INPP @OHIO UNIVERSITY INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

EFB 22

Momentum Space Coulomb Distorted Matrix Elements for Heavy Nuclei

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

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What Reactions are we interested in?

$^{3}He(d,p)^{4}He$

Degrees of Freedom ?

Light nuclei

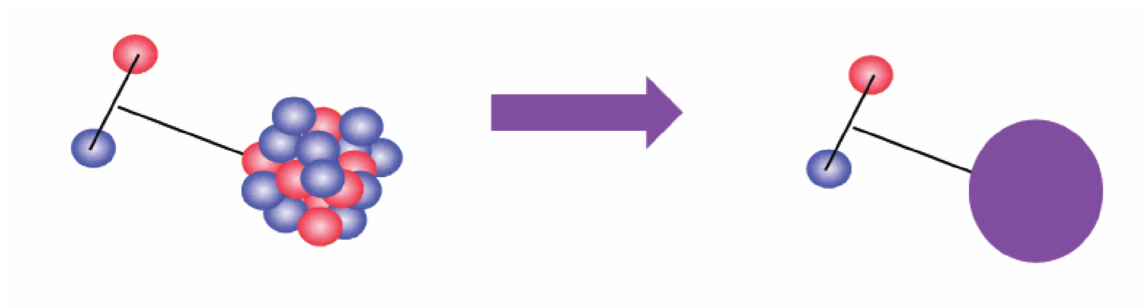
Heavy Nuclei

$^{140}Sn(d,p)^{141}Sn$

?

Reactions: Elastic Scattering, Breakup, Transfer

Reduce Many-Body to Few-Body Problem



Task:

- **Isolate important degrees of freedom in a reaction**
- **Keep track of important channels**
- **Connect back to the many-body problem**

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + \underbrace{V_{nA} + V_{pA}}_{\text{Optical potentials p+A and n+A}}$$

np interaction

Three-Body Problem

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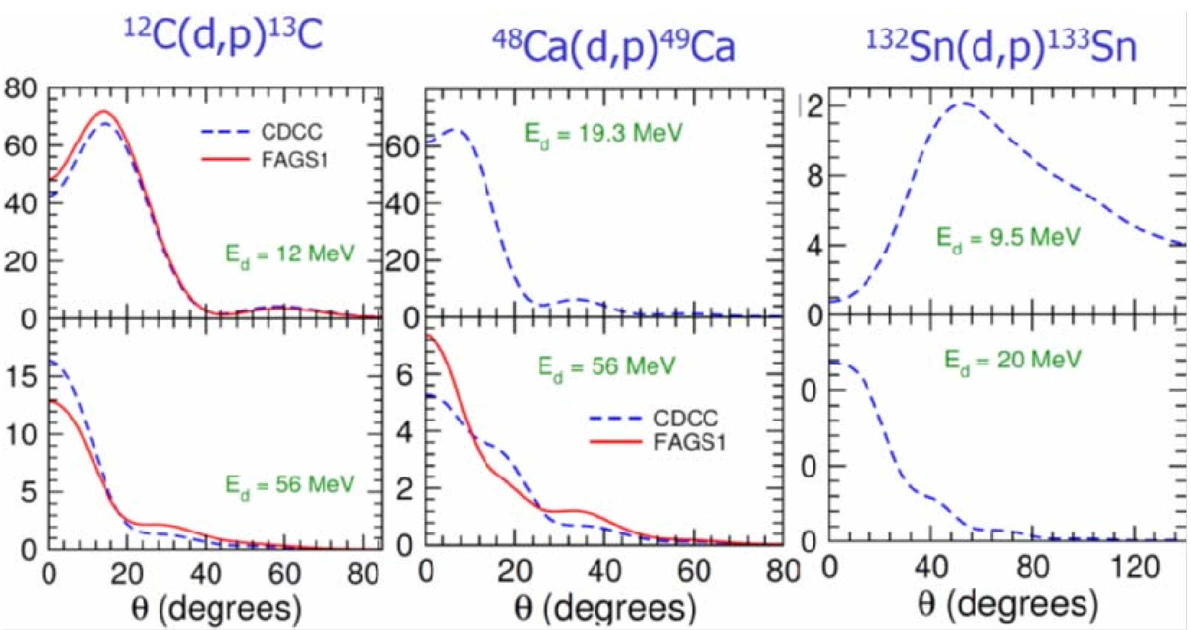
(d,p) Reactions as three-body problem

Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issue: current momentum space implementation of Coulomb interaction (shielding) does **not** converge for Z ≥ 20

Courtesy: F.M. Nunes

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Solve Faddeev equations in Coulomb basis (no screening)

Implies integrals like

$$\boxed{Z_l^C(p,p_\alpha) = \int \frac{dp'p'^2}{2\pi} U_l(p,p')\psi_l^C, p_\alpha(p')}$$

If $$U_l(p,p') = \sum_{i,j} u_{l,i}^*(p)\,(M_l)_{i,j}\,u_{l,j}(p')$$

Integral contains smooth function $u_{l,i}(p')$ **and** $\psi_{p_\alpha l}^C(p')$

Coulomb wave function in momentum space and pw decomposition

Very nasty! "pole" at $p_\alpha = p'$

Suggestion is new & needs to be tested

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First Test in Two-Body System

Calculate two-body Coulomb distorted nuclear matrix element

Separable nuclear Optical Potential

$$u_l(p'_\alpha, p_\alpha) = \sum_{ij} u^*_{li}(p'_\alpha)\,[M_l]_{ij}\,u_{lj}(p_\alpha)$$

$u_{li}(p_\alpha)$ is the nuclear potential form factor.

Compute: **Coulomb distorted nuclear form factor**

$$\boxed{u_l^C(p_\alpha) = \frac{1}{2\pi^2}\int dp\,p^2 u_l(p)\,\psi^C_{p_\alpha l}(p)}$$

$\psi^C_{p_\alpha l}(p)$ is the Coulomb scattering wave function

Challenges:

$$\psi^{C}_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$

$$\times\ \mathrm{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right]$$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha .$$

- Compute special functions of complex arguments
- ${}_2F_1$ (a,b;c,z) requires two different representations for pole and non-pole regions
- **“oscillatory” singularity at $p = p_\alpha$**
- **Gel’fand-Shilov regularization**
  - Reduce integrand around pole by subtracting 2 terms of the Taylor series

Gel'fand-Shilov Regularization:

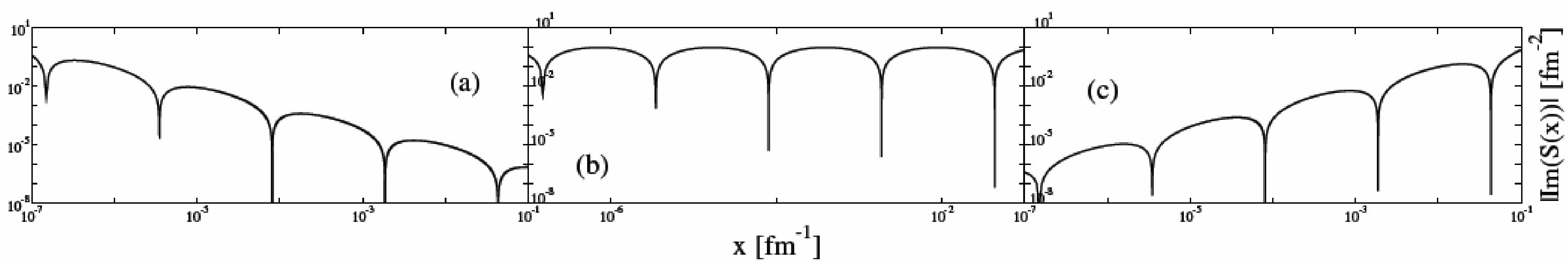
Generalization of Principal value regularization

