



INPP

INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

@OHIO UNIVERSITY

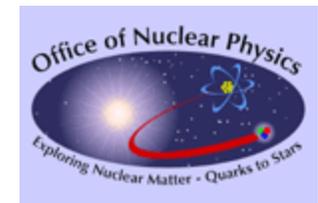
Nuclear Reaction Challenge and Opportunity for Few- and Many-Body Theory

Ch. Elster

TORUS collaboration

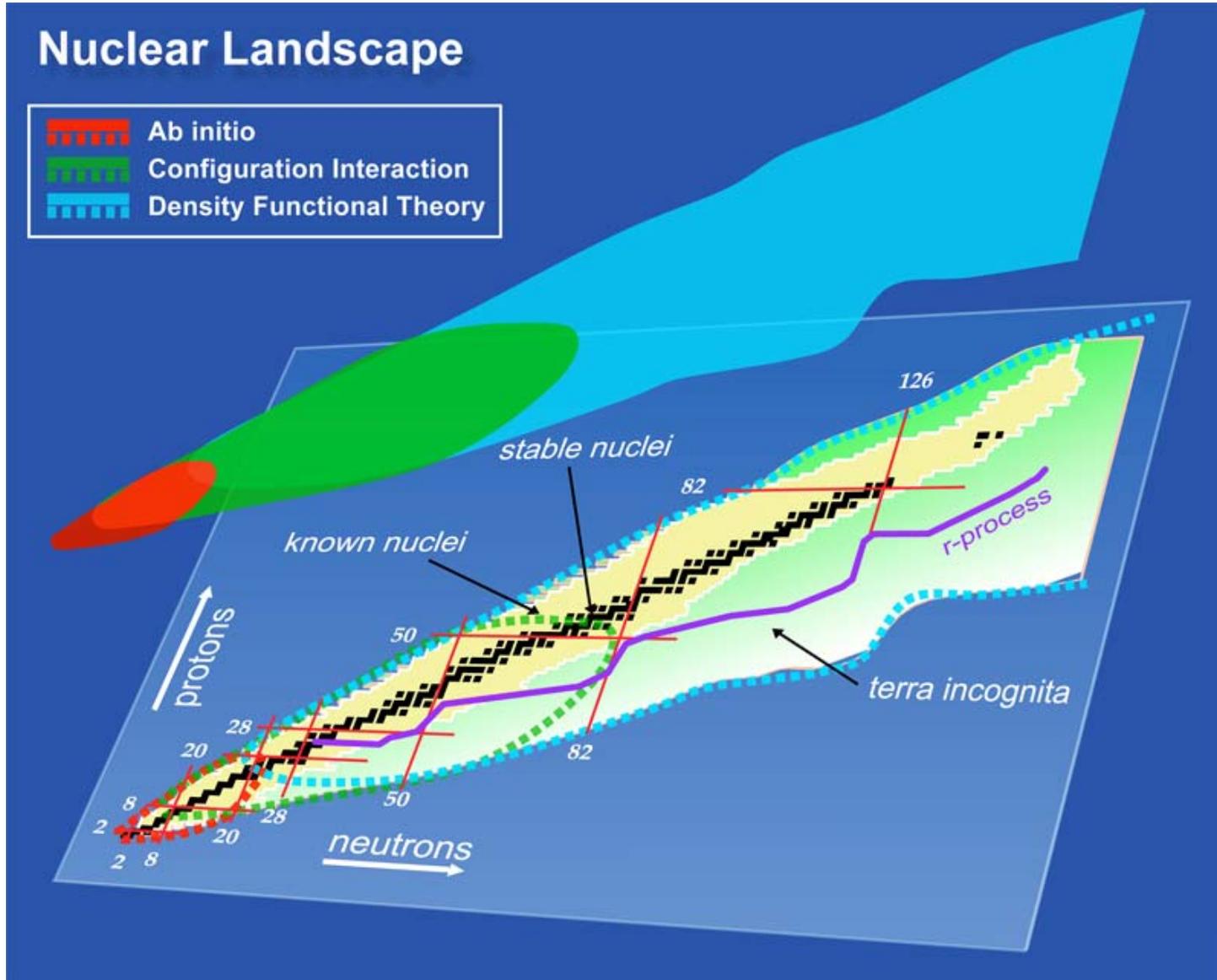
11/7/2010

Supported by: U.S. DOE



Nuclear Landscape

- Ab initio
- Configuration Interaction
- Density Functional Theory



Witold Nazarewicz

Reactions at FRIB

- High energy beams
 - Knock-out reactions (one or two nucleons)
 - Break up
 - Charge exchange
- Reaccelerated beams
 - Transfer reactions (one or two nucleons)
 - Transfer to the continuum
 - Excitations
 - Elastic
 - Fusion

