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The Coulomb Problem in Momentum Space without Screening

Ch. Elster

V. Eremenko, L. Hlophe, N.J. Upadhyay,
F. Nunes, G. Arbanas, J. E. Escher, I.J. Thompson

(The TORUS Collaboration)

Department of physics + astronomy

Office of Nuclear Physics — Exploring Nuclear Matter - Quarks to Stars

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Physics Problem: Nuclear Reactions dominated by few degrees of freedom

Reactions: Elastic Scattering, Breakup, Transfer

$^{3}He(d,p)^{4}He$

Light nuclei

$^{140}Sn(d,p)^{141}Sn$

Heavy Nuclei

?

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Physics Problem: Nuclear Reactions dominated by few degrees of freedom

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Many Body Problem !

$^{140}Sn(d,p)^{141}Sn$

Heavy Nuclei

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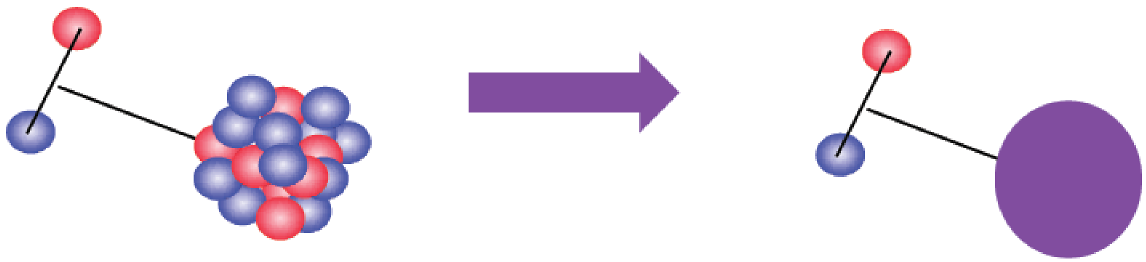
Reduce Many-Body to Few-Body Problem



Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body poblem:

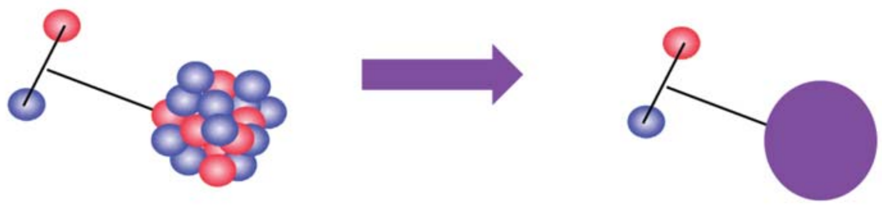
$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction (V_{np})

Optical potentials p+A and n+A ($V_{nA} + V_{pA}$)

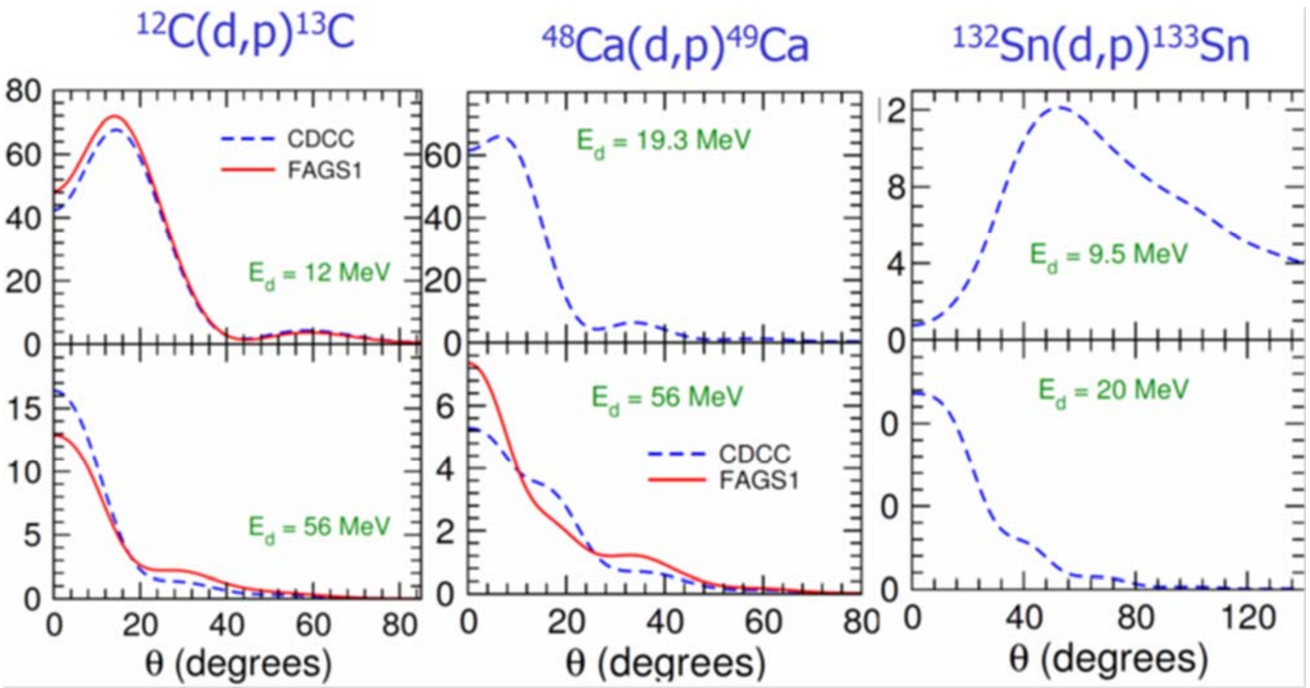
Effective Three-Body Problem → Faddeev calculation

(d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issue: current momentum space implementation of Coulomb interaction (shielding) does not converge for Z ≥ 20

Courtesy: F.M. Nunes

Theoretical Foundation for a remedy:

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Generalized Faddeev formulation of (d,p) reactions with:
(a) explicit inclusion of target excitations
(b) no screening of the Coulomb interaction

Faddeev formulation → momentum space

Suggestions:

- **Target excitations:**
  Including specific excited states → Formulation with separable interactions

- **No screening of the Coulomb interaction:**
  Formulation of Faddeev equations in Coulomb basis instead of plane wave basis

Both need preparatory work!

Roadmap for Proof of Principle

- **Separable interactions for**
  - neutron-proton (literature)
  - neutron-nucleus → **phenomenological optical potentials**
  - proton-nucleus → **Woods-Saxon functions in r-space**
- **Coulomb distorted neutron-nucleus formfactor**
  - Coulomb wave functions in momentum space
  - Integration over oscillatory singularities
- **Coulomb distorted proton-nucleus formfactor**
  - Proton-nucleus problem needs to be solved

Hamiltonian: $H = H_0 + V_{np} + V_{nA} + V_{pA}$

V_{np} : **NN interaction** -- momentum space ✔

V_{nA} : **Optical potential**

Phenomenological optical potentials fitted to data from ^{12}C to ^{208}Pb given in coordinate space and parameterized in terms of Woods-Saxon functions

$$U_{nucl}(r) = V(r) + i\left[W(r) + W_s(r)\right] + V_{ls}(r)\,\mathbf{l}\cdot\boldsymbol{\sigma}$$

$$\begin{aligned} V(r) &= -V_r f_{ws}(r, R_0, a_0) \\ W(r) &= -W_v f_{ws}(r, R_w, a_w) \\ W_s(r) &= -W_s(-4a_w) f'_{ws}(r, R_w, a_w) \\ V_{ls}(r) &= -(V_{so} + iW_{so})(-2) g_{ws}(r, R_{so}, a_{so}), \end{aligned}$$

$$\begin{aligned} f_{ws}(r, R, a) &= \frac{1}{1 + \exp(\frac{r-R}{a})} \\ f'_{ws}(r, R, a) &= \frac{d}{dr} f_{ws}(r, R, a) \\ g_{ws}(r, R, a) &= f'(r, R, a)/r \end{aligned}$$