Idea: reduce value of integrand near singularity

$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}}$$

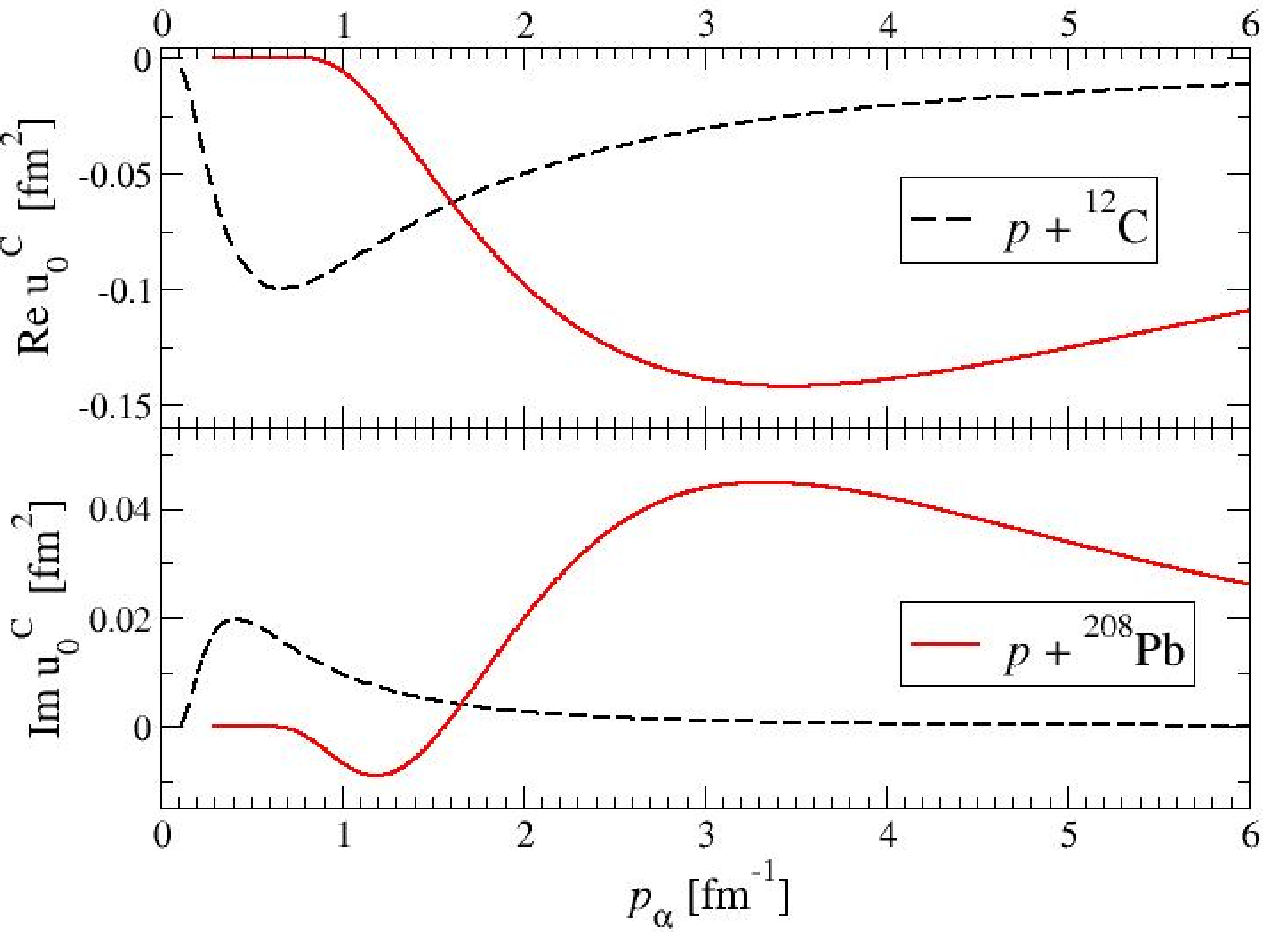
simplified

$$- \frac{i\varphi(0)}{\eta}[\Delta^{-i\eta} - (\Delta)^{-i\eta}] + \dots$$



I. M. Gel'fand and G. E. Shilov. "Generalized Functions". Vol. 1. Academic Press, New York and London. 1964.

With Yamaguchi-type test form factor

