs} \otimes \nu_{f7/2}$	0.86 ± 0.16	0.64 ± 0.10
854	$3/2^-$	$^{132}Sn_{gs} \otimes \nu_{p3/2}$	0.92 ± 0.18	5.61 ± 0.86
$1,363 \pm 31$	$(1/2^-)$	$^{132}Sn_{gs} \otimes \nu_{p1/2}$	1.1 ± 0.3	2.63 ± 0.43
2,005	$(5/2^-)$	$^{132}Sn_{gs} \otimes \nu_{f5/2}$	1.1 ± 0.2	$(9 \pm 2) \times 10^{-4}$

[K. Jones et al, Nature 2010]

a selection of topics focused on (d,p)

- distorted wave (DWBA) versus adiabatic (ADWA)
- combined method for transfer reactions
- adiabatic finite range
- transfer and breakup: CDCC versus Faddeev
- transfer versus knockout (recent Ar(d,p) data)

(d,p) reactions: deuteron breakup

DWBA: distorted wave Born approximation (1st order)
includes deuteron g.s. only (no breakup)

ADWA: adiabatic wave approximation
takes into account deuteron breakup to all orders
(present implementation neglects remnant and
uses zero range approximation)

[Johnson and Soper, Phys. Rev. C 1, 976(1970)]

A(d,p)B reaction amplitude

adiabatic distorted wave approximation (ADWA)

$$T = \left\langle \chi_f^{(-)} I_{AB} \left| \Delta V \right| I_{pd} \tilde{\chi}_i^{(+)} \right\rangle$$

proton distorted wave
$U_p(R)$

deuteron distorted wave
$U_d(R) = U_n(R) + U_p(R)$

Nucleon Optical potentials

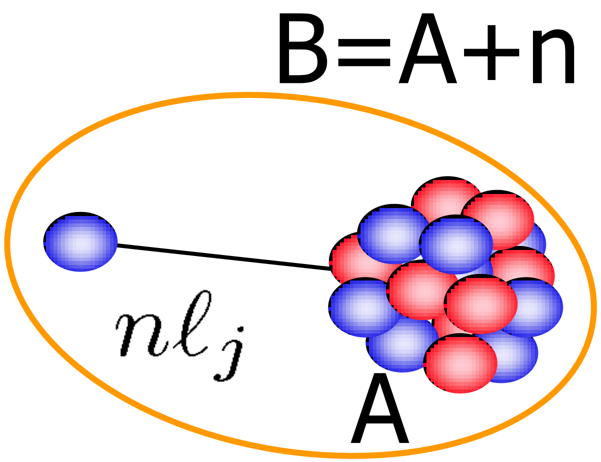
A(d,p)B reaction amplitude

adiabatic distorted wave approximation (ADWA)

$$T = \left\langle \chi_f^{(-)} I_{AB} \middle| \Delta V \middle| I_{pd}\, \tilde{\chi}_i^{(+)} \right\rangle$$

What about the overlap function?

meeting point between reactions and structure



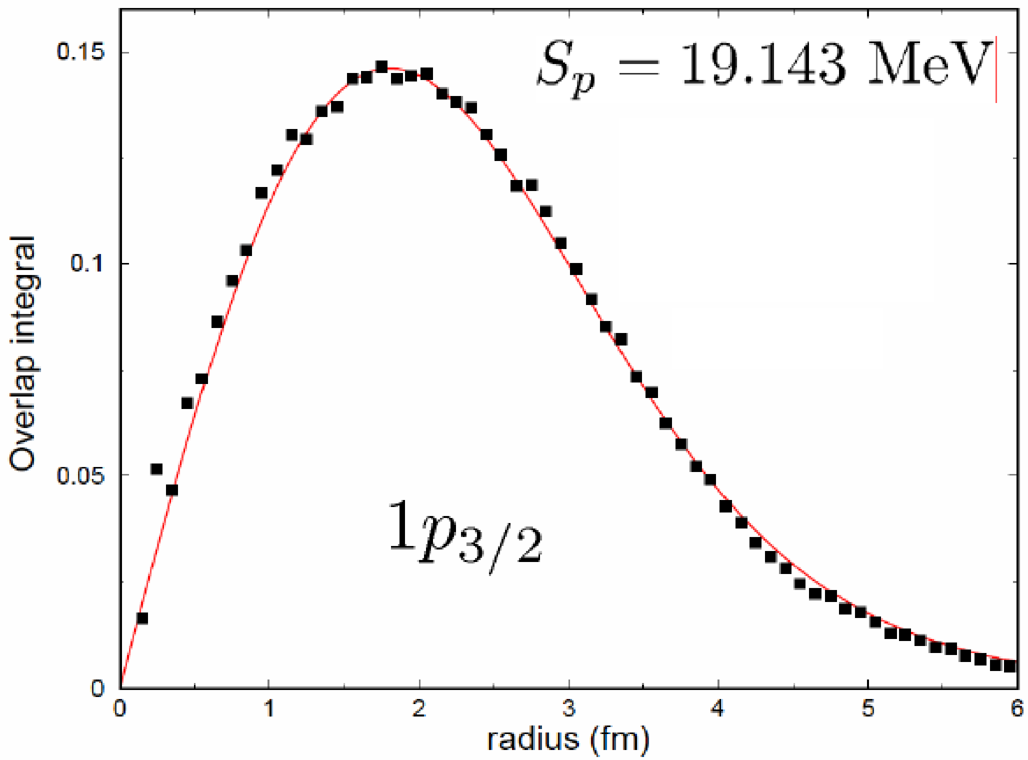
overlap function

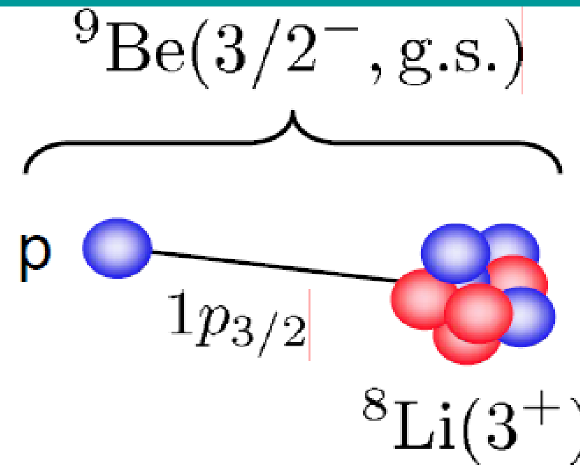
$$I_{I_A:I_B}(\mathbf{r}) = \langle \Phi^A_{I_A}(\xi_A) | \Phi^B_{I_B}(\xi_A, \mathbf{r}) \rangle$$

spectroscopic factor (S_{nlj}):
norm of overlap function

microscopic one nucleon overlap functions

$$\langle \vec{r}, {}^8\mathrm{Li}(3^+) | {}^9\mathrm{Be}(3/2^-, \mathrm{g.s.}) \rangle$$