Not useful in this form

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Historical remarks on separable potentials

- **Considerable work on NN potentials**
  - Goal: application in 3-nucleon calculations
  - Most sophisticated: Graz group
- **Formfactors parameterized in terms of Yukawa function**
  - Yukawa functions well suited for NN interaction
- **Not too useful for heavy nuclei**
  - Woods-Saxon function more appropriate

How to proceed:

First: Woods-Saxon functions have a semi-analytic Fourier transform: (fast converging series expansion)

Central term:

$$\overline{V}(\mathbf{q}) = \frac{V_r}{\pi^2}\left\{ \frac{\pi a_0 e^{-\pi a_0 q}}{q\left(1-e^{-2\pi a_0 q}\right)^2}\left[R_0\left(1-e^{-2\pi a_0 q}\right)\cos\left(qR_0\right) - \pi a_0\left(1+e^{-2\pi a_0 q}\right)\sin\left(qR_0\right)\right] \right.$$
$$\left. -a_0^3 e^{-\frac{R_0}{a_0}}\left[\frac{1}{(1+a_0^2q^2)^2} - \frac{2e^{-\frac{R_0}{a_0}}}{(4+a_0^2q^2)^2}\right]\right\}$$

Surface term:

$$\overline{W}_s(\mathbf{q}) = -4a_w\,\frac{W_s}{\pi^2}\left\{\frac{\pi a_w e^{-\pi a_w q}}{\left(1-e^{-2\pi a_w q}\right)^2}\right.$$
$$\left[\left(\pi a_w\left(1+e^{-2\pi a_w q}\right) - \frac{1}{q}\left(1-e^{-2\pi a_w q}\right)\right)\cos(qR_w) + R_w\left(1-e^{-2\pi a_w q}\right)\sin(qR_w)\right]$$
$$\left. + a^2e^{-R_w/a_w}\left[\frac{1}{(1+a_w^2q^2)^2} - \frac{4e^{-R_w/a_w}}{(4+a_w^2q^2)^2}\right]\right\}.$$

Details: L. Hlophe et al.: Phys.Rev. C88 (2013) 064608

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We can solve a momentum space scattering equation

Details: L. Hlophe et al.: Phys.Rev. C88 (2013) 064608

Second: Ernst-Shakin-Thaler (EST) Phys. Rev. C8, 46 (1973)

Define separable potential as

(here V Hermitian !)

$$\mathcal{V} = \frac{V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(+)}_{k_E}|V}{\langle\Psi^{(+)}_{k_E}|V|\Psi^{(+)}_{k_E}\rangle}$$

fixed energy

Then partial wave t-matrix :

$$\langle p'|t(E)|p\rangle = \frac{\langle p'|V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(+)}_{k_E}|V|p\rangle}{\langle\Psi^{(+)}_{k_E}|V - V g_0(E) V|\Psi^{(+)}_{k_E}\rangle}$$

Reminder:

$$V|\Psi^{(+)}_{k_E}\rangle := t|k_E\rangle$$

The EST construction guarantees:

At a given scattering energy E_{kE} the scattering wave functions obtained with the original potential V and the separable potential $\mathcal{V}$ are identical . ➔ the half-shell t-matrices are identical

Optical Potentials == Complex Potentials

Generalizaton of EST necessary

L. Hlophe et al.: Phys.Rev. C88 (2013) 064608

Definition with In-state necessary to fulfill reciprocity theorem

$$U = \frac{V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(-)}_{k_E}|V}{\langle\Psi^{(-)}_{k_E}|V|\Psi^{(+)}_{k_E}\rangle}$$

For time reversal operator $\mathscr{K}$ potential must fulfill:

$$\mathscr{K} U \mathscr{K}^{-1} = U^{\dagger}$$

Technical details:

Let $|f_{l,kE}>$ be a radial wave function and $K\,|f_{l,kE}> = |f^{*}_{l,kE}>$

Rank-1 separable t-matrix: $\langle p'|t(E)|p\rangle = \dfrac{\langle p'|u|f_{l,k_E}\rangle\langle f^{*}_{l,k_E}|u|p\rangle}{\langle f^{*}_{l,k_E}|u - ug_0(E)u|f_{l,k_E}\rangle}$

With $t(p',k_E,E_{k_E}) = \langle f^{*}_{l,k_E}|u|p'\rangle$ and $t(p,k_E,E_{k_E}) = \langle p|u|f_{l,k_E}\rangle$

$$\langle p'|t(E)|p\rangle = \frac{t(p',k_E,E_{k_E})\; t(p,k_E,E_{k_E})}{\langle f^{*}_{l,k_E}|u(1-g_0(E)u)|f_{l,k_E}\rangle} \equiv t(p',k_E,E)\;\tau(E)\;t(p,k_E,E)$$

and

$$\begin{aligned}\tau(E)^{-1} &= t(k_E,k_E,E_{k_E})\\ &+2\mu\left[\mathcal{P}\int dpp^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_E^2-p^2} - \mathcal{P}\int dpp^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_0^2-p^2}\right]\\ &+i\pi\mu\left[k_0 t(k_0,k_E,E_{k_E})t(k_0,k_E,E_{k_E}) - k_E\; t(k_E,k_E,E_{k_E})t(k_E,k_E,E_{k_E})\right].\end{aligned}$$

Generalization to arbitrary rank

$$U = \sum_{i,j} u|f_{l,k_{E_i}}\rangle\langle f_{l,k_{E_i}}|M|f^*{}_l, k_{E_j}\rangle\langle f^*_{l,k_{E_j}}|u$$

with $$\delta_{ik} = \sum_j \langle f_{l,k_{E_i}}|M|f^*_{l,k_{E_j}}\rangle\langle f^*_{l,k_{E_j}}|u|f_{l,k_{E_k}}\rangle = \sum_j \langle f^*_{l,k_{E_i}}|u|f_{l,k_{E_j}}\rangle\langle f_{l,k_{E_j}}|M|f^*_{l,k_{E_k}}\rangle$$

t-matrix $$t(E) = \sum u|f_{l,k_{E_i}}\rangle\tau_{ij}(E)\langle f^*_{l,k_{E_j}}|u$$

$$\sum_j \tau_{ij}(E)\ \langle f^*_{l,k_{E_j}}|u - u g_0(E) u|f_{l,k_{E_k}}\rangle = \delta_{ik}$$

Compute and solve system of linear equations

- Instead of analytic functions, form factors are given numerically
- Solving LS equation for n+A takes into account dependence on the different nuclei

Guideline for Rank of Separable Representation (up to ~50 MeV)

system	partial wave(s)	rank	EST support point(s) [MeV]
n+^{48}Ca	$l \geq 10$	1	40
	$l \geq 8$	2	29, 47
	$l \geq 6$	3	16, 36, 47
	$l \geq 0$	4	6, 15, 36, 47
n+^{132}Sn and n+^{208}Pb	$l \geq 16$	1	40
	$l \geq 13$	2	35, 48
	$l \geq 11$	3	24, 39, 48
	$l \geq 6$	4	11, 21, 36, 45
	$l \geq 0$	5	5, 11, 21, 36, 47