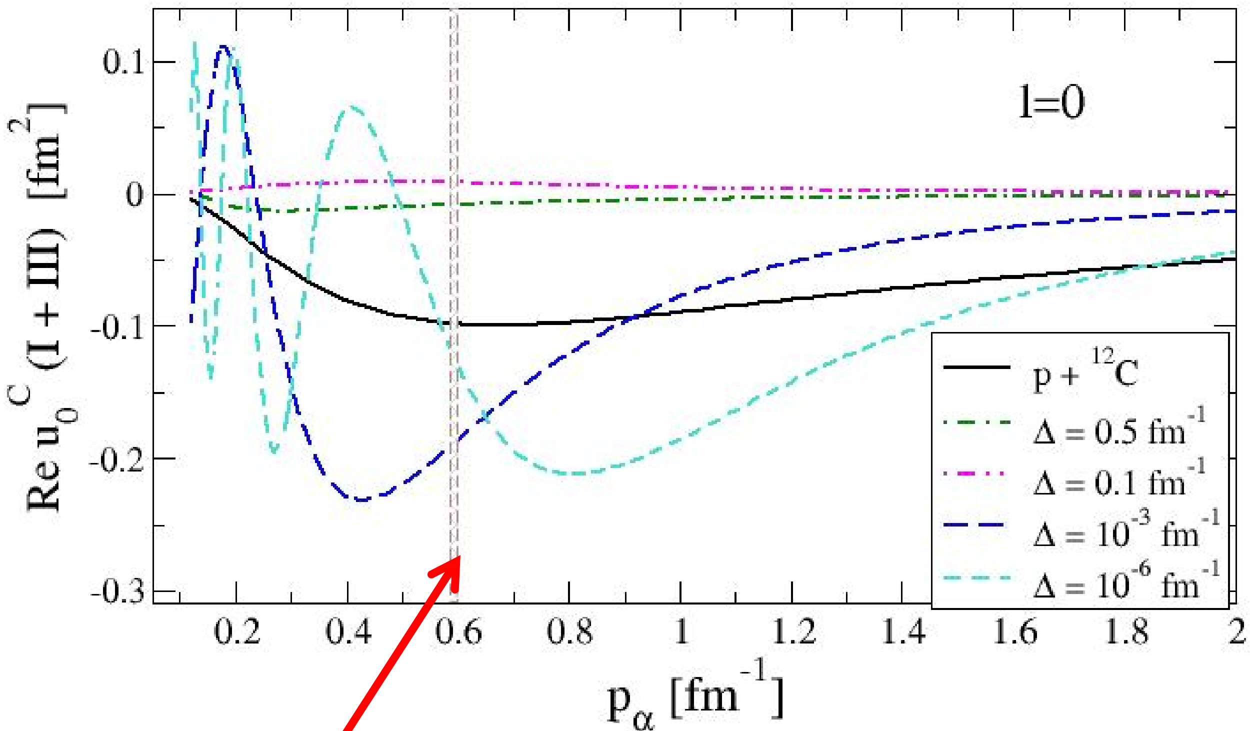


First calculation of Coulomb distorted ^{208}Pb formfactor in momentum space !

p + ^{12}C

$$u_l^C(p_\alpha) = \overbrace{\int_0^{p_\alpha-\Delta} \frac{dp}{2\pi^2}\, p^2 u_l(p)\, \psi^C_{p_\alpha l}(p)}^{I} + \overbrace{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \dots}^{II} + \overbrace{\int_{p_\alpha+\Delta}^{\infty} \dots}^{III}$$