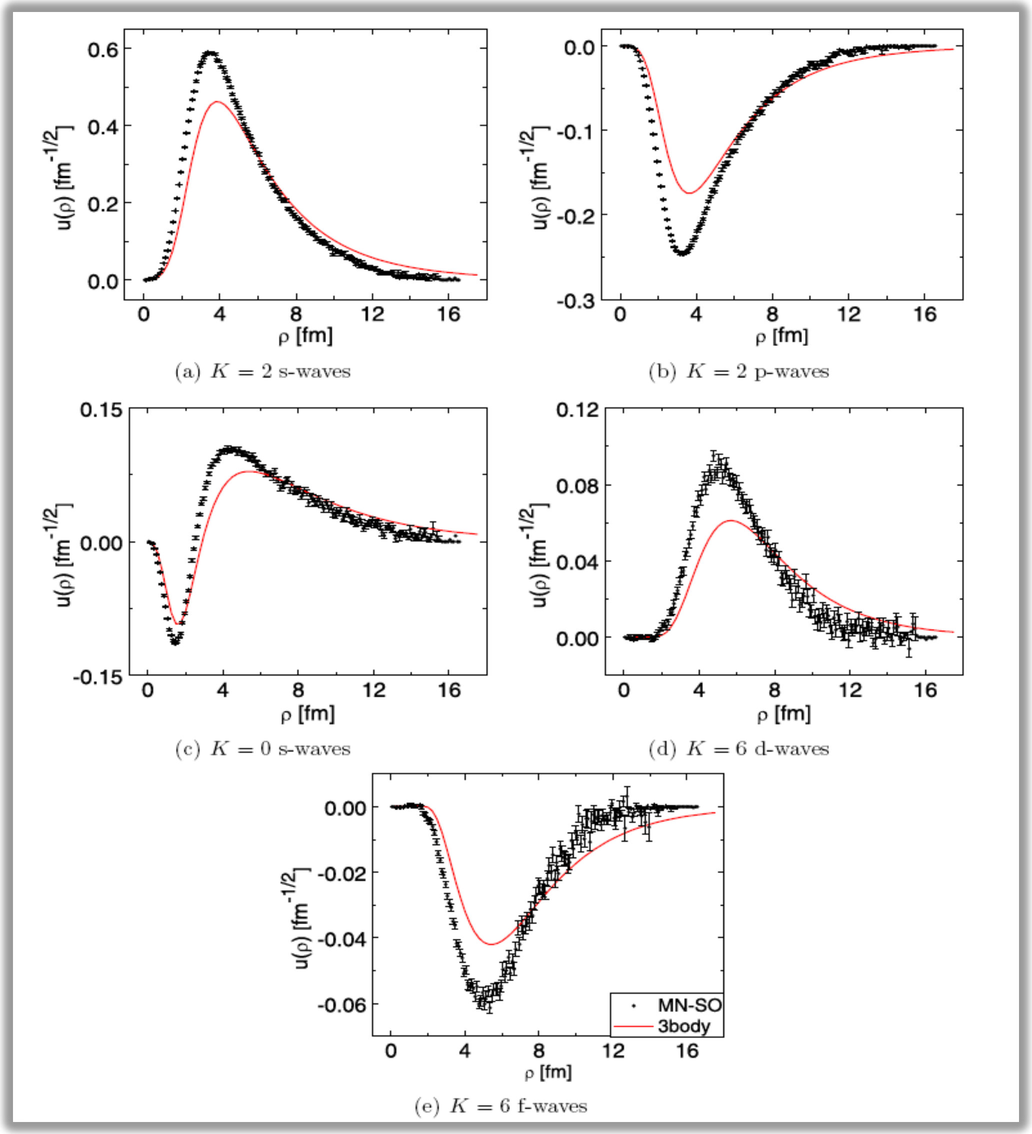


- <u>Microscopic</u> overlap from Argonne 9- and 8-body wave functions (*Bob Wiringa et al.*) Available for a few cases

Normalised bound state in Woods-Saxon potential well x $(0.23)^{1/2}$ <u>Spectroscopic factor</u>

$$r_V = r_{so} = \text{fitted}, \;\; a_V = a_{so} = \text{fitted}, \;\; V_{so} = 6.0$$

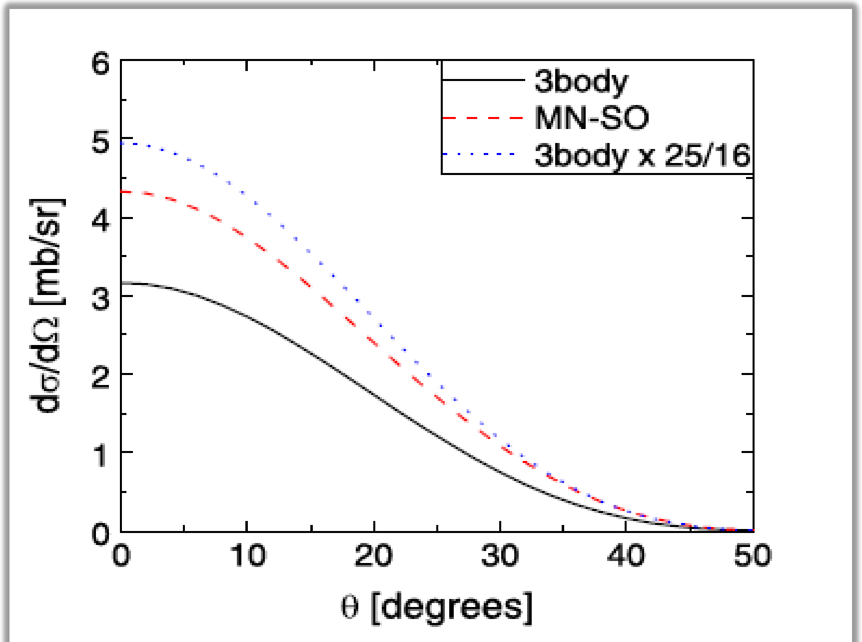
microscopic 2n overlap functions



(a) $K = 2$ s-waves
(b) $K = 2$ p-waves
(c) $K = 0$ s-waves
(d) $K = 6$ d-waves
(e) $K = 6$ f-waves