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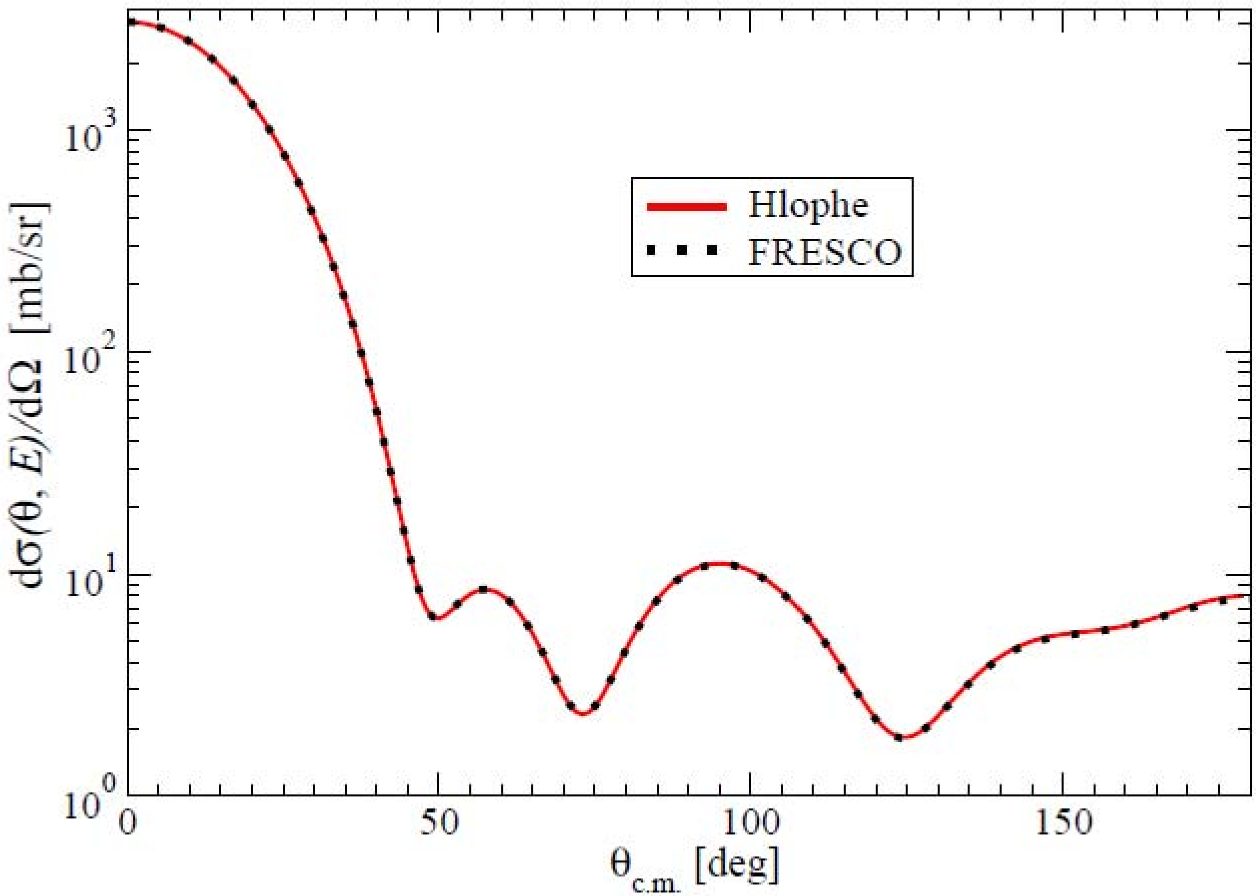
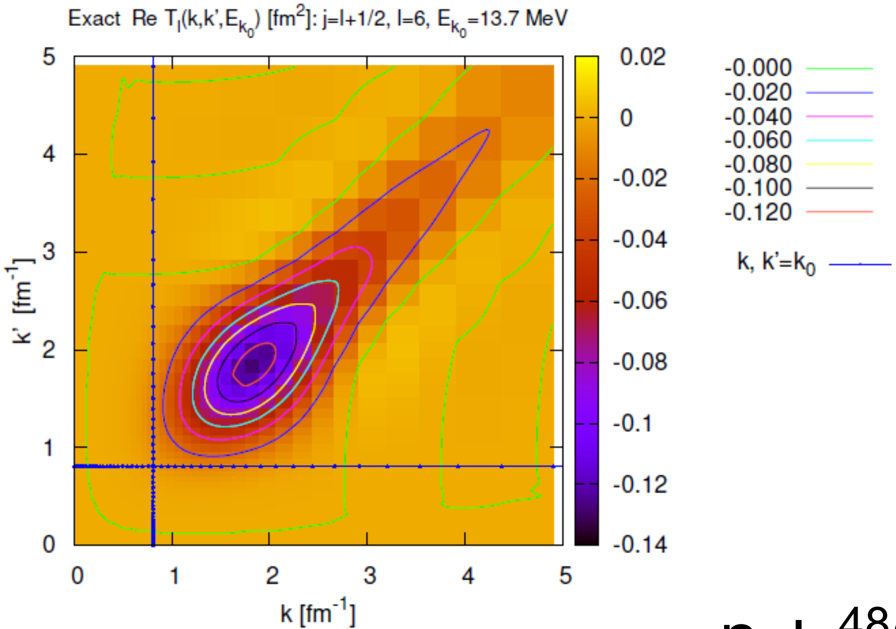
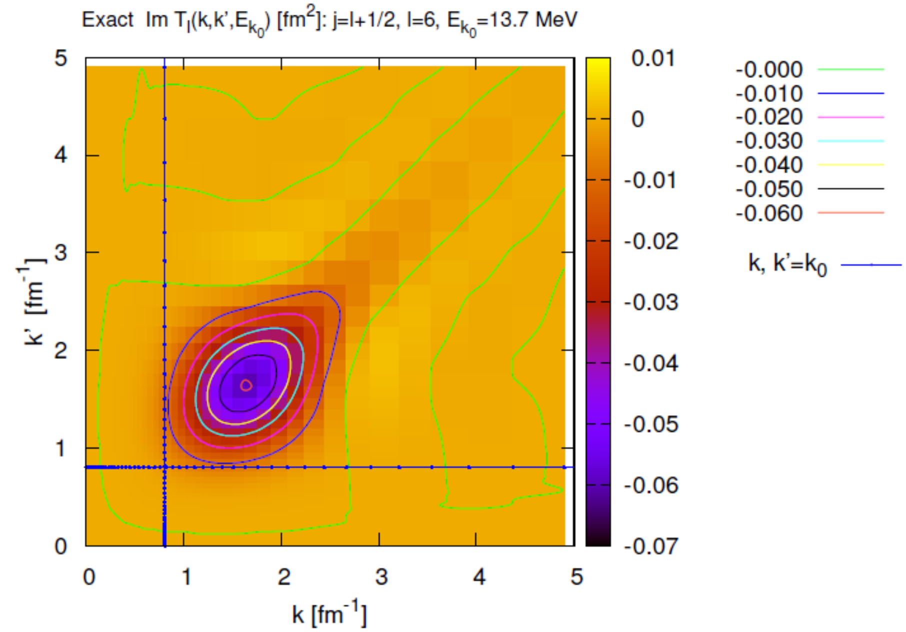
Comparison with r-space calculation: n+^{48}Ca @ 12 MeV