Important: Projectile can be

- close to dripline

- heavy neutron rich system



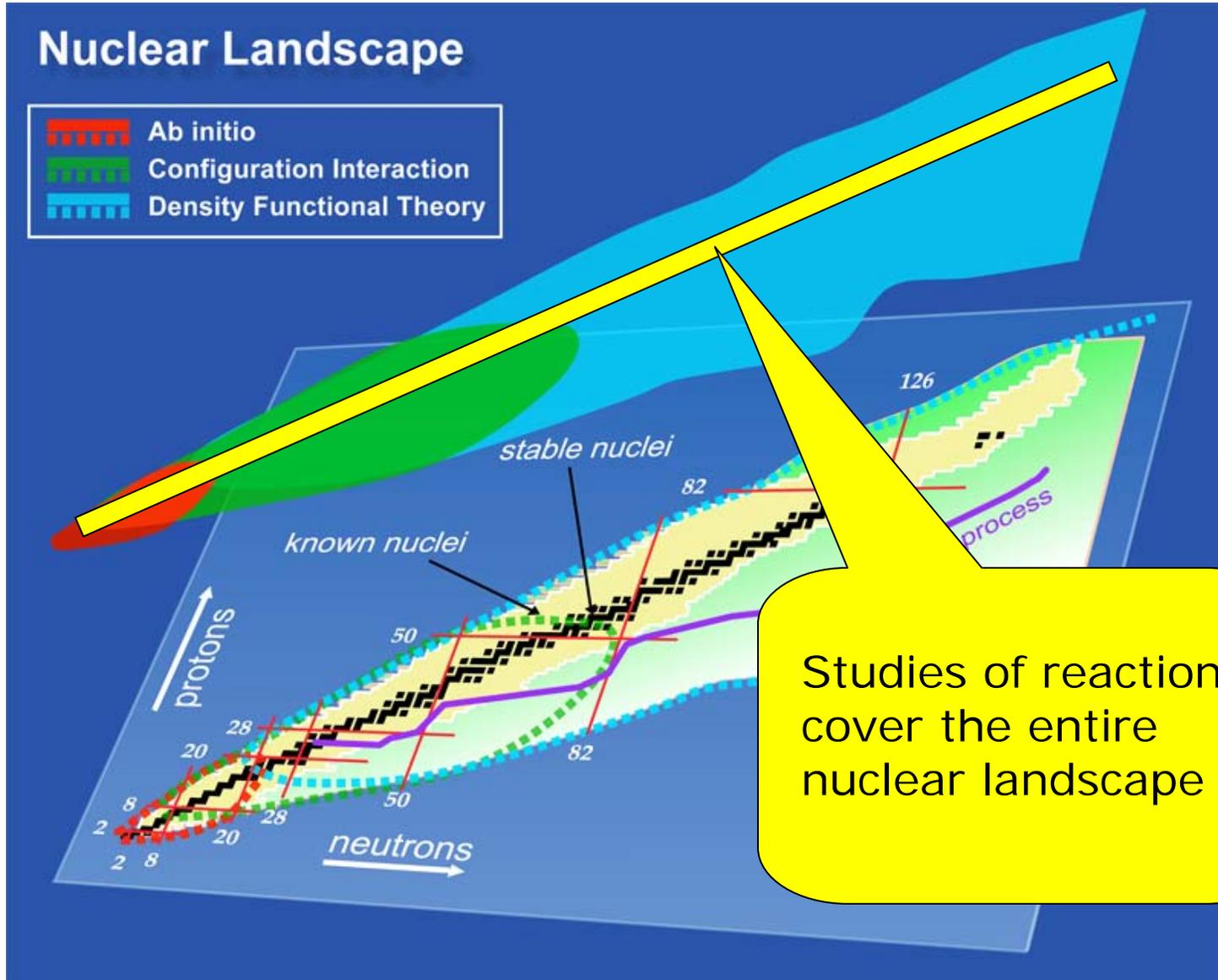
Separation
energy

~ 100 keV

~ 6 MeV

Nuclear Landscape

- Ab initio
- Configuration Interaction
- Density Functional Theory



Studies of reactions cover the entire nuclear landscape

Example: Transfer reactions

*traditionally used to
extract spin, parity
and spectroscopic
factors*

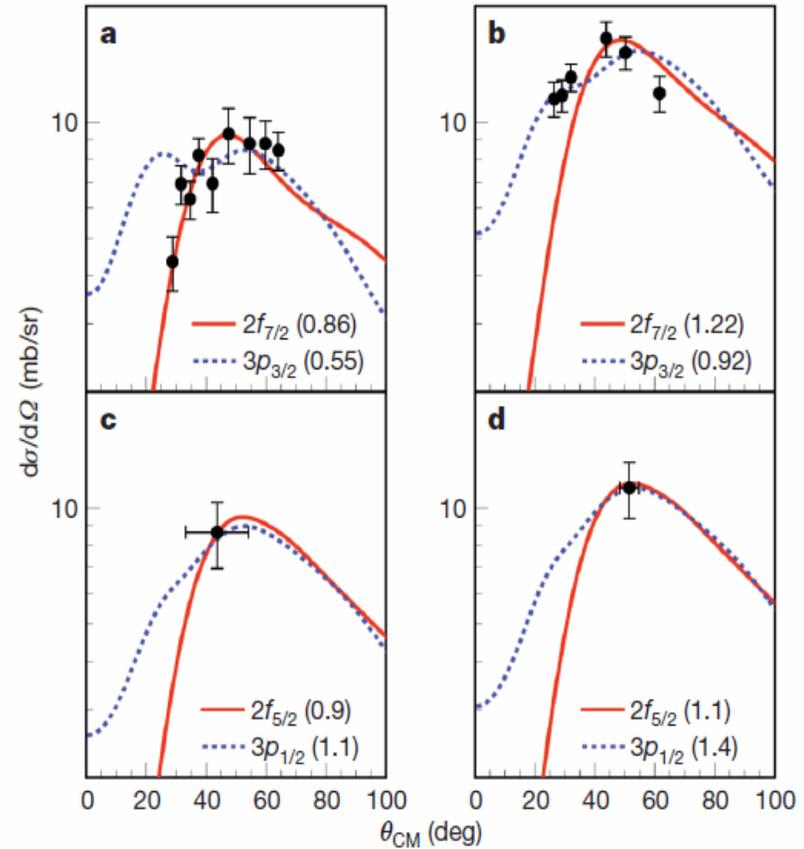


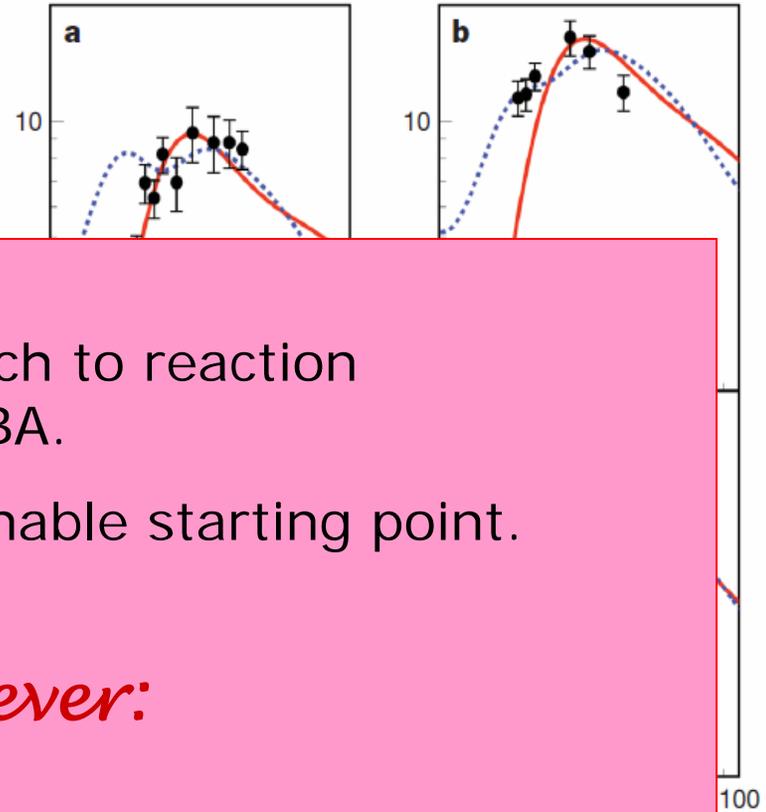
Table 1 | Properties of the four single-particle states populated by the $^{132}\text{Sn}(d,p)^{133}\text{Sn}$ reaction

E_x (keV)	J^π	Configuration	S	C^2 (fm^{-1})
0	$7/2^-$	$^{132}\text{Sn}_{\text{gs}} \otimes \nu_{f7/2}$	0.86 ± 0.16	0.64 ± 0.10
854	$3/2^-$	$^{132}\text{Sn}_{\text{gs}} \otimes \nu_{p3/2}$	0.92 ± 0.18	5.61 ± 0.86
$1,363 \pm 31$	$(1/2^-)$	$^{132}\text{Sn}_{\text{gs}} \otimes \nu_{p1/2}$	1.1 ± 0.3	2.63 ± 0.43
2,005	$(5/2^-)$	$^{132}\text{Sn}_{\text{gs}} \otimes \nu_{f5/2}$	1.1 ± 0.2	$(9 \pm 2) \times 10^{-4}$

[K. Jones et al, to appear in Nature 2010]

Example: Transfer reactions

traditionally used to extract spin parity and spectroscopic factors



A historical approach to reaction calculations is DWBA.