^{6}He 2n overlap functions

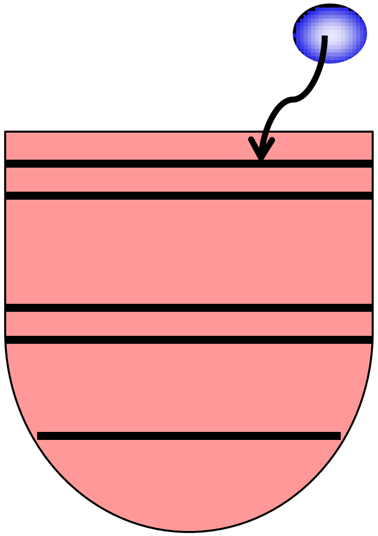
^{6}He(p,t)^{4}He @ 25 MeV



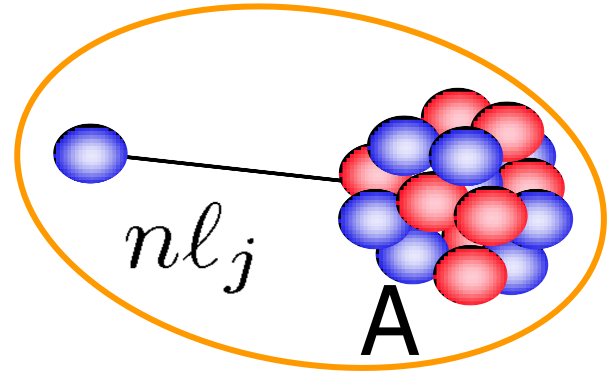
(b) Linear scale for small angles.

[Brida, PhD thesis, MSU 2009]
[Brida and Nunes, NPA in press]

single particle approximation



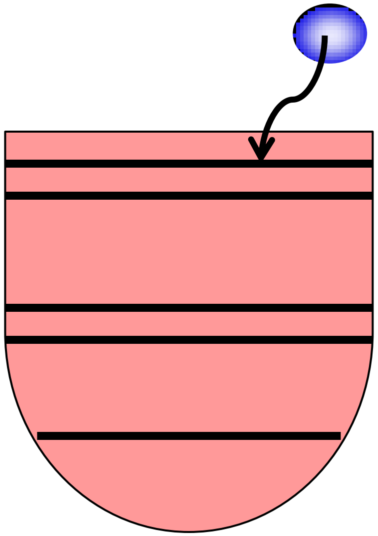
$$(T_r + V_{nA} + \varepsilon)\varphi_{nlj}(r) = 0$$



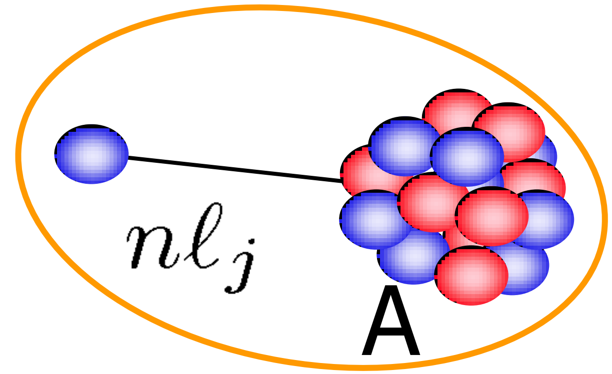
nucleons feels mean field generated by core nucleons V_{nA}

- specific n,l,j and separation energy
- assumptions about single particle parameters

single particle approximation



$$S_{nlj}^{B} = A_{nlj}^{2} = \frac{C_{lj}^{2}}{b_{nlj}^{2}}$$



Same radial dependence at large distances:

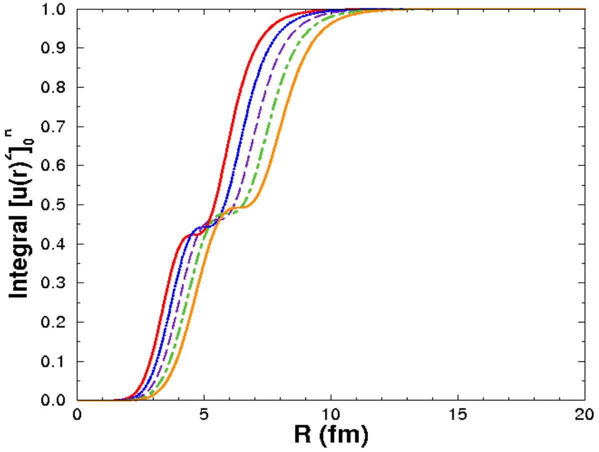
$$I_{AB}(r) \xrightarrow{r>R_N} C_{lj}\, i\kappa \quad h_l(i\kappa r) \qquad \varphi_{nlj}(r) \xrightarrow{r>R_N} b_{nlj}\, i\kappa \quad h_l(i\kappa r)$$

Extend that assumption within the range of the interaction:

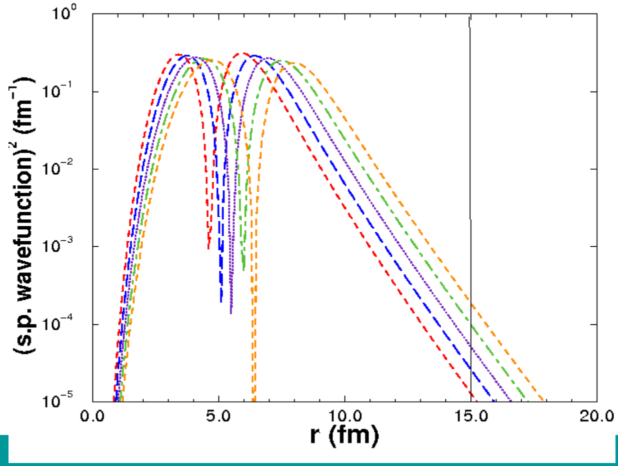
$$I_{AB}(r) = A_{nlj}\, \varphi_{nlj}(r)$$

Overlap function, SF and ANC

S (spectroscopic factor) – **volume** property



C (asymptotic normalization coefficient) – **asymptotic** property



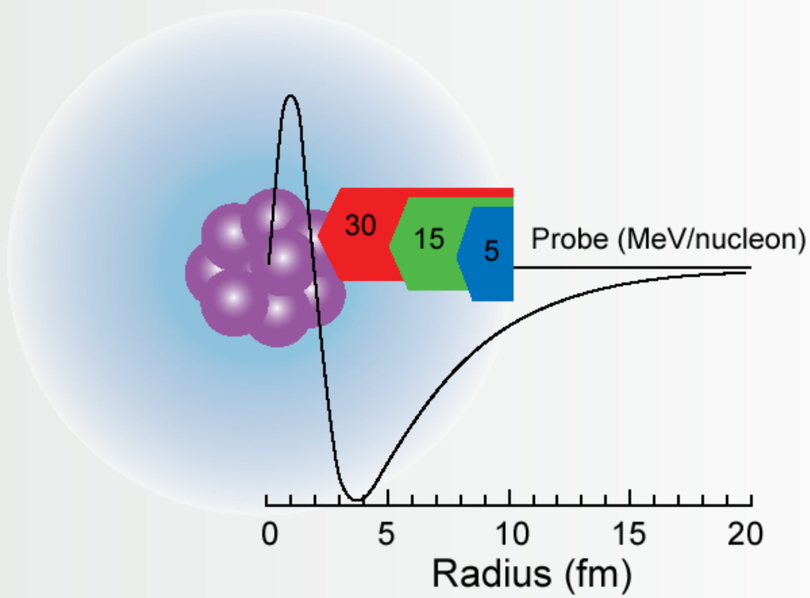
combined method

From sub-Coulomb transfer reaction obtain ANC $\sigma^{th}(b) = C^2 T_{out}{}^2$

From higher energy transfer reaction obtain SF consistent with ANC

$$\sigma^{th}(b) = (ST_{in}(b) + CT_{out})^2$$

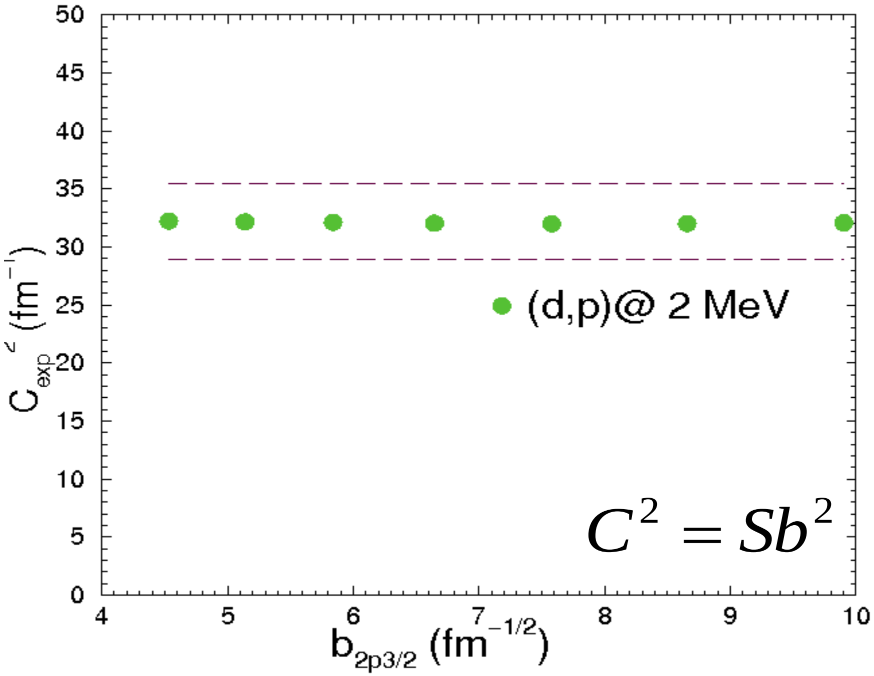
$$C^2 = Sb^2$$



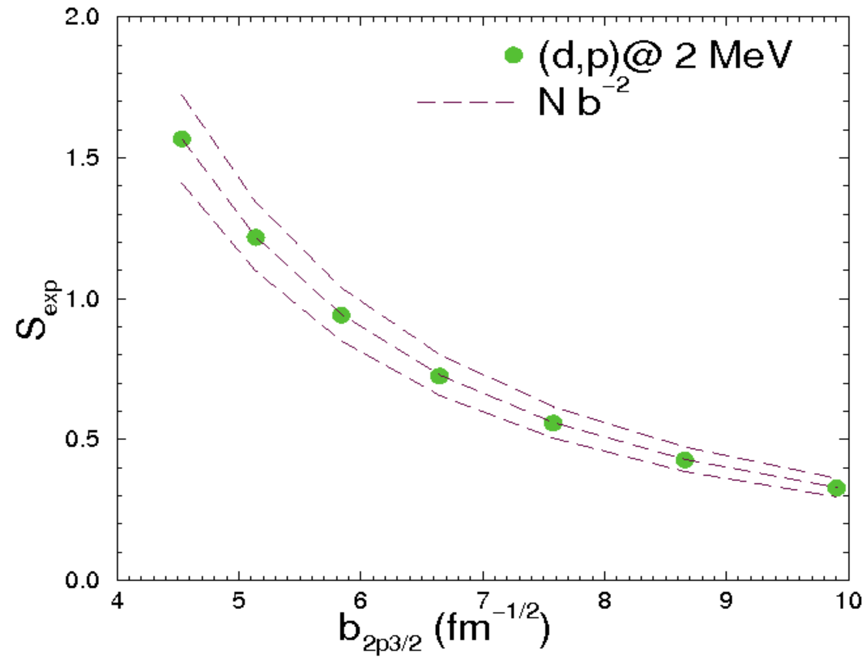
The overlap function for $^{19}C \rightarrow n+^{18}C$ in arbitrary units. The radial sensitivity of the $^{18}C(d,p)^{19}C$ cross section is represented by the colored bars for different beam energies.