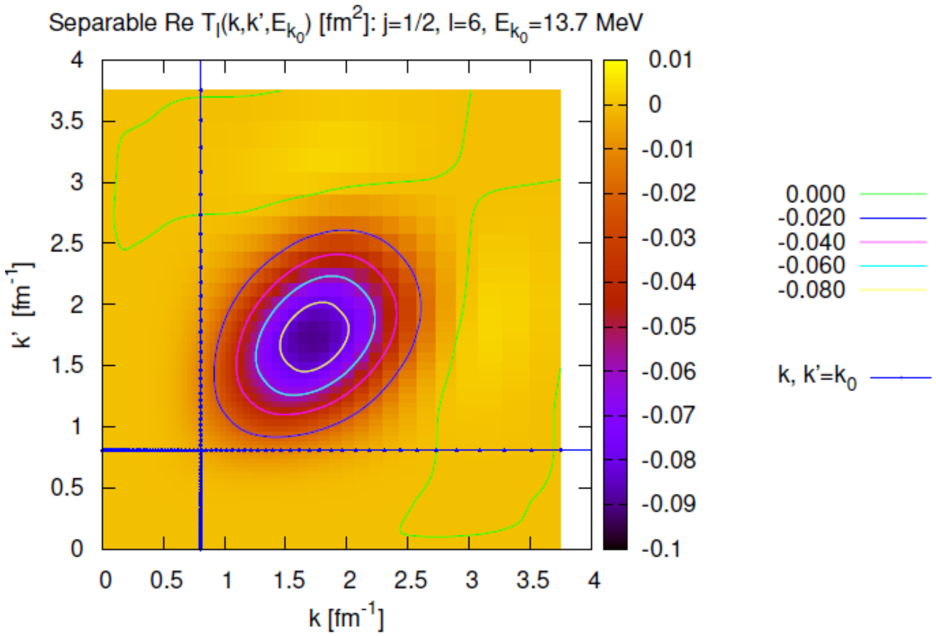


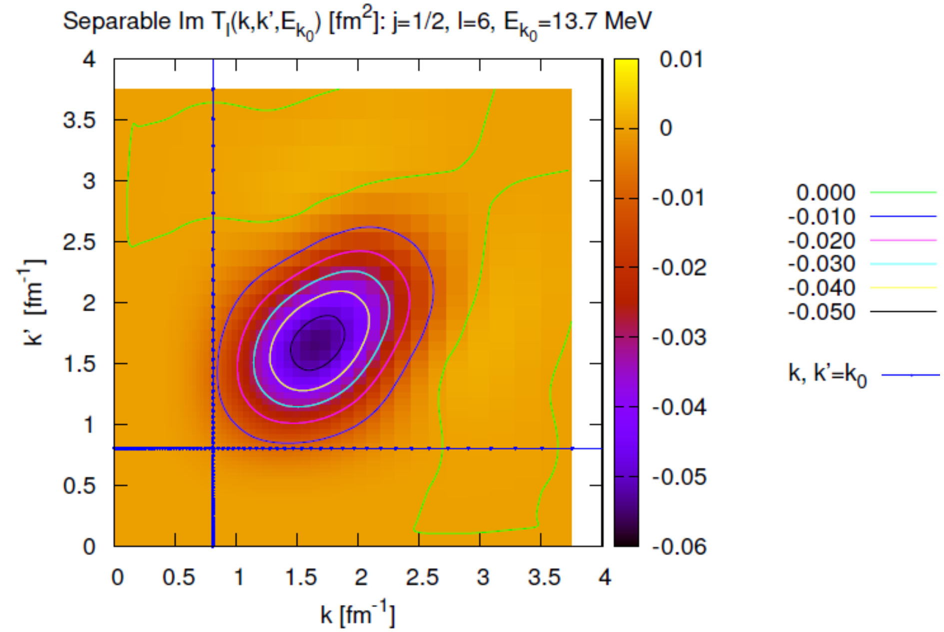
Off-Shell t-matrix elements: $t_l\,(k',k;\,E_{k0})$





n + ^{48}Ca





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Coulomb distorted neutron-nucleus formfactor

Separable t-matrix derived from p+A optical potential (generalized EST scheme)

$$t_l(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}(E) \langle f^*_{l,k_{E_j}}|u$$

Nuclear matrix elements $\langle p|t_l(E)|p'\rangle$

$$\langle p|u|f_{l,k_E}\rangle = t_l(p, k_E; E_{k_E}) \equiv u_l(p)$$
$$\langle f^*_{l,k_E}|u|p'\rangle = t_l(p', k_E; E_{k_E}) \equiv u_l(p')$$

Coulomb distorted nuclear matrix element

$$\langle \psi^C_{l,p}|u|f_{l,k_E}\rangle = \int_0^\infty \frac{dq\, q^2}{2\pi^2}\, u_l(q)\psi^C_{l,p}(q)^\star \equiv u^C_l(p)$$
$$\langle f^*_{l,k_E}|u|\psi^C_{l,p}\rangle = \int_0^\infty \frac{dq\, q^2}{2\pi^2}\, u_l(q)\, \psi^C_{l,p}(q) \equiv u^C_l(p)^\dagger$$

$\psi^C_{p_\alpha l}(p)$ is the Coulomb scattering wave function

Challenge I: momentum space Coulomb functions

$\eta = Z_1 Z_2\, e^2 \mu/q$

General: $$\psi^{C(+)}_{\mathbf{q},\eta}(\mathbf{p}) = \lim_{\gamma\to+0} \int d^3\mathbf{r}\, e^{-i\mathbf{p}\mathbf{r}-\gamma r}\, \psi^{C(+)}_{\mathbf{q},\eta}(r)$$

FT: A. Chan, MS thesis
U. Waterloo (2007)

$$= -4\pi e^{-\pi\eta/2}\Gamma(1+i\eta) \lim_{\gamma\to+0} \frac{d}{d\gamma} \left\{ \frac{[p^2-(q+i\gamma)^2]^{i\eta}}{[|\mathbf{p}-\mathbf{q}|^2+\gamma^2]^{1+i\eta}} \right\}$$

Partial wave decomposition (Mukhamedzanov, Dolinskii) (1966)

$$\psi^{C(+)}_{\mathbf{q},\eta}(\mathbf{p}) \equiv \sum_{l=0}^{\infty}(2l+1)\psi^{C}_{l,q,\eta}(p)P_l(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}}), \qquad \psi^{C}_{l,q,\eta}(p) = \frac{1}{2}\int_{-1}^{1} dz\, P_l(z)\psi^{C(+)}_{\mathbf{q},\eta}(\mathbf{p}),$$

$$\frac{1}{2}\int_{-1}^{1} dz\, P_l(z)(\zeta - z)^{-1-i\eta} = \frac{e^{\pi\eta}}{\Gamma(1+i\eta)}(\zeta^2-1)^{-i\eta/2} Q_l^{i\eta}(\zeta)$$

$$\zeta \equiv \frac{p^2+q^2+\gamma^2}{2pq},$$

Essential:

$Q_l^{i\eta}(\zeta)$ has different representations depending on ζ

$Q_l^{i\eta}(\zeta)$ **has different representations in terms of the hypergeometric function $_2F_1$ (a;b;c;z) depending on ζ**

ζ large enough (p and q different) → **"regular" representation**

$$Q_l^{i\eta}(\zeta) = \frac{e^{-\pi\eta}\Gamma(l+i\eta+1)\Gamma(1/2)}{2^{l+1}\Gamma(l+3/2)}(\zeta^2-1)^{i\eta/2}\,\zeta^{-l-i\eta-1} \times {}_2F_1\left(\frac{l+i\eta+2}{2},\frac{l+i\eta+1}{2};l+\frac{3}{2};\frac{1}{\zeta^2}\right)$$

ζ ≈ 1 (p ≈ q) → **"pole-proximity" representation**

$$Q_l^{i\eta}(\zeta) = \frac{1}{2}e^{-\pi\eta}\left\{\Gamma(i\eta)\left(\frac{\zeta+1}{\zeta-1}\right)^{i\eta/2}{}_2F_1\left(-l,l+1;1-i\eta;\frac{1-\zeta}{2}\right) + \frac{\Gamma(-i\eta)\Gamma(l+i\eta+1)}{\Gamma(l-i\eta+1)}\left(\frac{\zeta-1}{\zeta+1}\right)^{i\eta/2}{}_2F_1\left(-l,l+1;1+i\eta;\frac{1-\zeta}{2}\right)\right\}$$

