INPP INSTITUTE OF NUCLEAR & PARTICLE PHYSICS @OHIO UNIVERSITY

Separable Optical Potentials for (d,p) Reaction Calculations

Ch. Elster

The TORUS Collaboration

Department of physics + astronomy

OHIO UNIVERSITY

What Reactions are we interested in?

$^{3}He(d,p)^{4}He$

Light nuclei

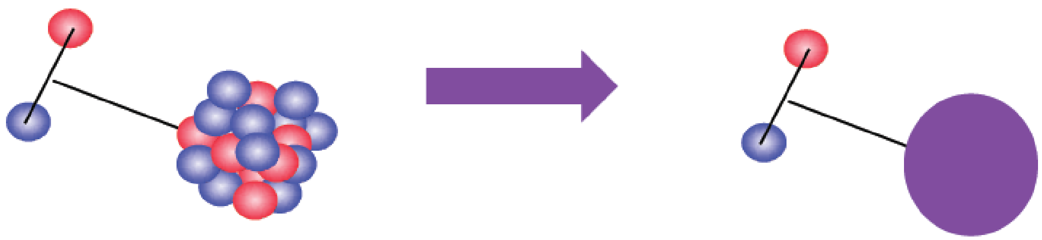
Heavy Nuclei

$^{140}Sn(d,p)^{141}Sn$

?

Reactions: Elastic Scattering, Breakup, Transfer

Reduce Many-Body to Few-Body Problem



Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

Hamiltonian for effective few-body poblem:

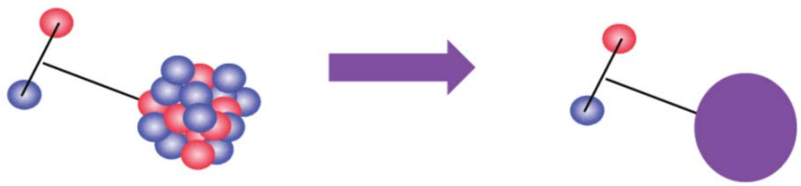
$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction

Optical potentials p+A and n+A

Three-Body Problem

(d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)

<u>Issue:</u> traditional Faddeev formulation does not contain target excitations

- **Especially important for reactions with exotic nuclei**
- **Forces between neutron (proton) and nucleus A**
- **Effective description with two-body optical potential**

TORUS Collaboration

www.reactiontheory.org

Department of physics + astronomy

OHIO UNIVERSITY

Theoretical Foundation :

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Generalized Faddeev formulation of (d,p) reactions with:
(a) explicit inclusion of target excitations
(b) explicit inclusion of the Coulomb interaction

Faddeev formulation → momentum space

Suggestions:

- **Target excitations:**
  Including specific excited states → Formulation with separable interactions
- **Explicit inclusion of Coulomb interaction**:
  Formulation of Faddeev equations in Coulomb basis instead of plane wave basis

Both need prep work!

Department of physics + astronomy

OHIO UNIVERSITY

Hamiltonian: $\mathbf{H = H_0 + V_{np} + V_{nA} + V_{pA}}$

V_{np} : **NN interaction** -- momentum space

V_{nA} : **Optical potential**

Phenomenological optical potentials fitted to data from ^{12}C to ^{208}Pb given in coordinate space and parameterized in terms of Woods-Saxon functions

$$U_{nucl}(r) = V(r) + i\left[W(r) + W_s(r)\right] + V_{ls}(r)\, \mathbf{l}\cdot\boldsymbol{\sigma}$$

$$\begin{aligned} V(r) &= -V_r f_{ws}(r, R_0, a_0) \\ W(r) &= -W_v f_{ws}(r, R_w, a_w) \\ W_s(r) &= -W_s(-4a_w) f'_{ws}(r, R_w, a_w) \\ V_{ls}(r) &= -(V_{so} + iW_{so})(-2) g_{ws}(r, R_{so}, a_{so}), \end{aligned}$$

$$\begin{aligned} f_{ws}(r, R, a) &= \frac{1}{1 + \exp(\frac{r-R}{a})} \\ f'_{ws}(r, R, a) &= \frac{d}{dr} f_{ws}(r, R, a) \\ g_{ws}(r, R, a) &= f'(r, R, a)/r \end{aligned}$$