In its time a reasonable starting point.

However:

In order to understand the richness of information obtained in FRIB experiments, theory needs to jump forward

^{132}Sn

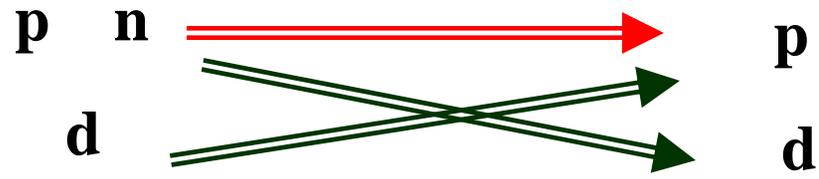
Table 1 | Properties of $^{132}\text{Sn}(d,p)^{133}\text{Sn}$ reactions

E_x (keV)	J^π	$^{132}\text{Sn}_{\text{gs}} \otimes \nu p_{1/2}$	1.1 ± 0.3	2.05 ± 0.45
0	$7/2^-$			
854	$3/2^-$			
$1,363 \pm 31$	$(1/2^-)$			
2,005	$(5/2^-)$	$^{132}\text{Sn}_{\text{gs}} \otimes \nu f_{5/2}$	1.1 ± 0.2	$(9 \pm 2) \times 10^{-4}$

[K. Jones et al, to appear in Nature 2010]

Reactions:

Projectile *Target*



${}^3\text{H}$ ${}^3\text{He}$

${}^3\text{He}$ ${}^3\text{H}$

4, 6, 8 He

4, 6, 8 He

${}^{56}\text{Ni}$ ${}^{78}\text{Ni}$

${}^{56}\text{Ni}$ ${}^{78}\text{Ni}$

${}^{132}\text{Sn}$

${}^{132}\text{Sn}$

...

...

Determination of NN forces

Three Nucleon Physics – Reactions: low energy to GeV regime
Development of Faddeev Formulations & Calculations, 3NF's

JOHN FADDEEV

Reactions:

Projectile Target

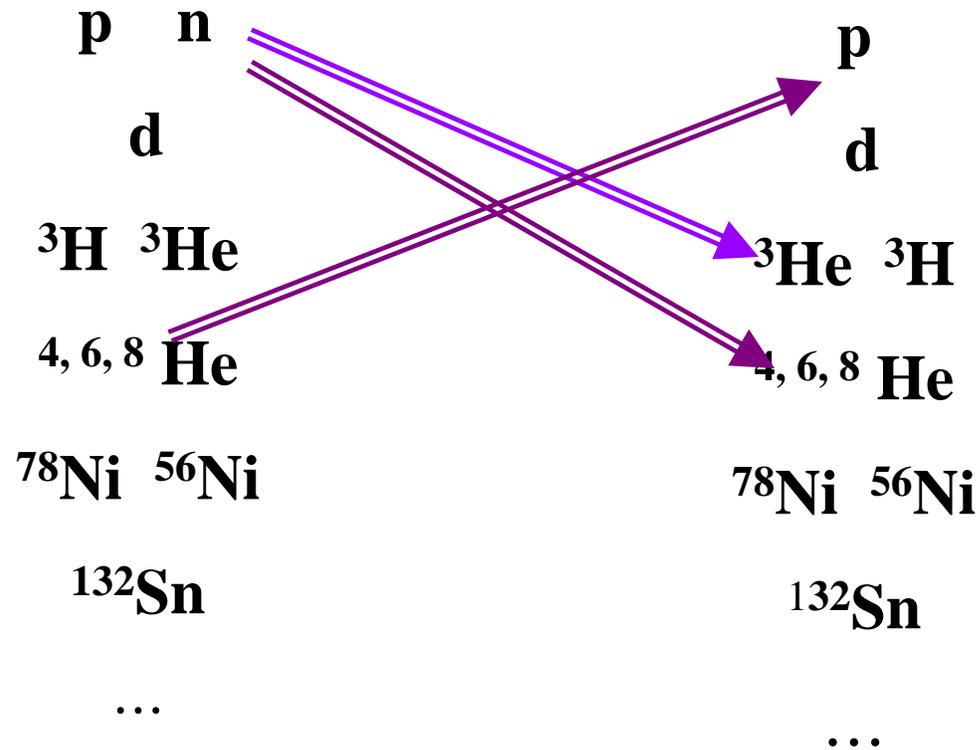


JOHN HUBBARD

Three Nucleon Physics – Reactions: low energy to GeV regime
Development of Faddeev Formulations & Calculations, 3NF's

Projectile

Target



Exact Few-Body Methods:

Faddeev-Yakubovski /

GFMC / Resonating Group / Hyperspherical Harmonics

Projectile

Target

p n
d
 ${}^3\text{H}$ ${}^3\text{He}$
4, 6, 8 He
 ${}^{78}\text{Ni}$ ${}^{56}\text{Ni}$
 ${}^{132}\text{Sn}$
...

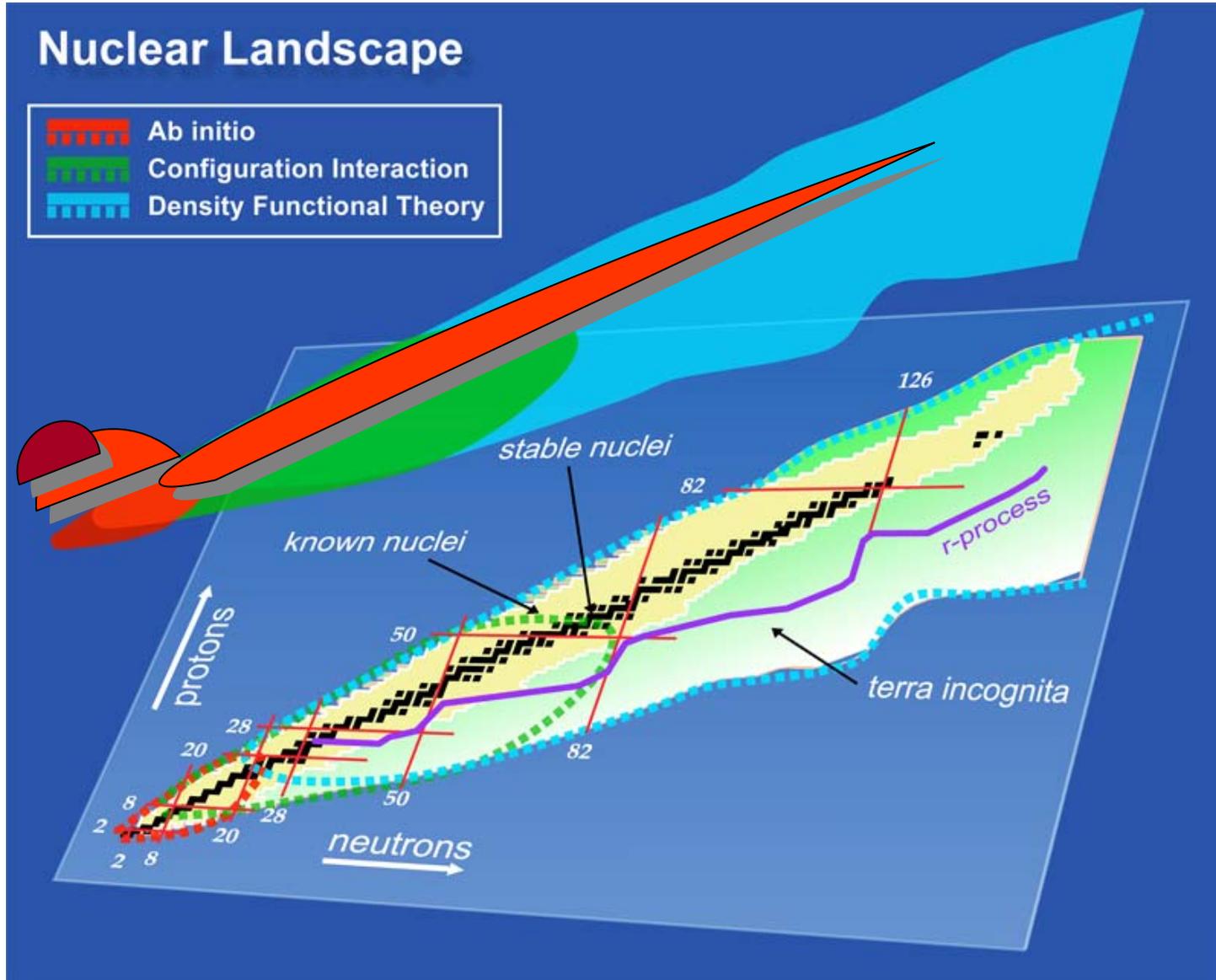
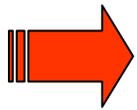
- Grenoble
- Lisbon
- ANL
- Los Alamos – Erlangen
- Kyoto
- LLNL - TRIUMF
- Pisa