Partial-wave momentum space Coulomb functions

"regular" representation

$$\psi^C_{l,q}(p) = -\frac{4\pi\eta e^{-\pi\eta/2} q (pq)^l}{(p^2+q^2)^{1+l+i\eta}} \left[\frac{\Gamma(1+l+i\eta)}{(1/2)_{l+1}}\right] \times {}_2F_1\left(\frac{2+l+i\eta}{2}, \frac{1+l+i\eta}{2}; l+3/2; \frac{4q^2p^2}{(p^2+q^2)^2}\right) \times \lim_{\gamma\to 0}\left[p^2-(q+i\gamma)^2\right]^{-1+i\eta}$$

"pole-proximity" representation:

$$\psi^C_{l,q}(p) = -\frac{2\pi}{p}\exp(-\pi\eta/2+i\sigma_l)\left[\frac{(p+q)^2}{4pq}\right]^l \lim_{\gamma\to 0} 2\,\Im\mathrm{m}\,\mathcal{D}.$$

$$\mathcal{D} \equiv \frac{\Gamma(1+i\eta)e^{-i\sigma_l}(p+q)^{-1+i\eta}}{(p-q+i\gamma)^{1+i\eta}}\, {}_2F_1\left(-l, -l-i\eta; 1-i\eta; \frac{(p-q)^2}{(p+q)^2}\right)$$

Oscillatory singularity for $p \to q$

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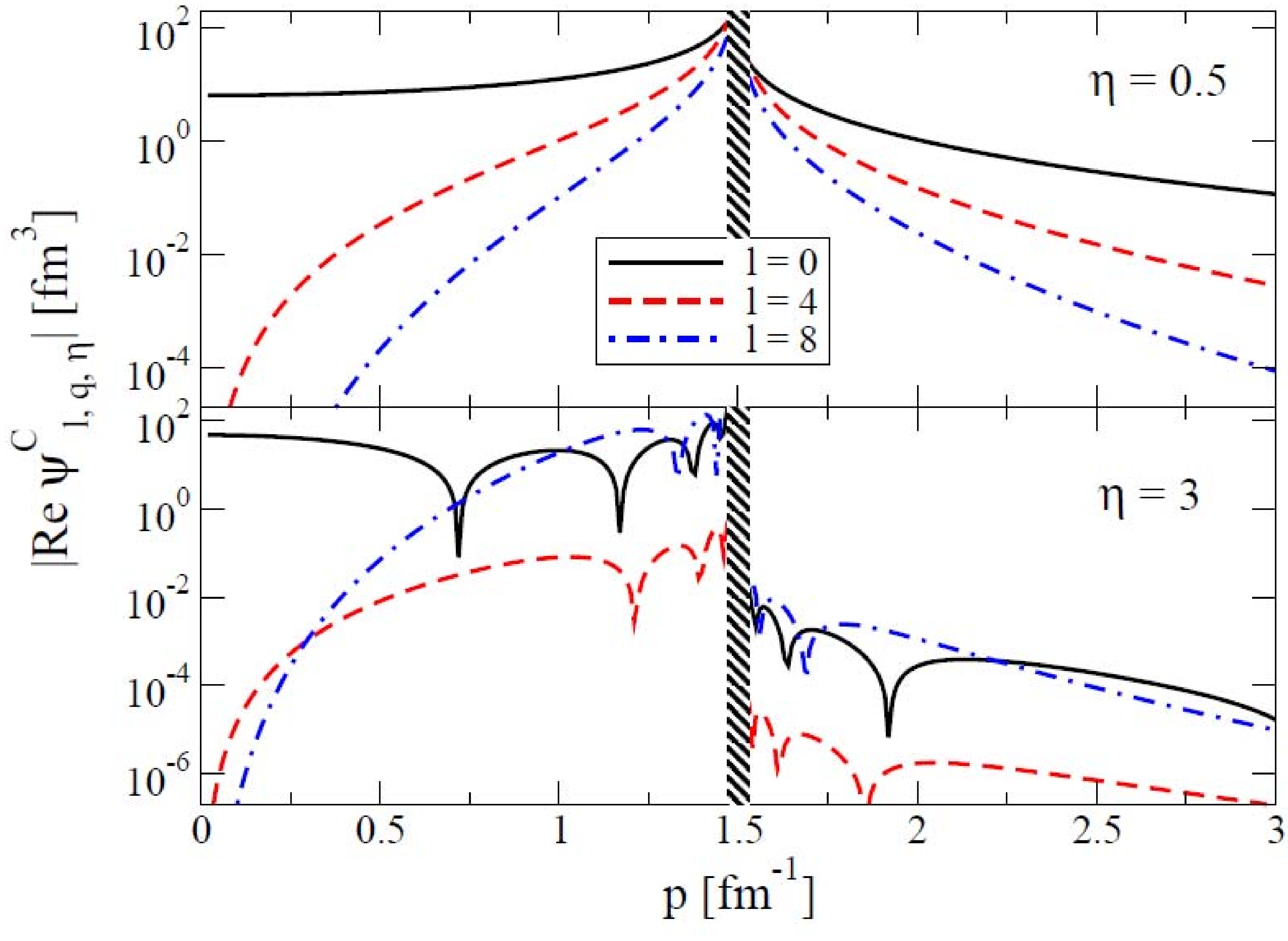
$q = 1.5\ fm^{-1}$



$\eta = 0.5$
l = 0
l = 4
l = 8
$\eta = 3$
$|\mathrm{Re}\,\psi^C_{l,q,\eta}|$ [fm^3]
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
0
0.5
1
1.5
2
2.5
3
p [fm^{-1}]

Challenge II:
Matrix elements with Coulomb basis functions

Separable t-matrix derived from p+A optical potential (generalized EST scheme)

$$t_l(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}(E) \langle f^*_{l,k_{E_j}}|u$$

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Coulomb distorted nuclear matrix element

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$$\langle f^*_{l,k_E}|u|\psi^C_{l,p}\rangle = \int_0^\infty \frac{dq\, q^2}{2\pi^2}\, u_l(q)\, \psi^C_{l,p}(q) \equiv u^C_l(p)^\dagger$$

"oscillatory" singularity at $q = p$: $\lim_{\gamma\to+0} \frac{1}{(q-p+i\gamma)^{1+i\eta}}$

Gel'fand-Shilov Regularization:

Generalization of Principal value regularization
Idea: reduce value of integrand near singularity

simplified

$$\int_{-\Delta}^{\Delta} dx \frac{\varphi(x)}{x^{1+i\eta}} = \int_{-\Delta}^{\Delta} dx \frac{\varphi(x) - \varphi(0) - \varphi'(0)x}{x^{1+i\eta}}$$

$$- \frac{i\varphi(0)}{\eta}[\Delta^{-i\eta} - (\Delta)^{-i\eta}] + \ldots$$

- **Reduce integrand around pole by subtracting 2 terms of the Laurent series**

Gel’fand-Shilov Regularization:

Generalization of Principal value regularization
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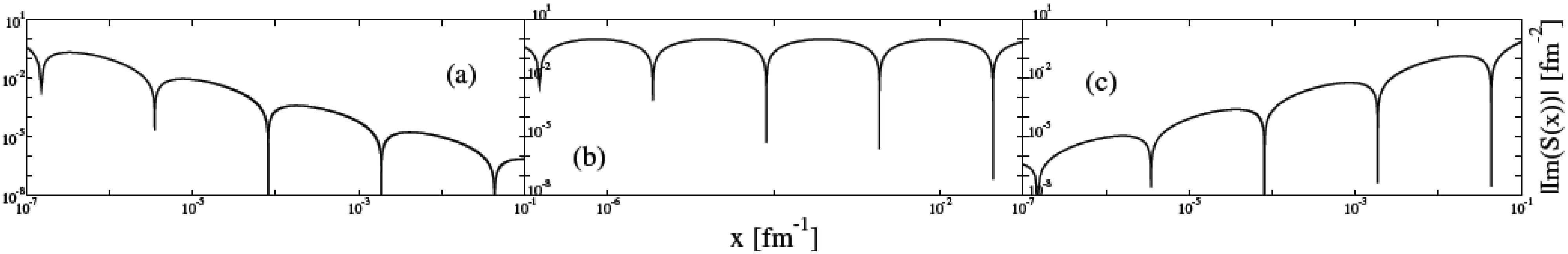
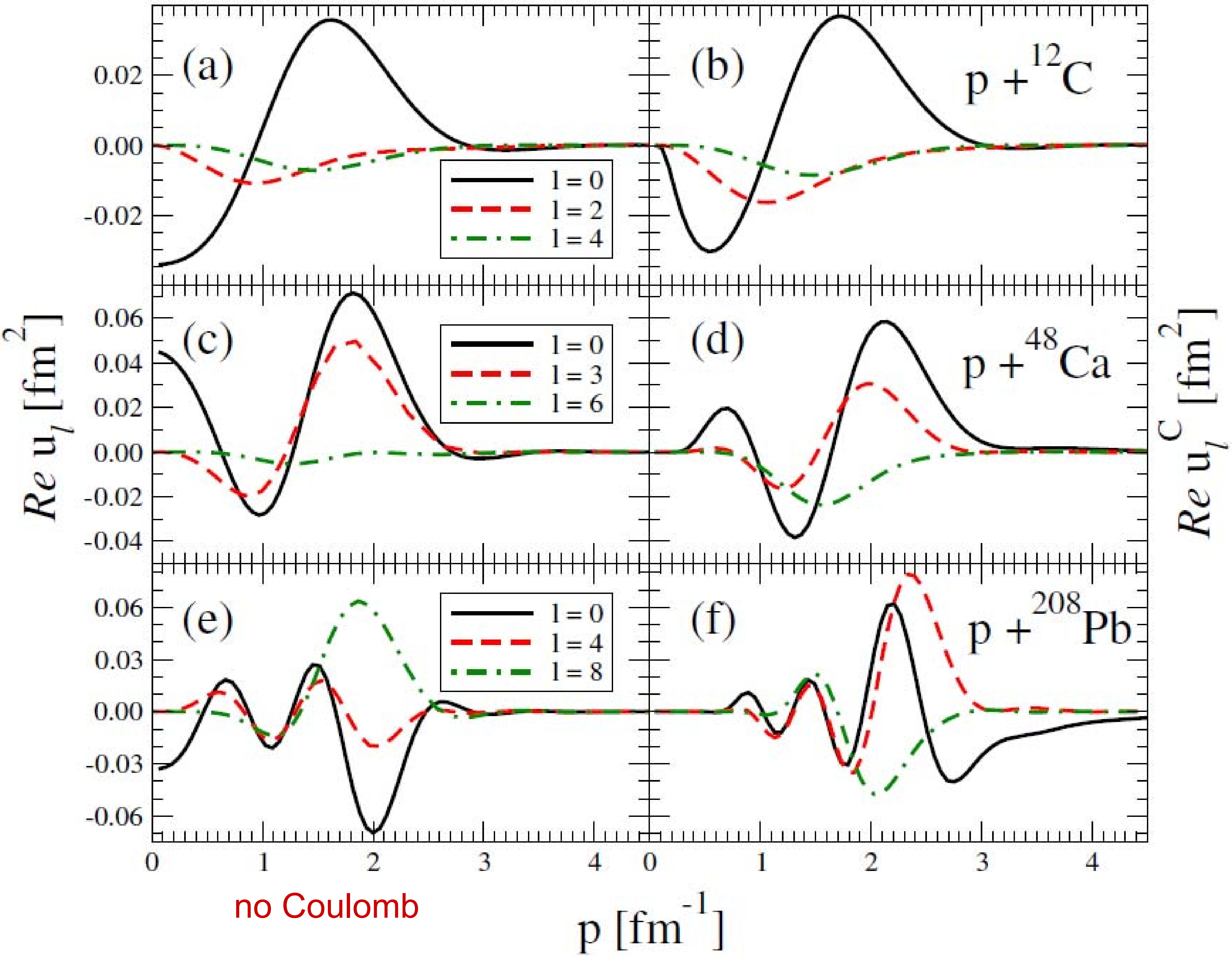
simplified

$$- \frac{i\varphi(0)}{\eta}\left[\Delta^{-i\eta} - (\Delta)^{-i\eta}\right] + \dots$$

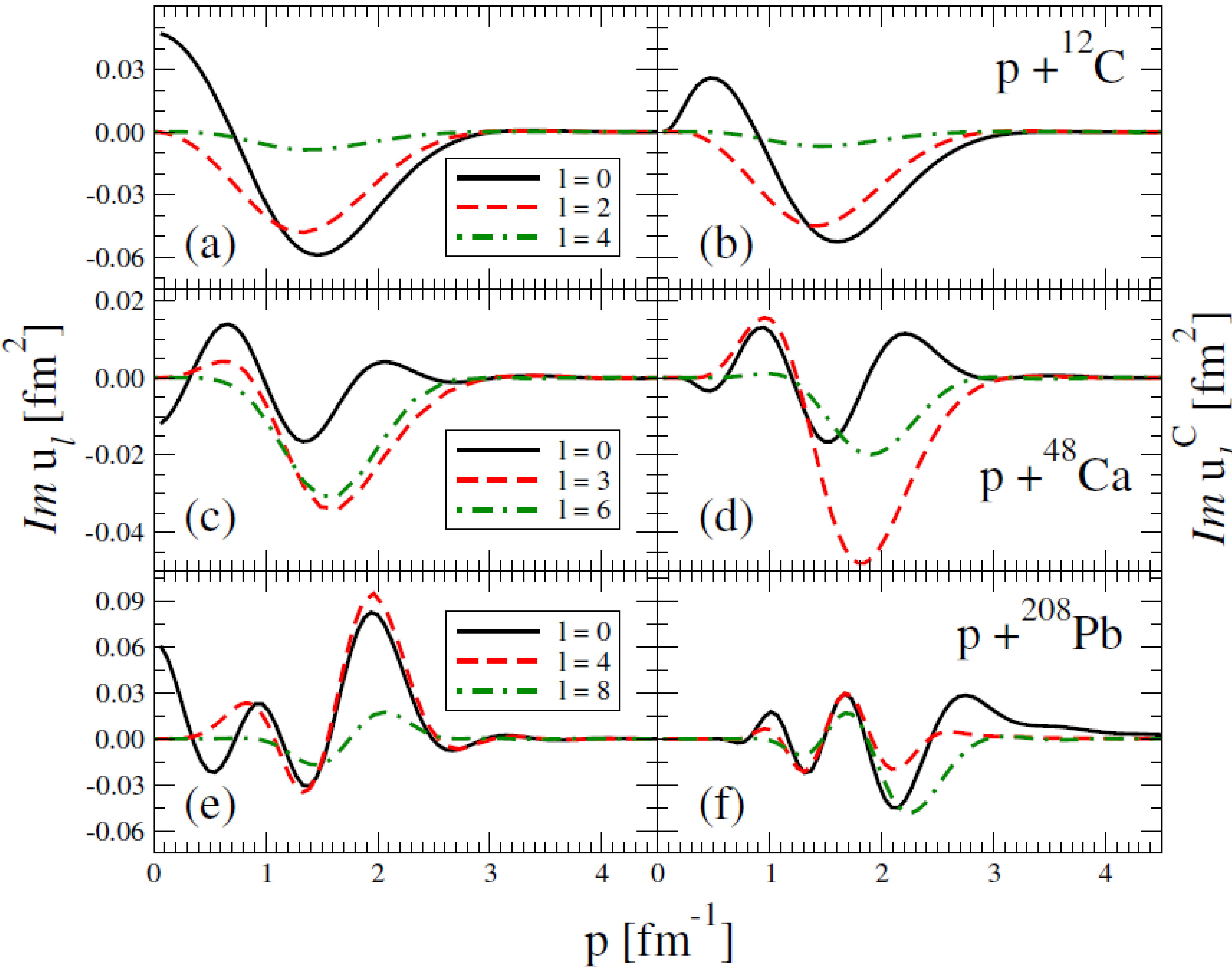
- Reduce integrand around pole by subtracting 2 terms of the Laurent series