Combined method provides a handle on single particle parameters!

combined method



$^{48}Ca(d,p)^{49}Ca$ sub-Coulomb



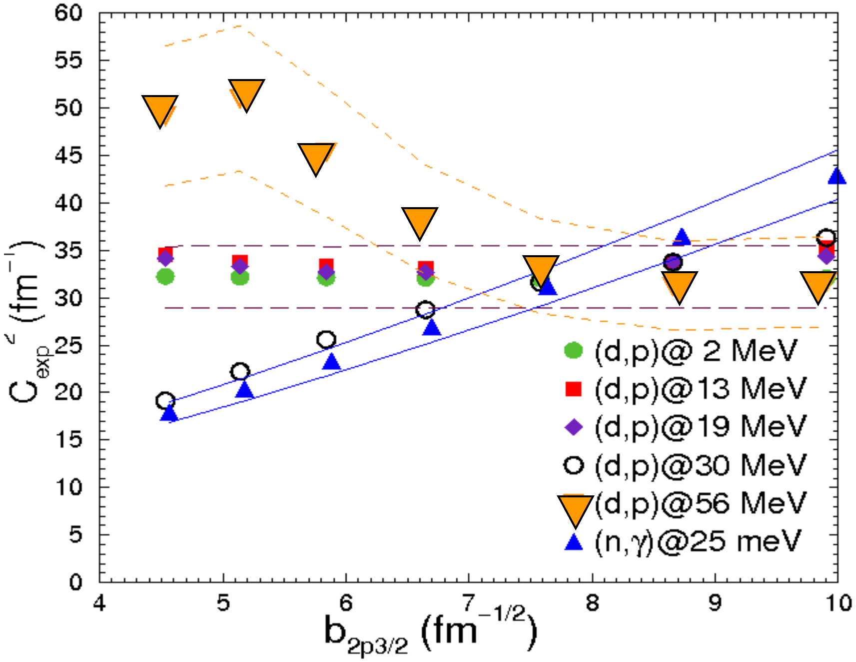
Mukhamedzhanov and FN, Phys. Rev. C 72, 017602 (2005)

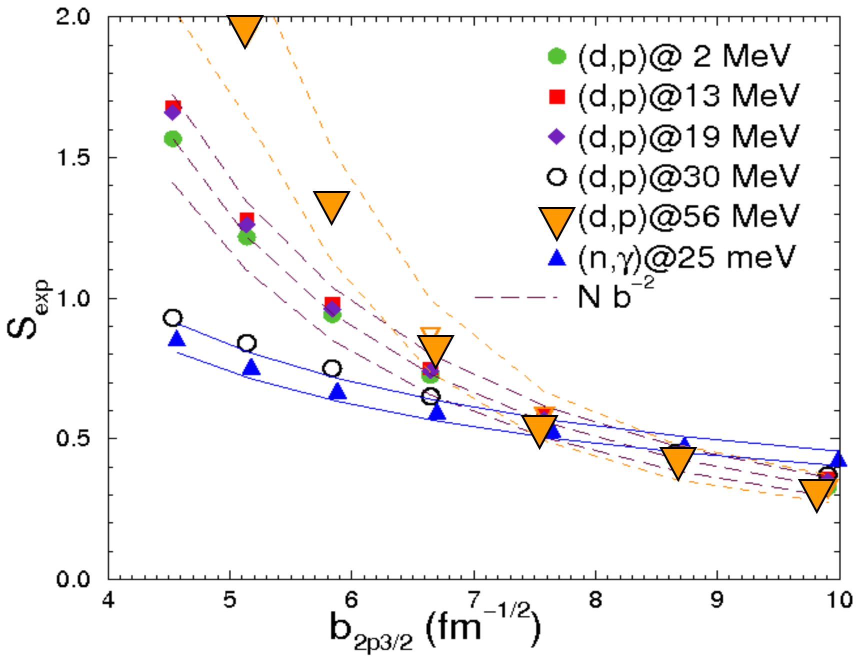
Pang, Mukhamedzhanov and FN, Phys. Rev. C 75, 024601 (2007)

Mukhamedzhanov, FN and Mohr, Phys. Rev. C 77, 051601R (2008)

combined method

SFs and ANCs from $^{48}Ca(d,p)^{49}Ca$ and $^{48}Ca(n,\gamma)^{49}Ca$





(d,p)@ 2 MeV and 56 MeV

$$SF = 0.55 \pm 0.25$$

(n,γ)@ 25 meV

$$SF = 0.53 \pm 0.11$$

[Mukhamedzhanov, FN and Mohr, Phys. Rev. C 77, 051601R (2008)]

(d,p) reactions: deuteron breakup

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(present implementation neglects remnant and
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[Johnson and Soper, Phys. Rev. C 1, 976(1970)]

how good is the approximation?
is it adequate for experiments at ISOL facilities?
is it adequate for experiments at fragmentation facilities?