Exact Few-Body Methods:

Faddeev-Yakubovskii

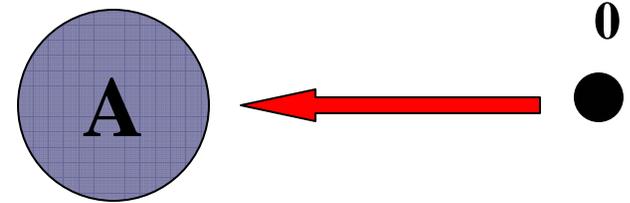
GFMC / Resonating Group / Hyperspherical Harmonics

Research



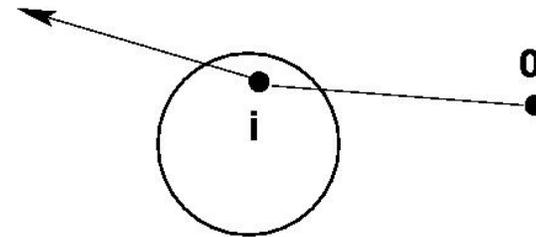
Scattering Problem :

Simplest: $p \rightarrow A$

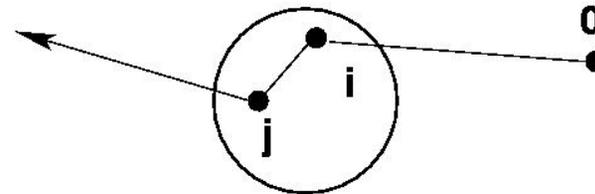


- Hamiltonian: $H = H_0 + V$
- Free Hamiltonian: $H_0 = h_0 + H_A$
- Assume: two-body interactions dominant
 - V : interactions between projectile '0' and target nucleons 'i'
$$V = \sum_{i=0}^A V_{0i}$$
- Transition Amplitude: $T = V + V G_0 T$
- **Multiple Scattering Expansion**

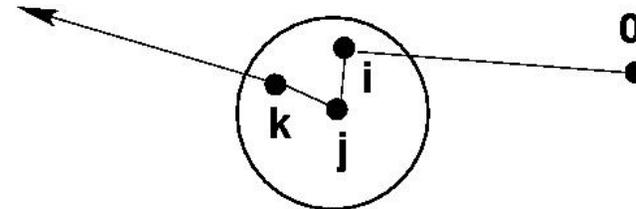
Spectator Expansion:



Single Scattering



Double Scattering



Triple Scattering

⋮

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Elastic Scattering

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $P = |\Phi_0\rangle\langle\Phi_0|$
 - With $1=P+Q$ and $[P,G_0]=0$
- For elastic scattering one needs
- $P T P = P U P + P U P G_0(E) P T P$
- Or

- $T = U + U G_0(E) P T$

- $U = V + V G_0(E) Q U \Leftarrow \text{optical potential}$

Optical Potential – phenomenological:



- Nucleus opaque \Rightarrow complex potential (removes flux)
- Most general form of optical potential
 - $\sum_i [V_{A,Z,N,E}(r) + i W_{A,Z,N,E}(r)] \text{Operator}_{(i)}$
- Best fit of elastic scattering data for a wide range of nuclei and energies
 - Cross sections, angular distributions, polarizations
- E.g.
 - Becchetti – Greenlees, Phys. Rev. 182, 1190 (1969)
 - Global: E.D. Cooper et al, PRC47, 297 (1993)
 - Koning – Delaroche, NP A713, 231 (2003)

Koning – Delaroche (2003)