I. M. Gel’fand and G. E. Shilov. “Generalized Functions”. Vol. 1. Academic Press, New York and London. 1964.


(a)
(b)
p + ^{12}C
l = 0
l = 2
l = 4
(c)
(d)
p + ^{48}Ca
l = 0
l = 3
l = 6
(e)
(f)
p + ^{208}Pb
l = 0
l = 4
l = 8
Re u_l [fm^2]
Re u_l^C [fm^2]
p [fm^{-1}]

no Coulomb


p + ^{12}C
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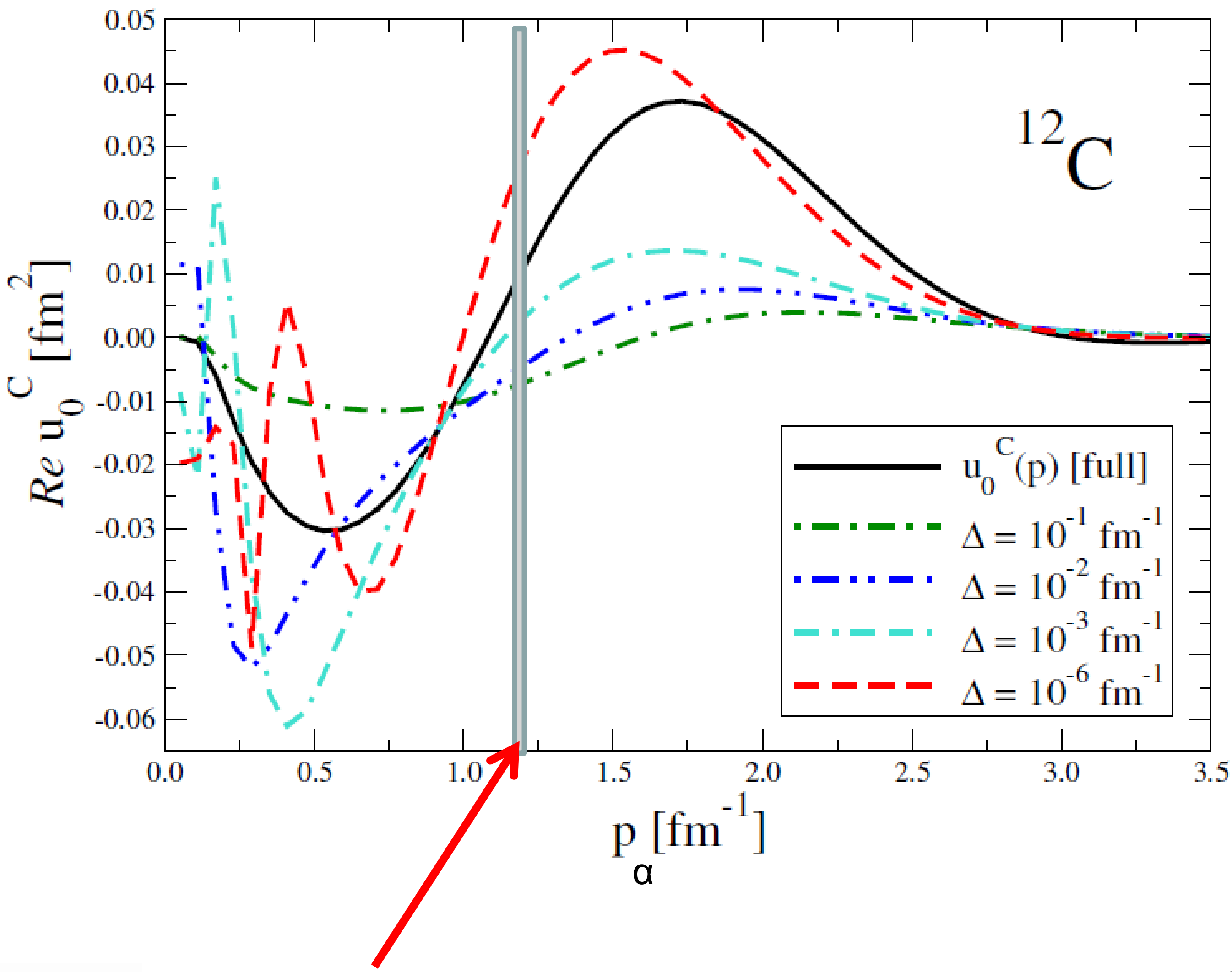
l = 0
l = 2
l = 4
l = 3
l = 6
l = 8
(a)
(b)
(c)
(d)
(e)
(f)
$Im\, u_l$ [fm^2]
$Im\, u_l^C$ [fm^2]
p [fm^{-1}]

p + ^{12}C

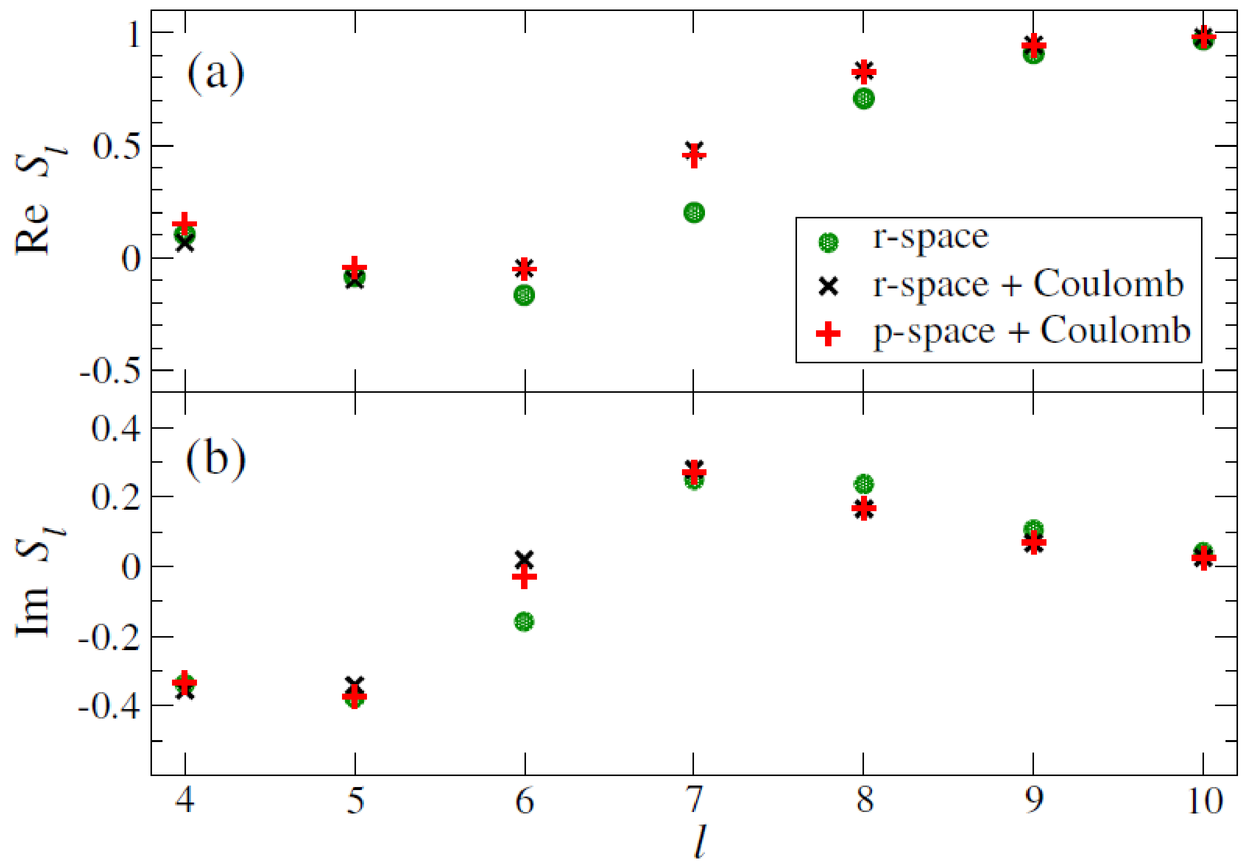
$$u_l^C(p_\alpha) = \overbrace{\int_0^{p_\alpha-\Delta}\frac{dp}{2\pi^2}\,p^2 u_l(p)\,\psi^C_{p_\alpha l}(p)}^{I} + \overbrace{\int_{p_\alpha-\Delta}^{p_\alpha+\Delta}\ldots}^{II} + \overbrace{\int_{p_\alpha+\Delta}^{\infty}\ldots}^{III}$$