ADWA as typically applied versus our work

effective potential for incoming deuteron including breakup

Zero range:
Johnson and Soper

Finite range:
Johnson and Tandy

evaluation of the transfer amplitude

zero range

$$T_{JS} = D_0 \int \mathrm{d}R \phi^*_{nA}(\vec{R}) \chi^*_{pB}(\vec{R}) \chi^{JS}_d(\vec{R})$$

or local energy approx **(Buttle and Goldfarb Proc. Phys. Soc 83, 701)**

finite range

$$T = < \phi^{(-)}_{nA} \chi_{pB} | V_{np} | \Psi^{(+)} >$$

[Nguyen, FN and Johnson, Phys. Rev. C (2010) in press]

finite range effect in (d,p) reactions

Target	E_d (MeV)	Δ(LEA)	Δ(FR-JS)	Δ(FR-JT)	Δ(JT-JS)
^{12}C	4	+5.6%	+5.5%	+4.5%	-1.04%
^{12}C	12	+2.6%	+2.9%	-1.5%	-4.31%
^{12}C	19.6	+11%	+12.5%	+7.7%	-4.23%
^{12}C	56	-37%	-27%	-36%	-12%
^{48}Ca	2	+6.5%	+6.3%	+2.6%	-3.52%
^{48}Ca	13	+4.9%	+3.8%	-2.8%	-6.22%
^{48}Ca	19	+5.0%	+4.0%	-0.3%	-4.13%
^{48}Ca	56	-5.2%	-6.5%	-24%	-18.6%
^{69}Ga	12	+4.3%	+4.7%	-1.1%	-5.49%
^{86}Kr	11	+4.8%	+5.5%	-0.4%	-5.63%
^{90}Zr	2.7	+6.2%	+7.3%	+5.5%	-1.68%
^{90}Zr	11	+5.4%	+5.0%	-0.9%	-5.56%
^{124}Sn	5.55	+6.1%	+10.6%	+7.5%	-2.8%
^{124}Sn	33.3	+2.9%	+4.6%	0%	-4.38%
^{124}Sn	70	+5.1%	-28.7%	-43.5%	-20.8%
^{208}Pb	8	+6.1%	+7.2%	+6.1%	-0.96%
^{208}Pb	12	+5.7%	+8.8%	+2.2%	-6.11%
^{208}Pb	20	+5.2%	+4.5%	-2.3%	-6.56%

[Nguyen, FN and Johnson, Phys. Rev. C (2010) in press]

finite range effect in (d,p) reactions

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^{208}Pb	20	+5.2%	+4.5%	-2.3%	-6.56%

[Nguyen, FN and Johnson, Phys. Rev. C (2010) in press]

(d,p) reactions: transfer and breakup

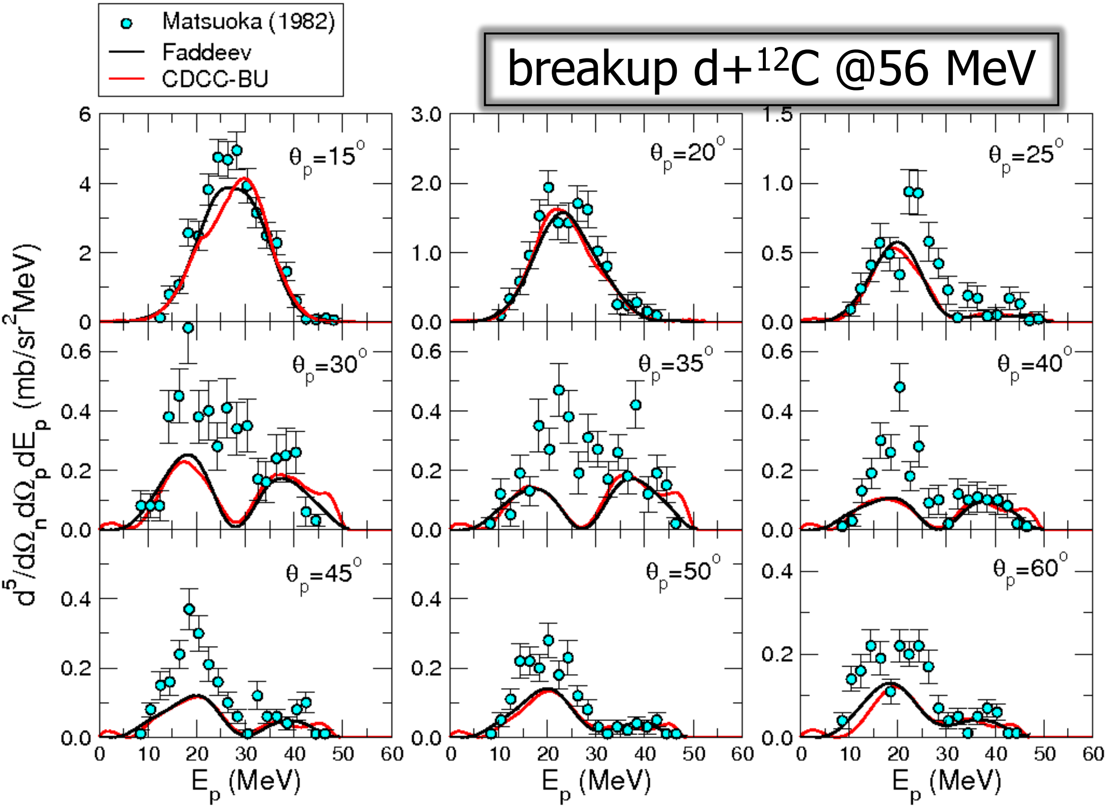
until recently best reaction theories for (d,p) consider breakup to all orders but transfer to first order.

is this a valid assumption?
when is it a valid assumption?