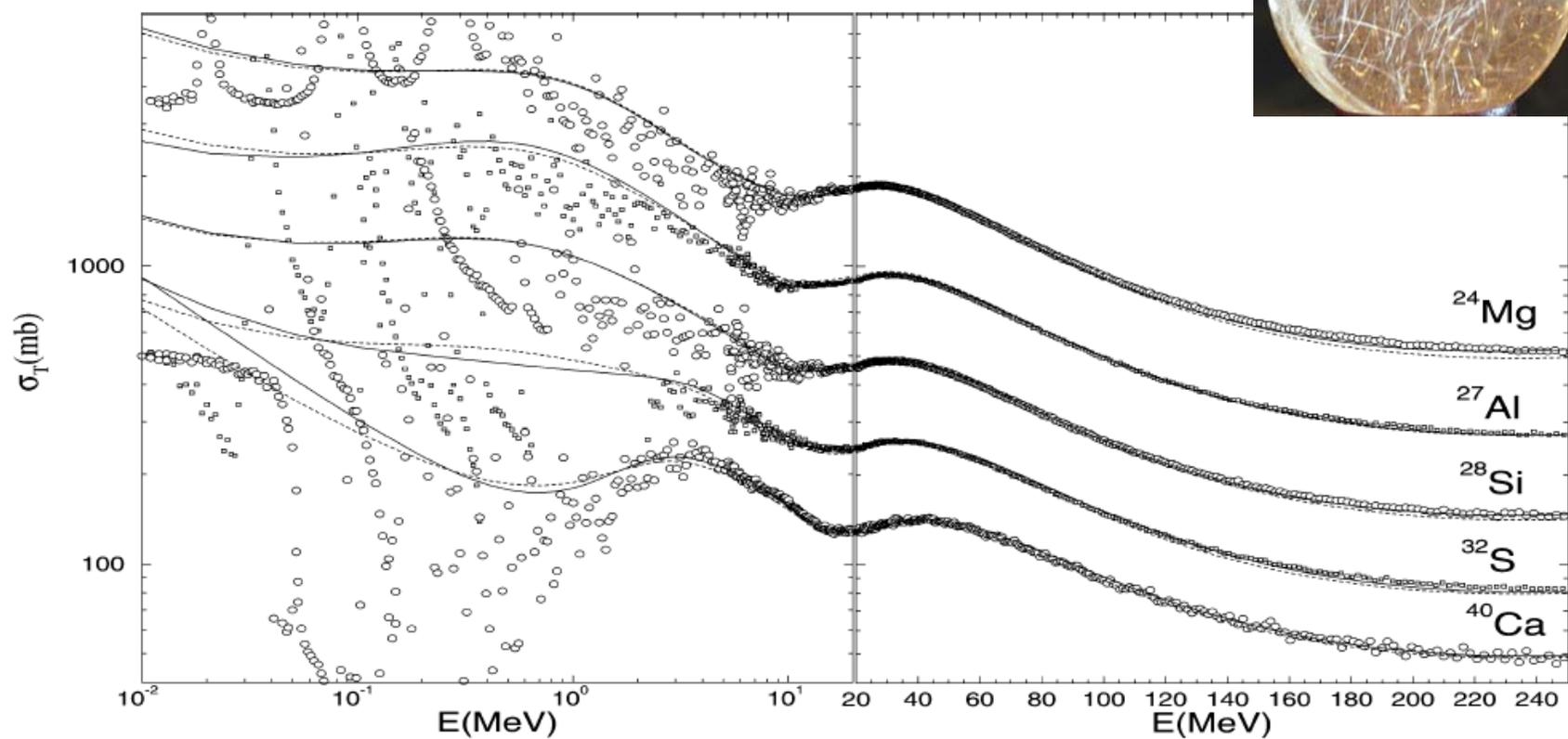


Fig. 2. Comparison of predicted neutron total cross sections and experimental data, for nuclides in the Mg–Ca mass region, for the energy range 10 keV–250 MeV.

Remark: Same importance as NN phase shift analysis

Talk: Steven Weppner

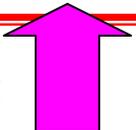
Elastic Scattering

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$$- \quad T = U + U G_0(E) P T$$

$$- \quad U = V + V G_0(E) Q U \quad \Leftarrow \text{optical potential}$$

Low Energies: 

Q-space contains e.g. coupling to resonances

\Rightarrow Take nuclear structure information explicitly into account

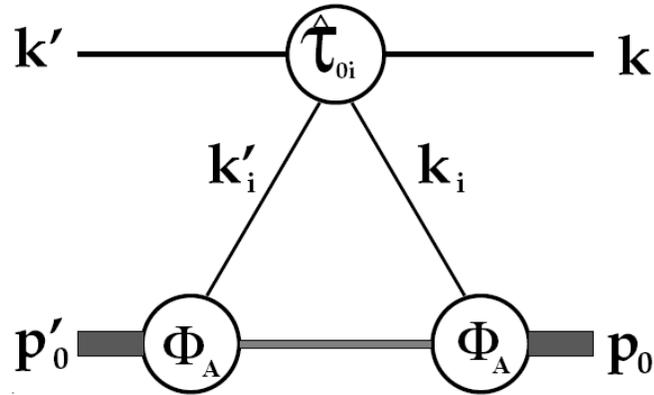
Talk: Ian Thompson

Microscopic :

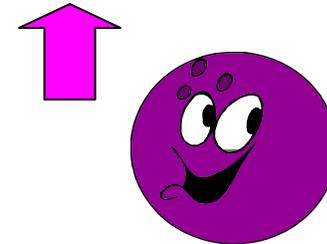
Chinn, Elster, Tandy, Redish, Thaler
 Crespo, Johnson, Tostevin
 Arrellano, Love

- **First order Optical Potential --- Full Folding**

$$\langle \mathbf{k}' | \langle \phi_A | PUP | \phi_A \rangle \mathbf{k} \rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=n,p} \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{0i}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$



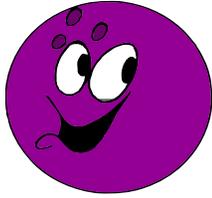
$$\langle \hat{\tau}_{01} \rangle \equiv \langle \mathbf{k}' | \langle \phi_A | \hat{\tau}_{01}(\mathcal{E}) | \phi_A \rangle \mathbf{k} \rangle$$



Proton scattering: $U_{el}(\mathbf{k}', \mathbf{k}) = Z \langle \hat{\tau}_{01}^{pp} \rangle + N \langle \hat{\tau}_{01}^{np} \rangle$

Optical Potential is non-local and depends on energy

Off-shell NN t-matrix and nuclear density matrix



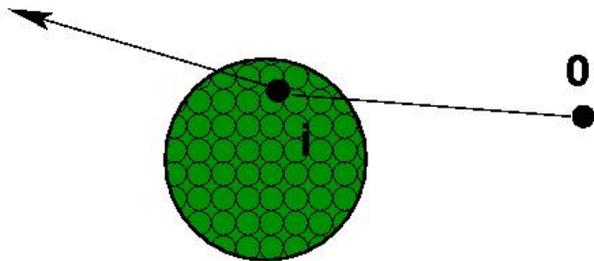
- Via $\langle \Phi_A | \Phi_A \rangle$ results from nuclear structure calculations enter
⇒ **Structure and Reaction calculations can be treated with similar sophistication**

Older microscopic calculations concentrated on closed shell spin-0 nuclei (ground state wave functions were not available)

- Today one can start to explore **importance of open-shells in light exotic nuclei** (full complexity of the NN interactions enters)
[Surrey group started work along this line]

Experimental relevance: Polarization measurements for ${}^6\text{He} \rightarrow p$
at RIKEN

“Nuclear Medium”



Single Scattering

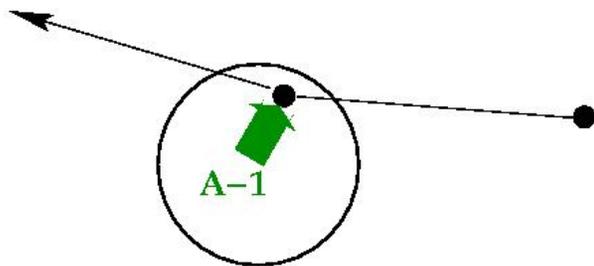
Propagator is (A+1) body operator:

$$G_0(E) = (E - h_0 - H_A + i\epsilon)^{-1}$$

H_A in a mean field view

$$H_A = h_i + \sum_{j \neq i}^A v_{ij} + H^i$$

$$\langle \sum_{j \neq i}^A v_{ij} \rangle \equiv U_i \equiv \text{mean field}$$



Single scattering is an implicit three-body problem:

Projectile + struck nucleon + (A-1) core

Chinn, Elster, Thaler PRC 48, 2956 (1993)

**Different consideration:
Folding the ground state wave function
with the nuclear matter g-matrix:**

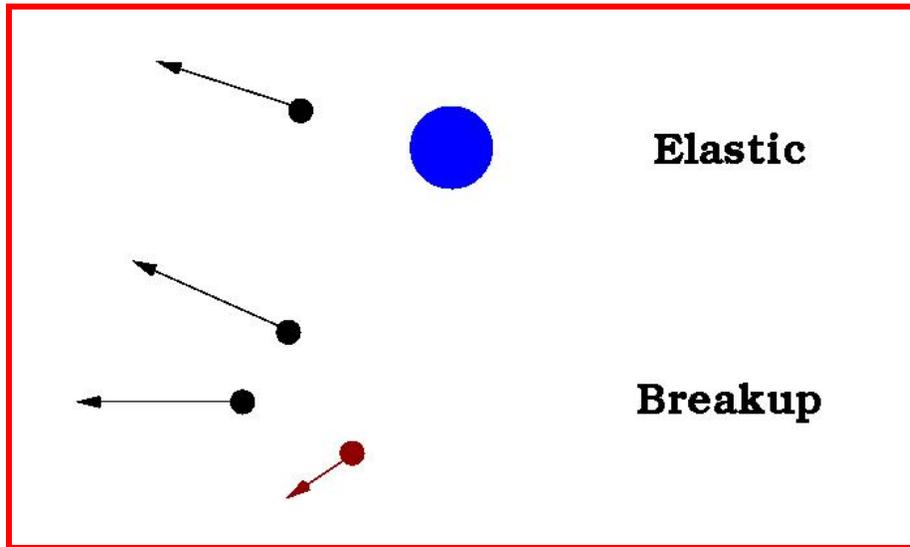
Pioneered by Arellano & Love – cont'd by Karataglidis & Amos

Again Single Scattering: (d,p) Reaction

p+d scattering :