Not useful in this form

Department of physics + astronomy

OHIO UNIVERSITY 1804

However:

Woods-Saxon functions have a semi-analytic Fourier transform: (fast converging series expansion)

Central term:

$$\overline{V}(\mathbf{q}) = \frac{V_r}{\pi^2}\left\{ \frac{\pi a_0 e^{-\pi a_0 q}}{q\left(1-e^{-2\pi a_0 q}\right)^2}\left[R_0\left(1-e^{-2\pi a_0 q}\right)\cos\left(qR_0\right) - \pi a_0\left(1+e^{-2\pi a_0 q}\right)\sin\left(qR_0\right)\right] \right.$$
$$\left. -a_0^3 e^{-\frac{R_0}{a_0}}\left[\frac{1}{(1+a_0^2q^2)^2} - \frac{2e^{-\frac{R_0}{a_0}}}{(4+a_0^2q^2)^2}\right]\right\}$$

Surface term:

$$\overline{W}_s(\mathbf{q}) = -4a_w\,\frac{W_s}{\pi^2}\left\{\frac{\pi a_w e^{-\pi a_w q}}{\left(1-e^{-2\pi a_w q}\right)^2}\right.$$
$$\left[\left(\pi a_w\left(1+e^{-2\pi a_w q}\right) - \frac{1}{q}\left(1-e^{-2\pi a_w q}\right)\right)\cos(qR_w) + R_w\left(1-e^{-2\pi a_w q}\right)\sin(qR_w)\right]$$
$$\left. + a^2 e^{-R_w/a_w}\left[\frac{1}{(1+a_w^2q^2)^2} - \frac{4e^{-R_w/a_w}}{(4+a_w^2q^2)^2}\right]\right\}.$$

physics + astronomy

Details: L. Hlophe et al.: arXiv:1310.8334

OHIO UNIVERSITY

Short Intro to Separable Potentials:

Consider Lippmann-Schwinger (LS) equation:

Hamiltonian: $H = h_0 + v$

$$t(E) = v + v\, g_0(E)\, t(E) \; = v + v g(E) v$$

free resolvent: $g_0(E) = 1/\,(\,E \; - h_0 + i\varepsilon)$

full resolvent: $g(E) = 1/\,(\,E \; - H + i\varepsilon)$

Spectrum of full resolvent: $$\mathbf{1} = \sum_B |\Psi_B\rangle\langle\Psi_B| + \int d^3k |\Psi_k^{(+)}\rangle\langle\Psi_k^{(+)}|$$

For $E \rightarrow E_B$ $$t(z) \simeq \frac{v|\Psi_B\rangle\langle\Psi_B|v}{z + E_B}.$$

t-matrix has pole with residue: $$\langle k|v|\Psi_B\rangle\langle\Psi_B|v|k'\rangle := h_B(k) h_B^*(k')$$

Define rank-1 Separable Potential: $V \equiv v|\Psi_B\rangle\lambda\langle\Psi_B|v$

General form of rank-1 separable t-matrix

$$t(z) = |h\rangle\tau(z)\langle h| = \frac{|h\rangle\langle h|}{\frac{1}{\lambda} - \langle h|g_0(z)|h\rangle}$$

With pole at: $$\frac{1}{\lambda} = \langle\Psi_B|vg_0(-E_B)v|\Psi_B\rangle = \langle\Psi_B|v\frac{1}{z+E_B}v|\Psi_B\rangle = \langle\Psi_B|v|\Psi_B\rangle$$

General form of rank-1 separable t-matrix (constructed with bound state wave function

$$t(z) = \frac{v|\Psi_B\rangle\langle\Psi_B|v}{\langle\Psi_B|(v - vg_0(z)v)|\Psi_B\rangle}$$

Unitary Pole Approximation (UPA)

Used many times …

physics + astronomy

OHIO UNIVERSITY

Ernst-Shakin-Thaler (EST)

Phys. Rev. C8, 46 (1973)

Extend to scattering states and define

$$\mathcal{V} = \frac{V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(+)}_{k_E}|V}{\langle\Psi^{(+)}_{k_E}|V|\Psi^{(+)}_{k_E}\rangle}$$

(here V Hermitian !)