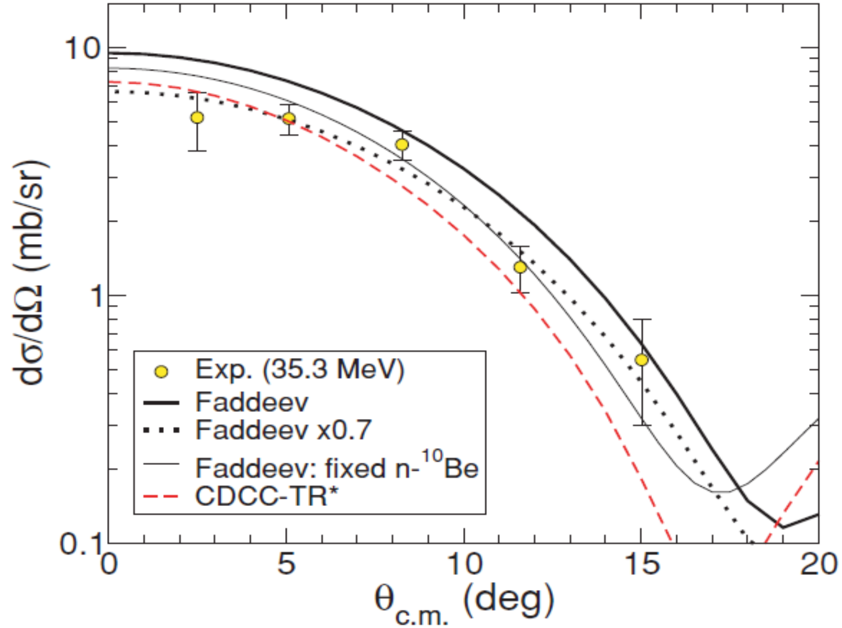
need full Faddeev calculation

comparing CDCC with Faddeev

breakup d+^{12}C @56 MeV

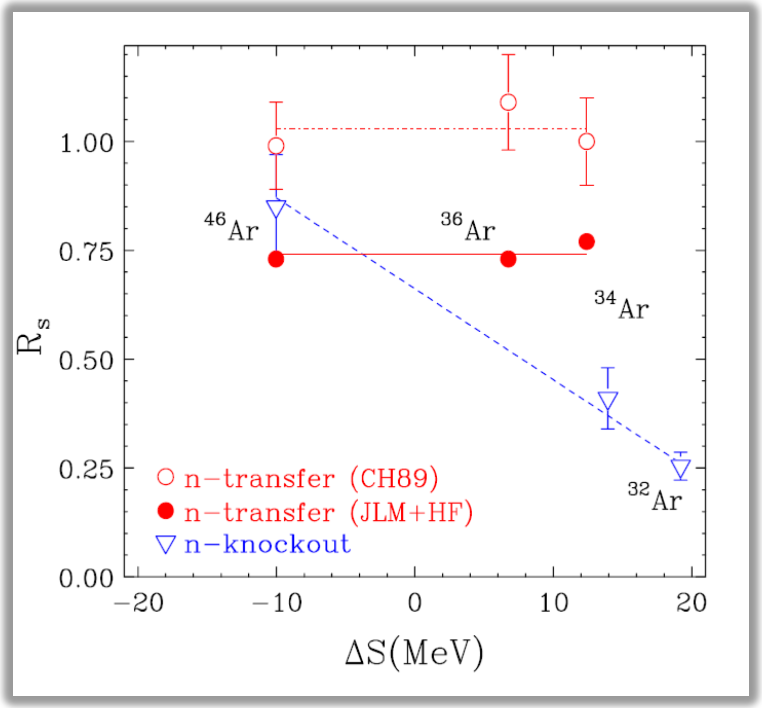


^{11}Be(p,d)^{10}Be at $E_p = 38.4$ MeV



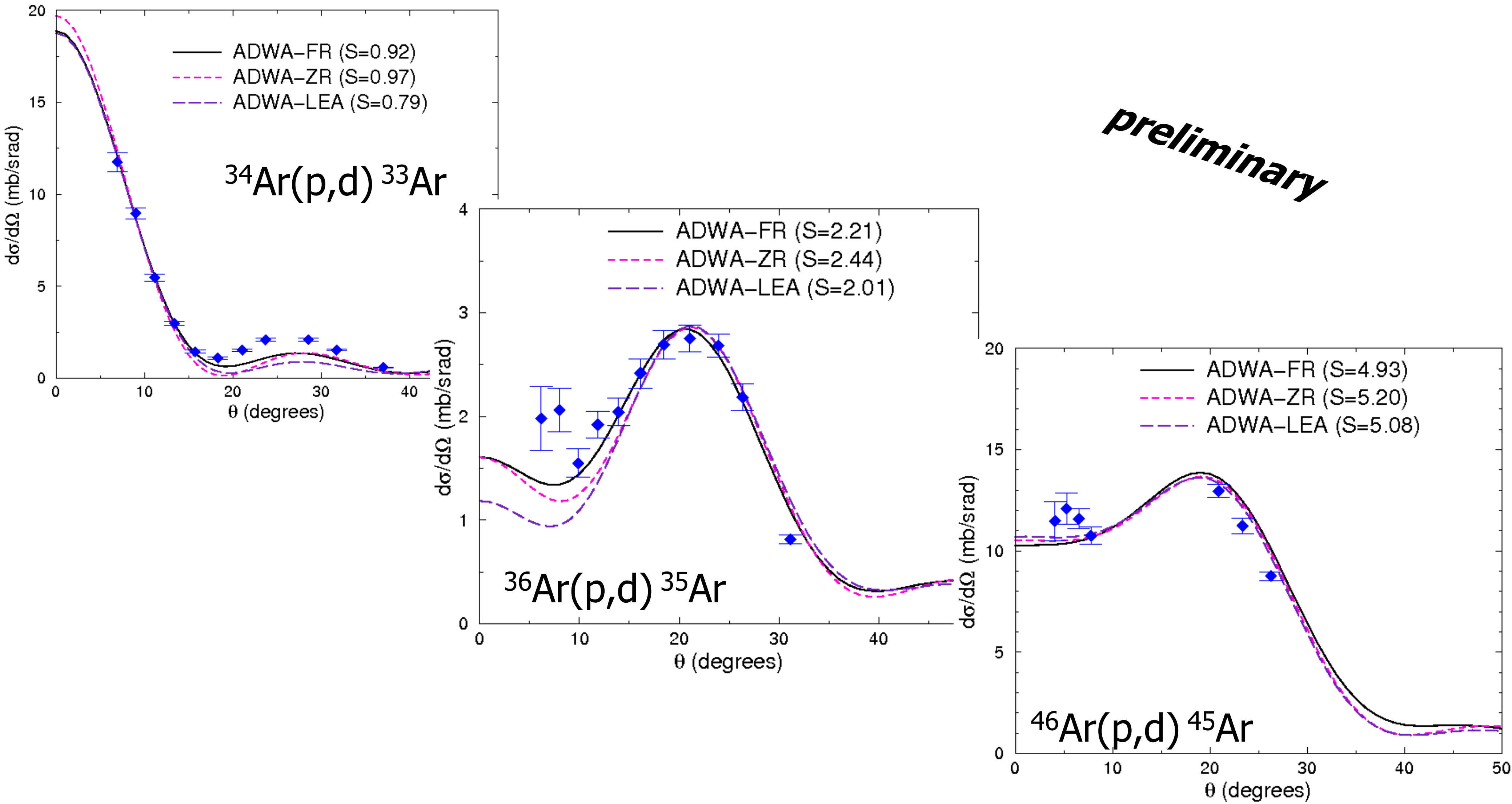
[Deltuva, Moro, Nunes and Fonseca, PRC76, 064602]