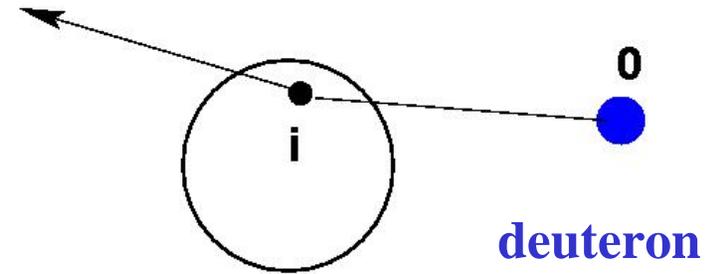


In



Elastic

Breakup

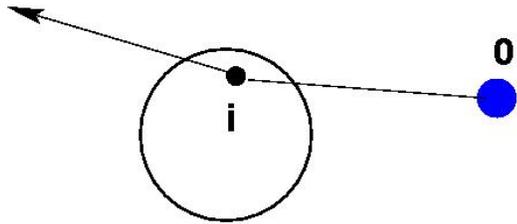


Single Scattering

*Faddeev Formulation
of (d,p) Reactions*



Faddeev Formulation of (d,p) Reactions



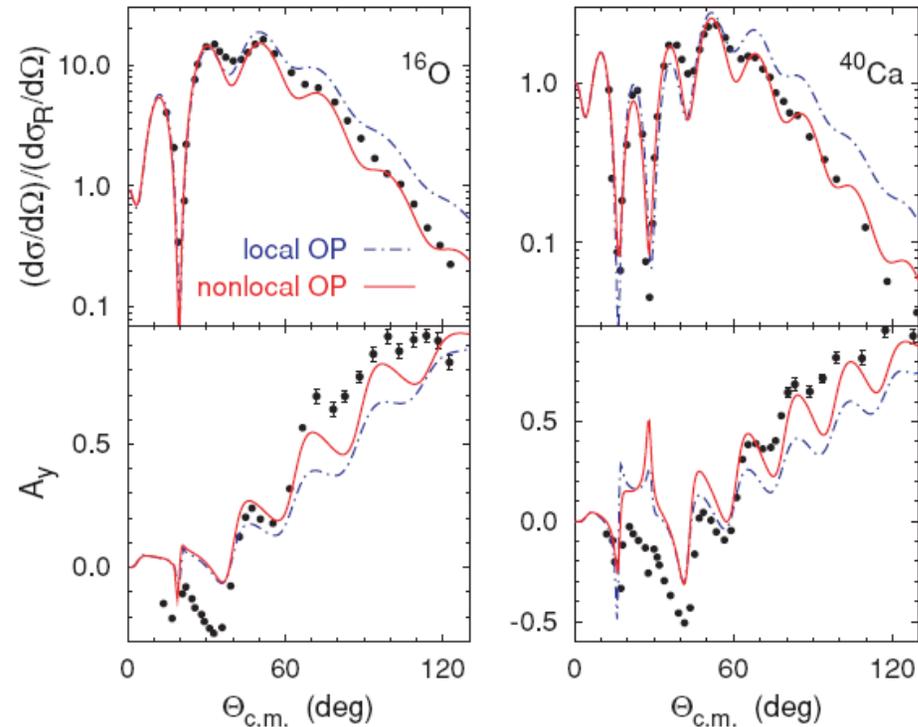
Single Scattering

Interactions:
phenomenological
optical potentials

Expected that
those capture the
important features
of a nucleon in the
nucleus

Deuteron: NN interaction
p(n) – nucleon i: optical potential

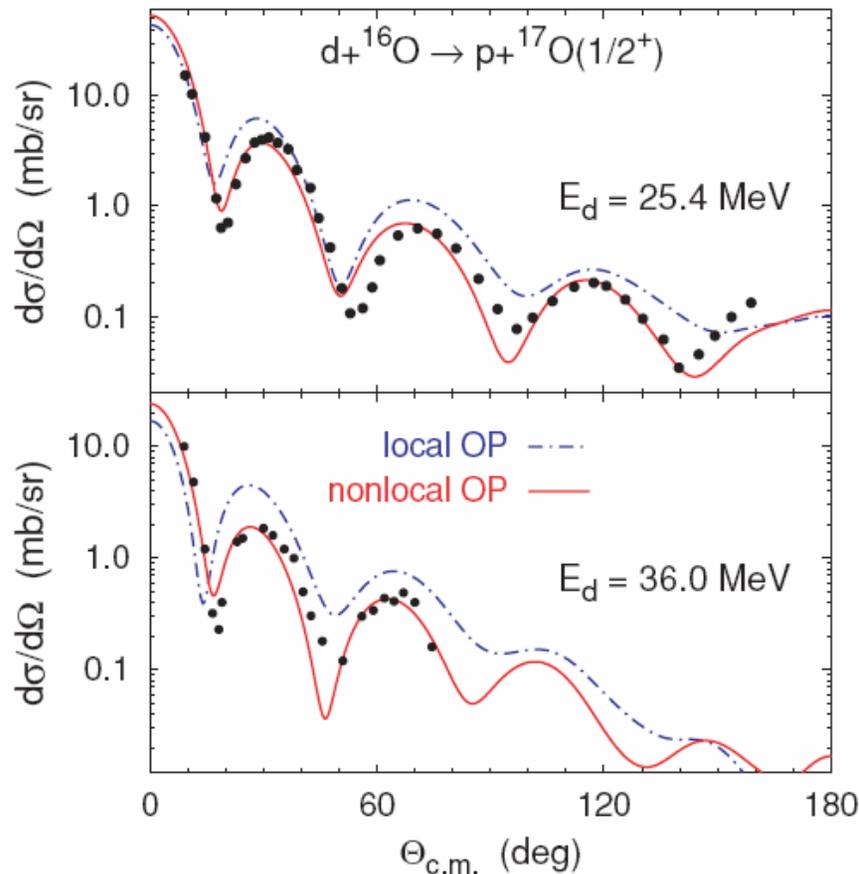
Elastic deuteron scattering @ 56 MeV



A. Deluva, PRC 79, 021602(R) (2009)

Faddeev Formulation of (d,p) Reactions cont'd

A. DELTUVA



Transfer Reaction
(breakup)

Further:

Considerations on optical potential similar to the ones in $p + A$ scattering?

Improvements in treating optical potential in Faddeev calculation?

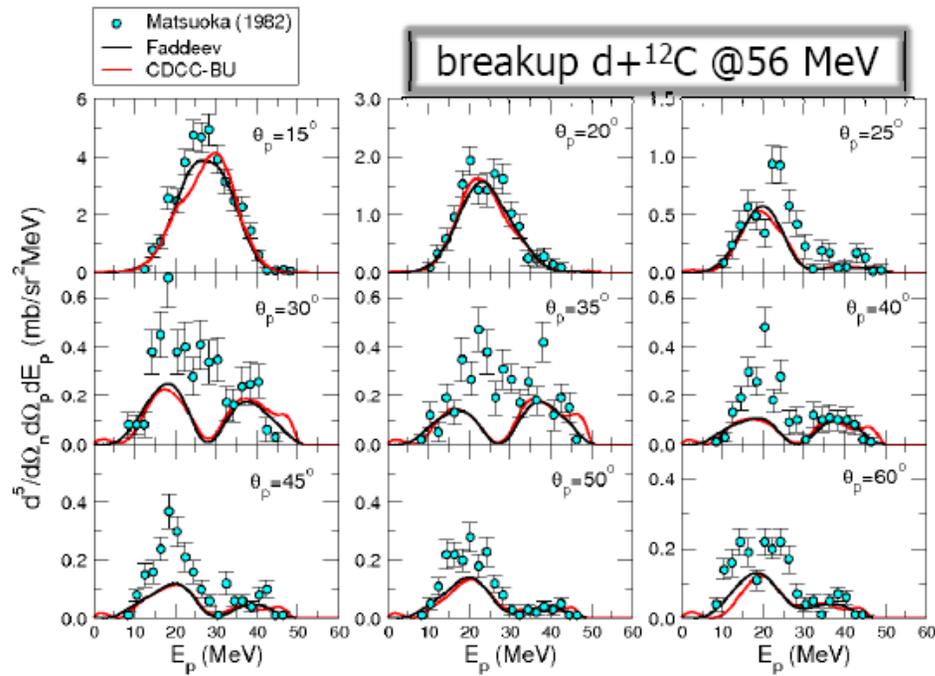
Talk: A. Fonseca

CDCC: Continuum discretized coupled channel approach

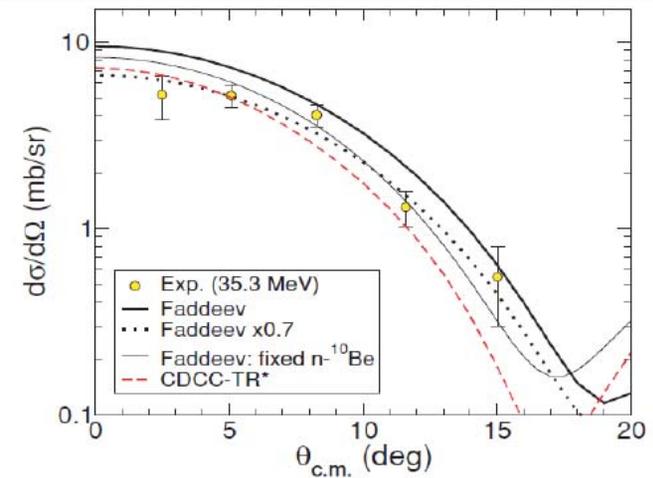
Developed to for calculating deuteron-nucleus reactions (~80s)

- Approximate treatment of the 3-body problem including breakup to all orders but -- assuming breakup-transfer couplings are small.
- Well developed and widely used for $d+A$
- **Expansion in scattering (continuum) states of target and projectile**
 - Core excitations can be included naturally
 - Convergence of expansion needs to be established for each reaction
- **Difficult to treat breakup and transfer on equal footing**
 - Assumptions can and need to be tested **today** with respect to a Faddeev approach.