Then partial wave t-matrix :

$$\langle p'|t(E)|p\rangle = \frac{\langle p'|V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(+)}_{k_E}|V|p\rangle}{\langle\Psi^{(+)}_{k_E}|V - V g_0(E) V|\Psi^{(+)}_{k_E}\rangle}$$

Reminder:

$$V|\Psi^{(+)}_{k_E}\rangle := t|k_E\rangle$$

The EST construction guarantees:

At a given scattering energy E_{kE} the scattering wave functions obtained with the original potential V and the separable potential $\mathcal{V}$ are identical . ➔ the half-shell t-matrices are identical

Optical Potentials == Complex Potentials
Generalizaton of EST necessary

L. Hlophe et al.: arXiv:1310.8334

Definition with In-state necessary to fulfill reciprocity theorem

$$U = \frac{V|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(-)}_{k_E}|V}{\langle\Psi^{(-)}_{k_E}|V|\Psi^{(+)}_{k_E}\rangle}$$

For time reversal operator $\mathcal{K}$ potential must fulfill:

$$\mathcal{K}\, U\, \mathcal{K}^{-1} = U^{\dagger}$$

Department of physics + astronomy

OHIO UNIVERSITY

Technical details:

Let $|f l_{,kE}>$ be a radial wave function and $K_0|f l_{,kE}> = |f^*_{l,kE}>$

Rank-1 separable t-matrix: $\langle p'|t(E)|p\rangle = \frac{\langle p'|u|f_{l,k_E}\rangle\langle f^*_{l,k_E}|u|p\rangle}{\langle f^*_{l,k_E}|u - ug_0(E)u|f_{l,k_E}\rangle}$

With $t(p', k_E, E_{k_E}) = \langle f^*_{l,k_E}|u|p'\rangle$ and $t(p, k_E, E_{k_E}) = \langle p|u|f_{l,k_E}\rangle$

$$\langle p'|t(E)|p\rangle = \frac{t(p', k_E, E_{k_E})\; t(p, k_E, E_{k_E})}{\langle f^*_{l,k_E}|u(1 - g_0(E)u)|f_{l,k_E}\rangle} \equiv t(p', k_E, E)\; \tau(E)\; t(p, k_E, E)$$

and

$$\begin{aligned}\tau(E)^{-1} &= t(k_E, k_E, E_{k_E}) \\ &+2\mu\left[\mathcal{P}\int dpp^2 \frac{t(p, k_E, E_{k_E})t(p, k_E, E_{k_E})}{k_E^2 - p^2} - \mathcal{P}\int dpp^2 \frac{t(p, k_E, E_{k_E})t(p, k_E, E_{k_E})}{k_0^2 - p^2}\right] \\ &+i\pi\mu\Big[k_0 t(k_0, k_E, E_{k_E})t(k_0, k_E, E_{k_E}) - k_E\; t(k_E, k_E, E_{k_E})t(k_E, k_E, E_{k_E})\Big]\,.\end{aligned}$$

Generalization to arbitrary rank

$$\mathrm{U} = \sum_{i,j} u|f_{l,k_{E_i}}\rangle\langle f_{l,k_{E_i}}|M|f^*{}_{l}, k_{E_j}\rangle\langle f^*_{l,k_{E_j}}|u$$

with $$\delta_{ik} = \sum_j \langle f_{l,k_{E_i}}|M|f^*_{l,k_{E_j}}\rangle\langle f^*_{l,k_{E_j}}|u|f_{l,k_{E_k}}\rangle = \sum_j \langle f^*_{l,k_{E_i}}|u|f_{l,k_{E_j}}\rangle\langle f_{l,k_{E_j}}|M|f^*_{l,k_{E_k}}\rangle$$