Fixed p_α

First Physics Check:

p + ^{48}Ca



Selected partial wave S-matrix elements S_{l+1} for p+^{48}Ca (CH89 optical potential) with Coulomb distorted n+^{48}Ca formfactors

Calculated with neutron-nucleus formfactors

Roadmap for Proof of Principle

- **Separable interactions for**
  - neutron-proton (literature)
  - neutron-nucleus → **phenomenological optical potentials**
  - proton-nucleus → **Woods-Saxon functions in r-space**
- **Coulomb distorted neutron-nucleus formfactor**
  - Coulomb wave functions in momentum space
  - Integration over oscillatory singularities
- **Coulomb distorted proton-nucleus formfactor**
  - Proton-nucleus problem needs to be solved

Coulomb distorted proton-nucleus formfactor

EST construction based on:

- solve the scattering problem in complete basis
- require that for a set energies E_i the wave functions (half-shell t-matrices) obtained with the original and separable potential coincide.

➔ **EST construction can be performed in the Coulomb basis**

$$t_l^{CN}(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}^C(E) \langle f^*_{l,k_{E_j}}|u$$

$$\sum_j \tau_{ij}^C(E) \langle f^*_{l,k_{E_j}}|u - u g_C(E) u|f_{l,k_{E_k}}\rangle = \delta_{ik}$$

$$\hat{g}_C(E_{p0}) = (E - \hat{H}^C + i\varepsilon)^{-1} \qquad \hat{H}^C = H_0 + \hat{V}^C$$

Coulomb Green's function

$$t_l^{CN}(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle \tau_{ij}^C(E)\langle f^*_{l,k_{E_j}}|u$$

Multiply from left and right with a Coulomb state:

$$\langle \psi^C_{l,k_E}|u|f_{l,k_E}\rangle = \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, u_l(p)\, (\psi^C_{l,k_E})^\star(p) \equiv u_l^C(k_E)$$

$$\langle f^*_{l,k_E}|u|\psi^C_{l,k_E}(k_E)\rangle = \int_0^\infty \frac{dp\, p^2}{2\pi^2}\, u_l(p)\, \psi^C_{l,k_E}(p) \equiv (u_l^C)^\dagger(k_E)$$

However: Not a consistent EST construction.

Previous figure used n-A t-matrix calculated in a plane wave basis as input.

→ **Wave functions obtained with original and separable potential are not the same at the support points if calculated in different bases.**

We need the p+A half-shell t-matrices
for a charged particle EST generalization

Non-trivial (due to "pinch singularity")

Use approach by Elster, Liu, Thaler, JPG 19, 2123 (1993)

Solve Lippmann-Schwinger equation in Coulomb distorted basis:

$$\langle k'|\tau_l(E)|k\rangle = \langle k'|U_l|k\rangle + \int \langle k'|U_l|k''\rangle \frac{4\pi k''^2 \, dk''}{E - E'' + i\varepsilon} \langle k''|\tau_l(E)|k\rangle$$

Looks like a regular LS equation if potential element

$$\langle k'|U_l|k\rangle = \langle (\phi_l^C)^{(+)}(k')|V^S|(\phi_l^C)^{(+)}(k)\rangle$$

This is the hard part!

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$$\langle k'|U_l|k\rangle = \langle(\phi_l^C)^{(+)}(k')|V^S|(\phi_l^C)^{(+)}(k)\rangle$$

Matrix elements exist and are well defined if V^S is finite-ranged ✔

We solve this as:

$$\langle k'|U_l|k\rangle = \int \langle\phi_l^C(k')|r'\rangle r'^2\,\mathrm{d}r'\,\underline{\langle r'|V_l^S|r''\rangle}\,r''^2\,\mathrm{d}r''\,\langle r''|\phi_l^C(k)\rangle$$

Woods-Saxon given in Coordinate space

Summary & Outlook

Faddeev-AGS framework in Coulomb basis passed first test!

- **Momentum space nuclear form factors obtained in a Coulomb distorted basis for high charges for the first time.**
- **"Oscillatory singularity" of $\psi_{p_\alpha,l}{}^{c}(p)$ at $p = p_\alpha$ successfully regularized.**
- **Algorithms to compute $\psi_{p_\alpha,l}{}^{c}(p)$ and the overlap integral successfully implemented**

In Progress:

Calculations with separable p+A optical potentials (EST type)

Near Future:

Implementation of Faddeev-AGS equations in the Coulomb basis to obtain (d,p) observables

TORUS: Theory of Reactions for Ustable iSotopes

A Topical Collaboration for Nuclear Theory

http://www.reactiontheory.org/

- Ch. Elster, V. Eremenko[†‡], and L. Hlophe[†]: Institute of Nuclear and Particle Physics, and Department of Physics and Astronomy, Ohio University, Athens, OH 45701.
- F. M. Nunes and N. J. Upadhyay[†]: National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824.
- G. Arbanas: Nuclear Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831.
- J. E. Escher and I. J. Thompson: Lawrence Livermore National Laboratory, L-414, Livermore, CA 94551.

† Post-Docs or Grad Students.

‡ M. V. Lomonosov Moscow State University, Moscow, 119991, Russia.

Some insights for momentum space Coulomb wave functions:

Pole:

$$\psi^C_{p_\alpha l}(p) = -\frac{4\pi}{p} e^{-\pi\eta/2}\Gamma(1+i\eta)e^{i\alpha_l}\left[\frac{(p+p_\alpha)^2}{4pp_\alpha}\right]^l$$

$$\times \operatorname{Im}\left[e^{-i\alpha_l}\frac{(p+p_\alpha+i0)^{-1+i\eta}}{(p-p_\alpha+i0)^{1+i\eta}}\,{}_2F_1\left(-l,-l-i\eta;1-i\eta;\zeta\equiv\frac{(p-p_\alpha)^2}{(p+p_\alpha)^2}\right)\right]$$

Switching point: $\zeta = \chi \approx 0.34$

$$\eta = Z_1 Z_2 e^2 \mu / p_\alpha$$

Non-Pole:

$$\psi^C_{p_\alpha l}(p) = -\frac{4\pi\eta e^{-\pi\eta/2} p_\alpha (pp_\alpha)^2}{(p^2+p_\alpha^2)^{l+1+i\eta}}\left[\frac{\Gamma(l+1+i\eta)\Gamma(1/2)}{\Gamma(l+3/2)}\right]$$

$$\times [p^2-(p_\alpha+i0)^2]^{-1+i\eta}\,{}_2F_1\left(\frac{l+2+i\eta}{2},\frac{l+1+i\eta}{2};l+\frac{3}{2};\chi\equiv\frac{4p^2p_\alpha^2}{(p^2+p_\alpha^2)^2}\right)$$

Type equation here.