Comparing CDCC and Faddeev Calculations



$^{11}\text{Be}(p,d)^{10}\text{Be}$ at $E_p = 38.4$ MeV



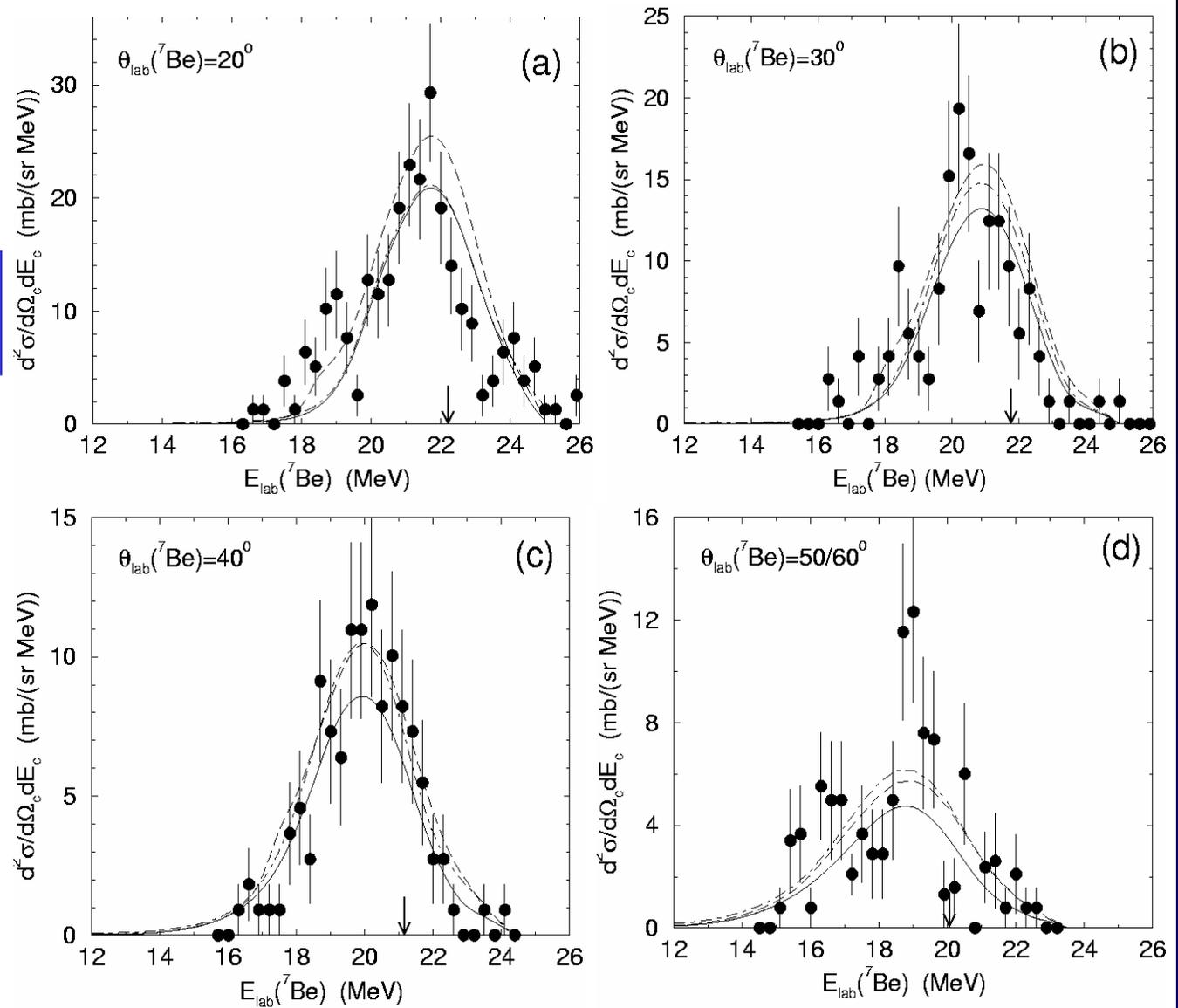
[Deltuva, Moro, Nunes and Fonseca, PRC76, 064602]

Detailed comparisons between CDCC and Faddeev approach needed to improve on short-comings of CDCC

Breakup of ${}^8\text{B}$

${}^8\text{B}$ breakup on ${}^{58}\text{Ni}$
($E_{\text{beam}} = 26 \text{ MeV}$)

Results of CDCC
calculations
assuming a single
particle structure
for ${}^8\text{B} = {}^7\text{Be} + p$



[Tostevin, FN, Thompson PRC (2001) 024617]

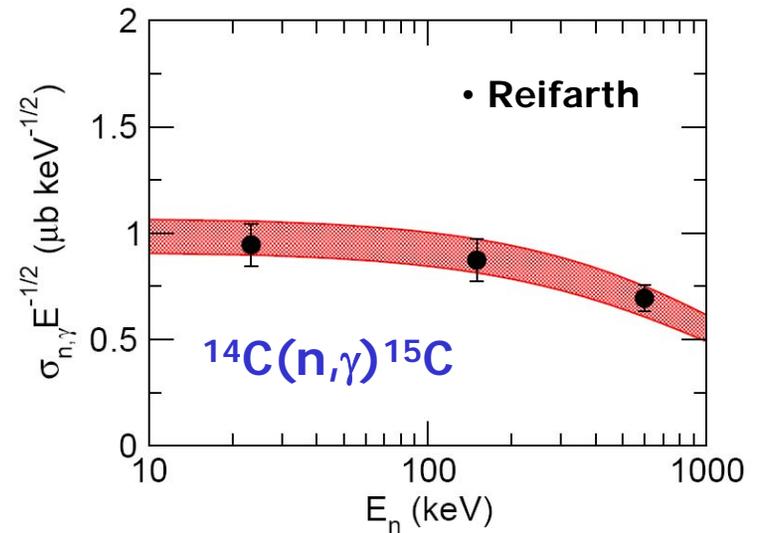
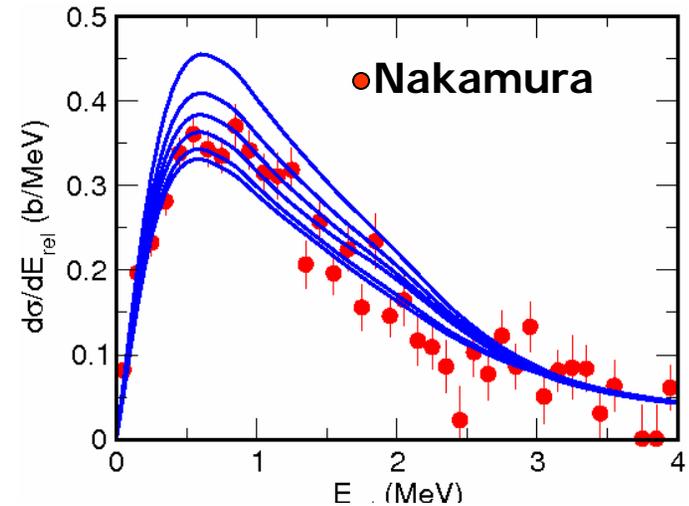
Breakup reactions and (n,γ) : methodology

- CDCC + set of single particle parameters
- extract ANC from χ^2 minimum
 - error from $\varepsilon = \chi_{\min}^2 + 1$
- Yao, JPG33 (2006) 1

$ANC = 1.32 \pm 0.07 \text{ fm}^{-1/2}$

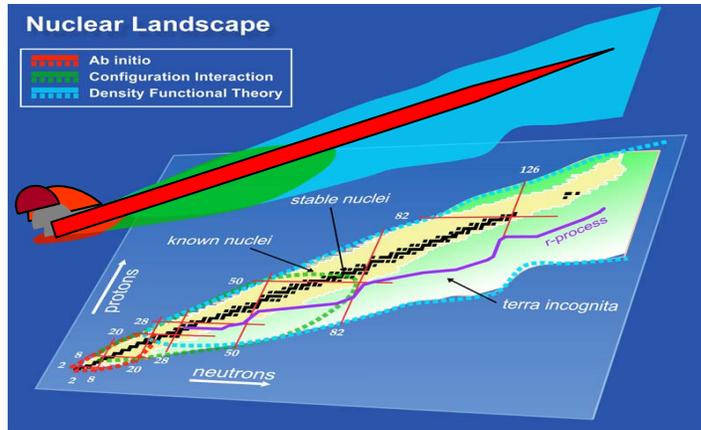
Summers and Nunes, PRC78(2009)069908

$^{208}\text{Pb}(^{15}\text{C}, ^{14}\text{C} + n)^{208}\text{Pb} @ 68 \text{ MeV/u}$



Nakamura et al, NPA722(2003)301c

Reifarth et al, PRC77,015804 (2008)



Goal for Reaction Theory:

Determine the topography of the nuclear landscape according to reactions described in definite schemes

At present `traditional' few-body methods are being successfully applied to a subset of nuclear reactions.

Establish overlaps, where different approaches can be firmly tested.

This `cross fertilization' of two different fields carries a lot promise for developing the theoretical tools necessary for FRIB physics.

It is an exciting time to participate in this endeavor.