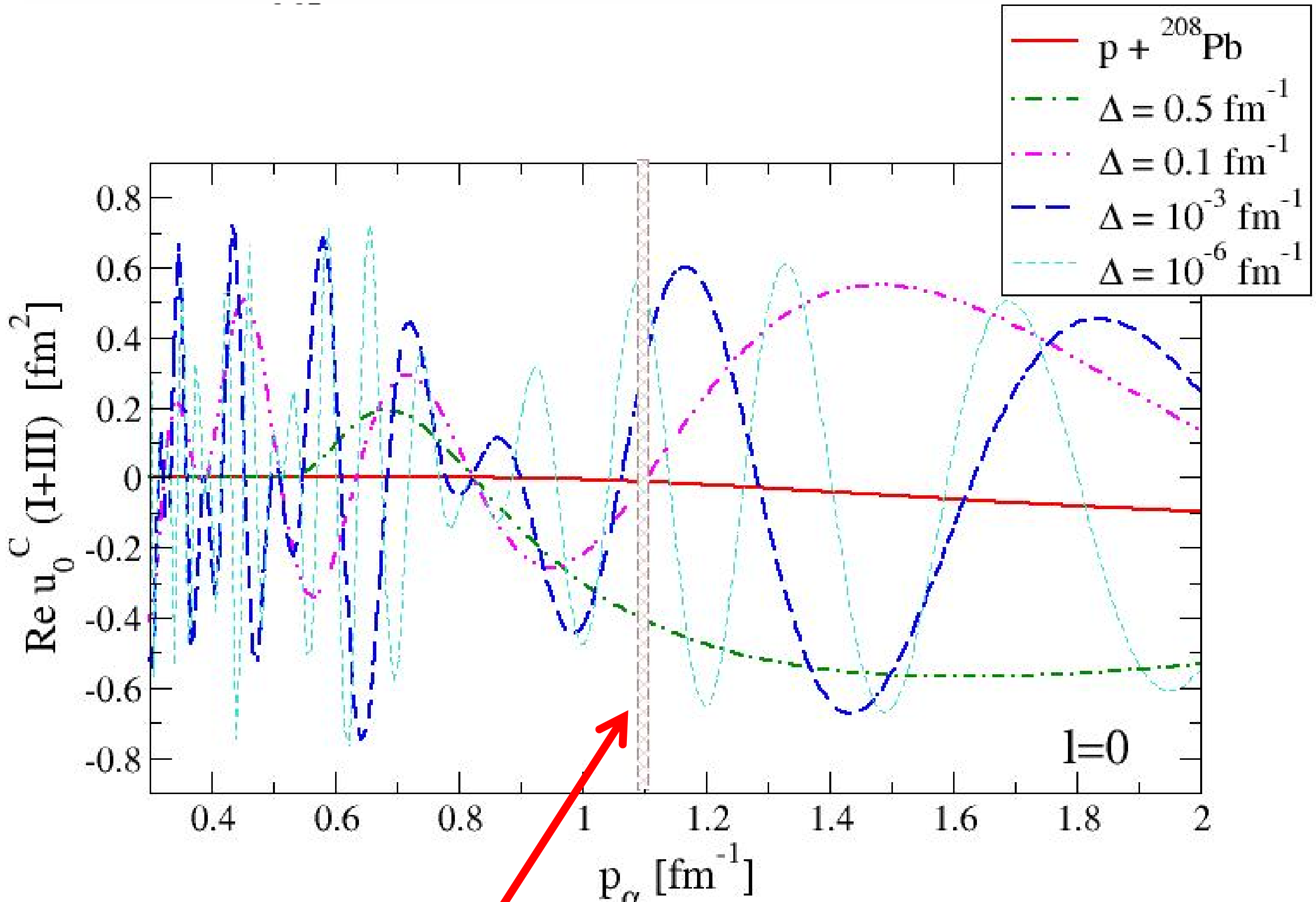
Pole region



Fixed p_α

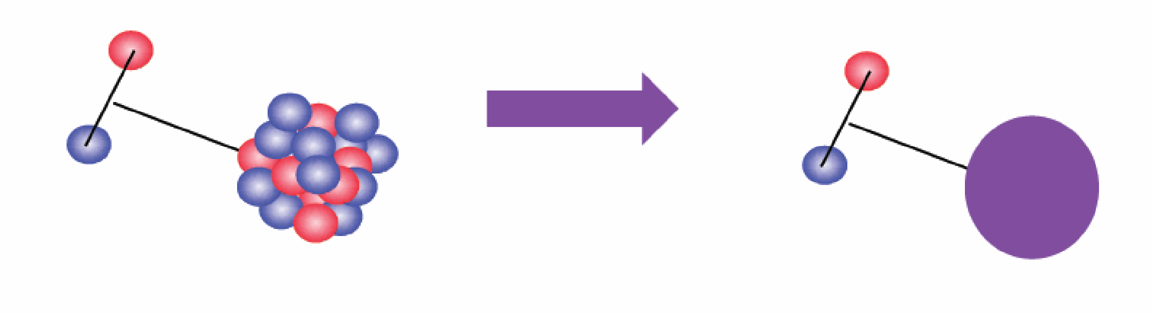
p + ^{208}Pb

$$u_l^C(p_\alpha) = \overbrace{\int_0^{p_\alpha-\Delta} \frac{dp}{2\pi^2}\, p^2 u_l(p)\psi^C_{p_\alpha l}(p)}^{I} + \overbrace{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \ldots}^{II} + \overbrace{\int_{p_\alpha+\Delta}^{\infty} \ldots}^{III}$$



Fixed p_α

Reduce Many-Body to Few-Body Problem



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Hamiltonian for effective few-body poblem:

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Three-Body Problem

Separable Representation of Optical Potentials

Starting point: Woods-Saxon Representation

Method of Ernst-Shakin-Thaler

BUT: Needs to be generalized for complex potentials

So that $\mathcal{K}U\mathcal{K}^{-1} = U^\dagger$ $\mathcal{K}$ is the time-reversal operator.

$$\mathrm{U} = \sum_{i,j} u|\Psi_i^{(+)}\rangle\langle\Psi_i^{(+)}|M|\Psi_j^{(-)}\rangle\langle\Psi_j^{(-)}|u$$

$$\delta_{ik} = \sum_j \langle\Psi_i^{(+)}|M|\Psi_j^{(-)}\rangle\langle\Psi_j^{(-)}|u|\Psi_k^{(+)}\rangle = \sum_j \langle\Psi_i^{(-)}|u|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|M|\Psi_k^{(-)}\rangle.$$

Definition with In/Out-states necessary to fulfill reciprocity theorem

t-matrix:

$$t(E) = \sum_{i,j} u|\Psi_i^{(+)}\rangle\tau_{ij}(E)\langle\Psi_j^{(-)}|u$$

$$\sum_j \tau_{ij}(E)\, \underbrace{\langle\Psi_j^{(-)}|u - ug_0(E)u|\Psi_k^{(+)}\rangle}_{\text{Compute and solve system of linear equations}} = \delta_{ik}\,.$$