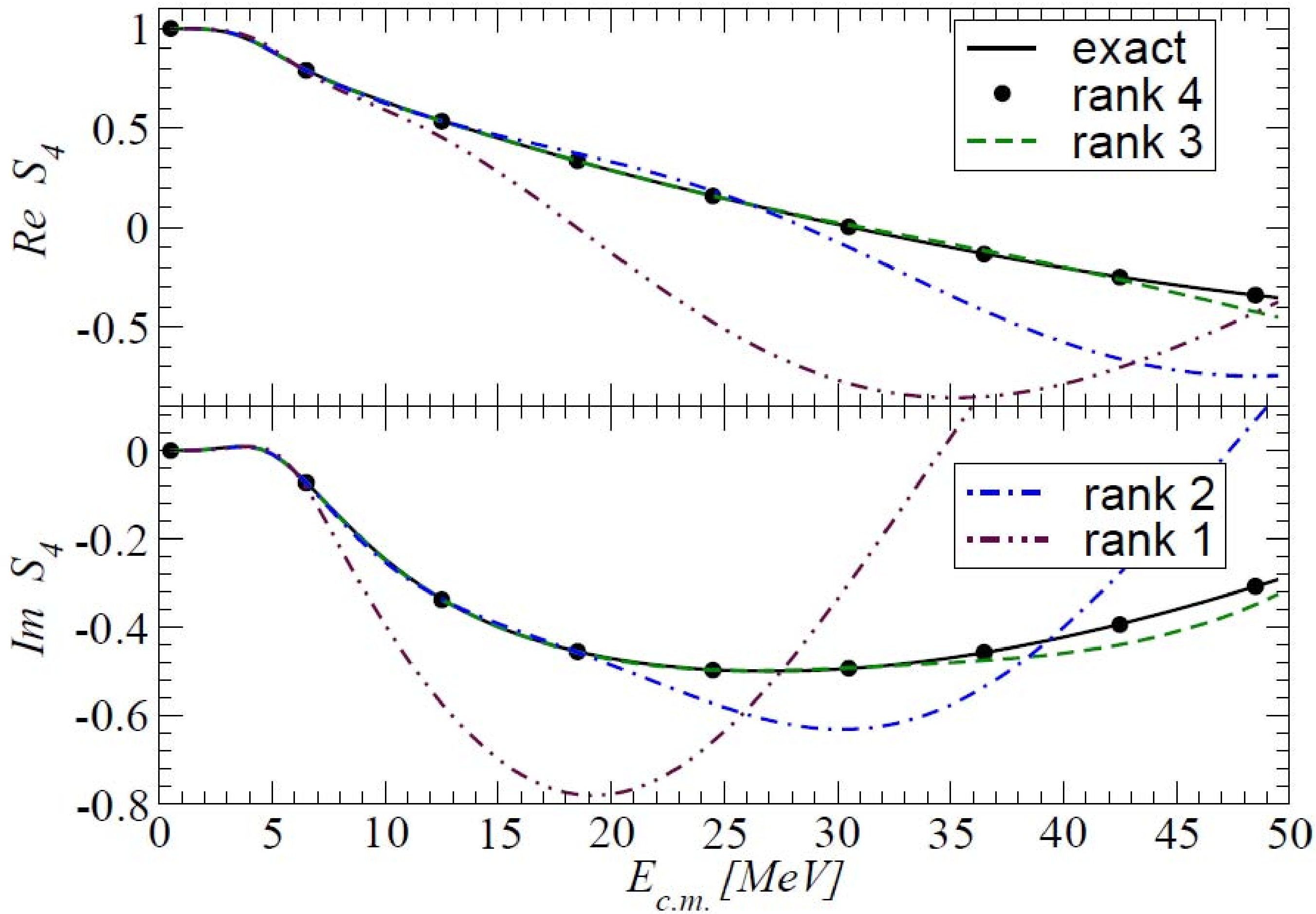
t-matrix $$t(E) = \sum_{i,j} u|f_{l,k_{E_i}}\rangle\tau_{ij}(E)\langle f^*_{l,k_{E_j}}|u$$

$$\sum_j \tau_{ij}(E)\ \langle f^*_{l,k_{E_j}}|u - ug_0(E)u|f_{l,k_{E_k}}\rangle = \delta_{ik}$$