transfer versus knockout



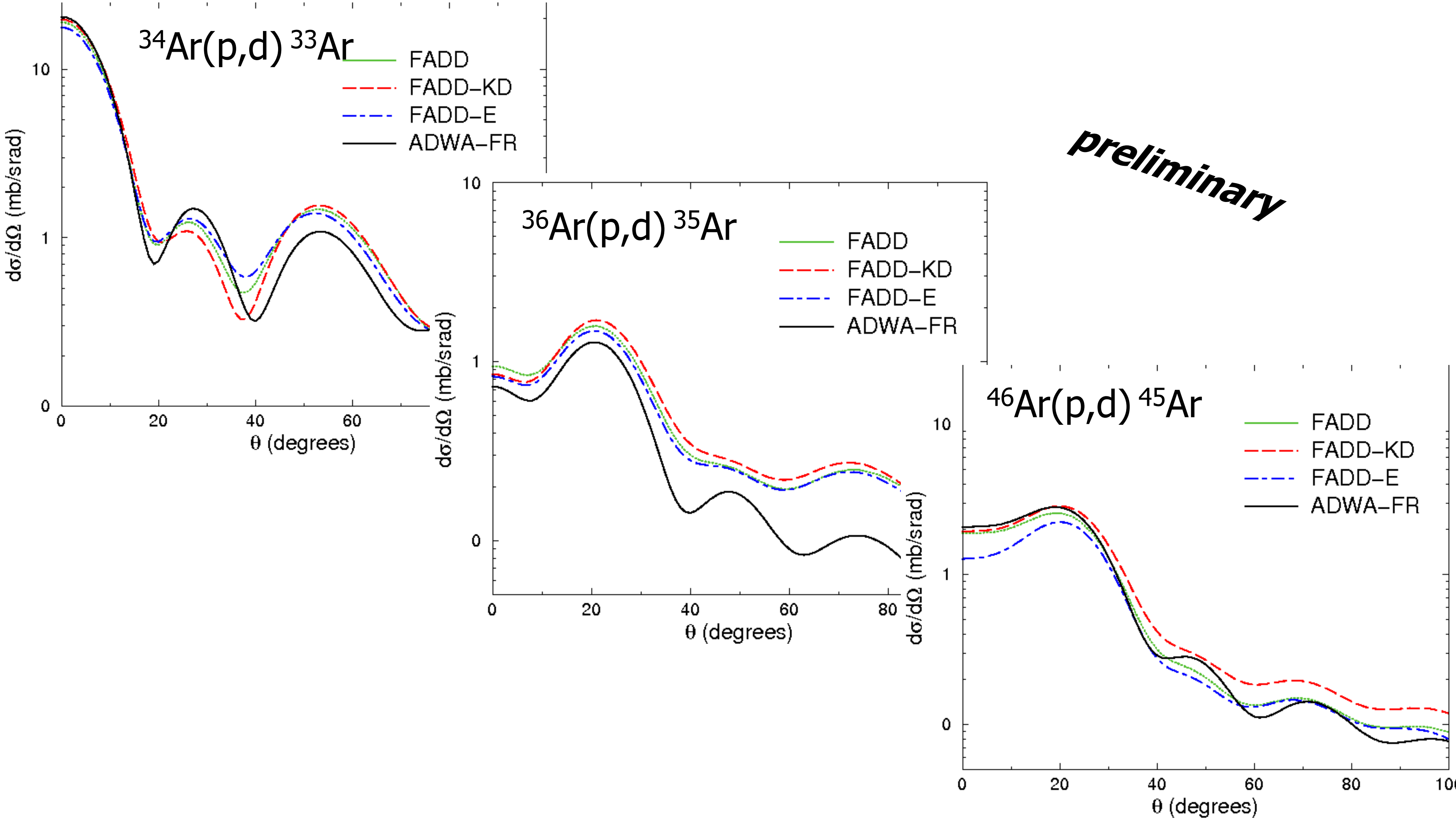
[Jenny Lee et al, PRL 104 (2010) 112701]

(p,d) reactions with $^{34,36,46}Ar$



[FN, Deltuva, Hong, 2010]

(p,d) reactions with $^{34,36,46}Ar$



[FN, Deltuva, Hong, 2010]

error from reaction theory

preliminary

TABLE II: Estimates of theoretical errors in the extracted spectroscopic factors due to approximations in the reaction model as well as experimental errors.

Errors	$\epsilon_{th}(^{34}\text{Ar})$	$\epsilon_{th}(^{36}\text{Ar})$	$\epsilon_{th}(^{46}\text{Ar})$
Optical potential	8 %	7 %	4 %
Faddeev	6 %	19 %	11 %
Experiment	10%	10%	10%
Total	14 %	23 %	15 %

[FN, Deltuva, Hong, 2010]

conclusion

Transfer reactions and combined method

- one benchmark with (n,γ) but many applications with future experiments
- finite range effects can be very important at intermediate energies

Testing CDCC against Faddeev

- disagreement needs to be better understood… new formalism?

Transfer reactions compared to knockout

- uncertainties in reaction theory have been quantified
- results move toward agreement

thankyou!

Collaborators: Ngoc Nguyen (MSU), June Hong(MSU), Ivan Brida(ANL),
Akram Mukhamedzhanov, Peter Mohr,
Arnas Deltuva, Antonio Moro, Ron Johnson

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