Compute and solve system of linear equations

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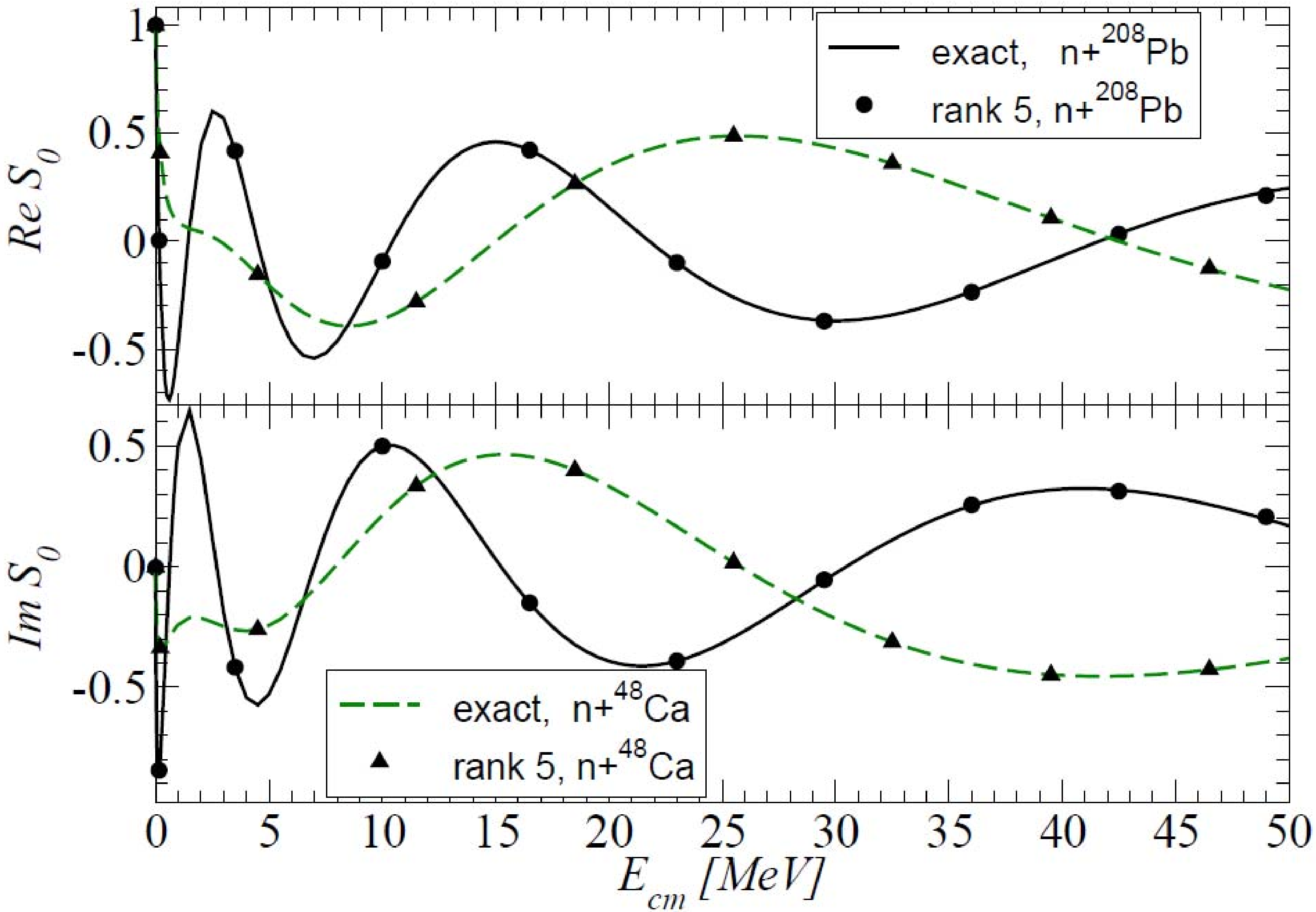
n + ^{48}Ca and n + ^{208}Pb : l=0

Chapel-Hill Optical Potential



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Summary & Outlook

Faddeev-AGS framework in Coulomb basis passed first test!

- Momentum space nuclear form factors obtained in a Coulomb distorted basis for high charges for the first time.
- "Oscillatory singularity" of $\psi_{p_\alpha,l}{}^{c}(p)$ at $p = p_\alpha$ successfully regularized.
- Algorithms to compute $\psi_{p_\alpha,l}{}^{c}(p)$ and the overlap integral successfully implemented

In Progress:

Calculations with separable p+A optical potentials (generalized EST scheme)

Near Future:

Implementation of Faddeev-AGS equations in t Coulomb basis to obtain (d,p) observables

TORUS: Theory of Reactions for Ustable iSotopes

A Topical Collaboration for Nuclear Theory

http://www.reactiontheory.org/

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Some insights for momentum space Coulomb wave functions:

$$\underline{\textbf{Pole:}} \qquad \psi^{C}_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$

$$\times\,\mathrm{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\zeta\equiv\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right]$$

Switching point: $\zeta = \chi \approx 0.34$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha$$

$$\underline{\textbf{Non-Pole:}} \qquad \psi^{C}_{p_\alpha l}(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p_\alpha (pp_\alpha)^2}{(p^2+p_\alpha^2)^{l+1+i\eta}}\left[\frac{\Gamma(l+1+i\eta)\Gamma(1/2)}{\Gamma(l+3/2)}\right]$$

$$\times\left[p^2-(p_\alpha+i0)^2\right]^{-1+i\eta}{}_2F_1\left(\frac{l+2+i\eta}{2},\frac{l+1+i\eta}{2};l+\frac{3}{2};\chi\equiv\frac{4p^2p_\alpha^2}{(p^2+p_\alpha^2)^2}\right)$$

Some insights on momentum space Coulomb wave functions: There are two representations for pole and non-pole regions

$$
\begin{aligned}
\psi^{C}_{p_\alpha l}(p') \;=\; & \frac{-4\pi\, e^{-\eta_\alpha \pi/2}}{p'} \left(\frac{(p'+p_\alpha)^2+\gamma^2}{4p'p_\alpha} \right)^{l} \times \Gamma(1+i\eta_\alpha)\, e^{i\alpha_l} \\
& \times \lim_{\gamma\to+0} \mathrm{Im} \left\{ \left[e^{-i\alpha_l} \frac{(p'+p_\alpha+i\gamma)^{i\eta_\alpha-1}}{(p'-p_\alpha+ii\gamma)^{i\eta_\alpha+1}} \right. \right. \\
& \left. \left. \times\, {}_2F_1\left(-l, -l-i\eta_\alpha;\, 1-i\eta_\alpha;\, \frac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}\right) \right] + \gamma\left[\,\ldots\ldots\,\right] \right\}
\end{aligned}
$$

Switch: $\dfrac{4{p'}^2 p_\alpha^2}{({p'}^2+p_\alpha^2+\gamma^2)^2} = \dfrac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}$

$\psi^{C}_{p_\alpha l}(p')$ at low & high mc

$$
\begin{aligned}
\psi^{C}_{p_\alpha l}(p') \;=\; & -2\pi\, e^{-\eta_\alpha \pi/2} (p'\, p_\alpha)^{l} \left[\frac{\Gamma(l+1+i\eta_\alpha)\Gamma(\frac{1}{2})}{\Gamma(l+\frac{3}{2})} \right] \\
\times\; & \lim_{\gamma\to+0} \left\{ \left[\left(\frac{2({p'}^2-(p_\alpha+i\gamma)^2)^{i\eta_\alpha}}{({p'}^2+p_\alpha^2+\gamma^2)^{l+i\eta_\alpha+1}} \right) \left(\frac{\eta_\alpha(p_\alpha+i\gamma)}{{p'}^2-(p_\alpha+i\gamma)^2} - \frac{\gamma(l+i\eta_\alpha+1)}{{p'}^2+p_\alpha^2+\gamma^2} \right) \right. \right. \\
& \left. \left. \times\, {}_2F_1\left(\frac{l+i\eta_\alpha+2}{2}, \frac{l+i\eta_\alpha+1}{2};\, l+\frac{3}{2};\, \frac{4{p'}^2 p_\alpha^2}{({p'}^2+p_\alpha^2+\gamma^2)^2}\right) \right] + \gamma\left[\,\ldots\ldots\,\right] \right\}
\end{aligned}
$$

Code will eventually be published

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