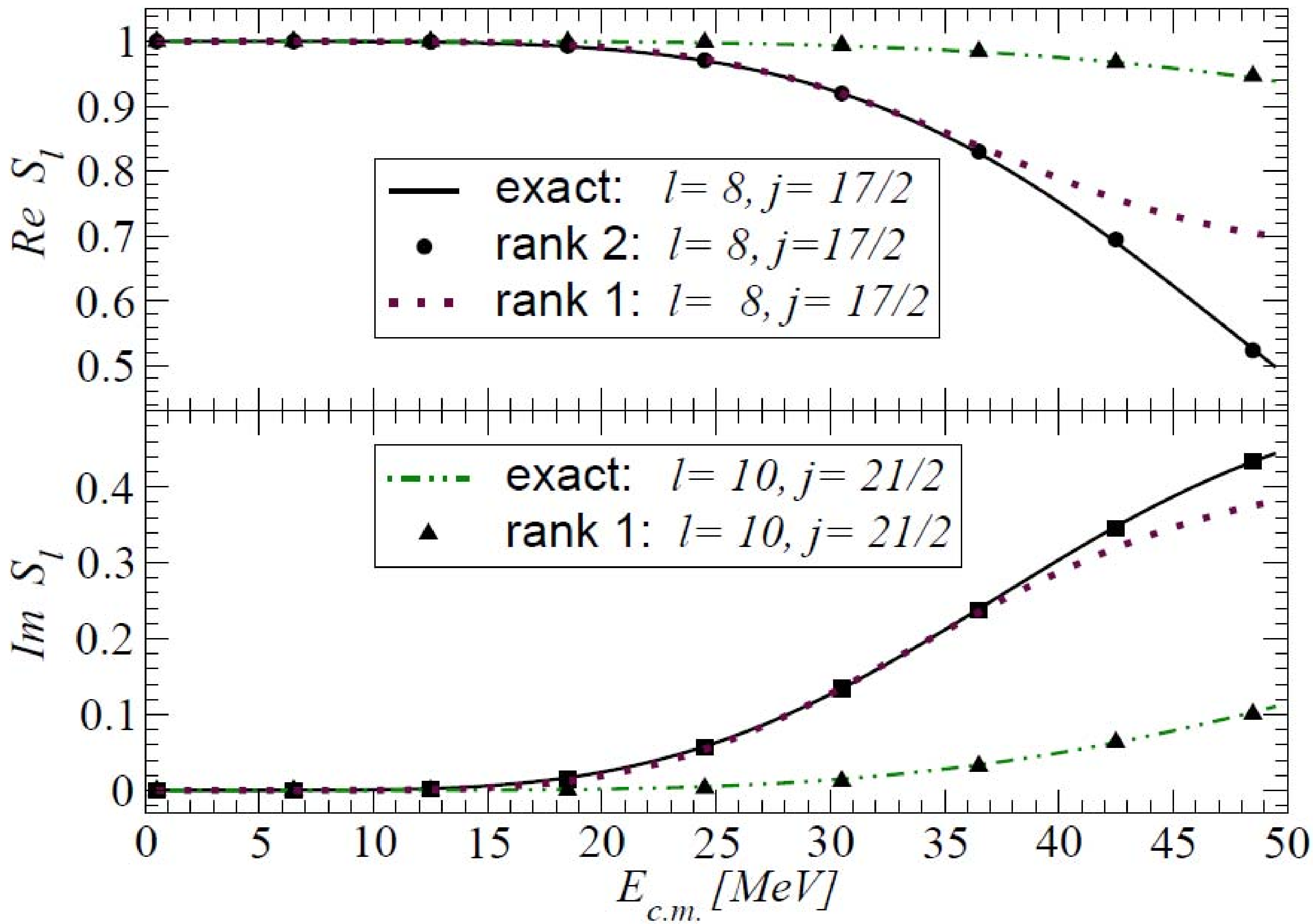
Compute and solve system of linear equations

n + ^{48}Ca : l=4, j=9/2

s-matrix elements



n + ^{48}Ca : higher partial waves

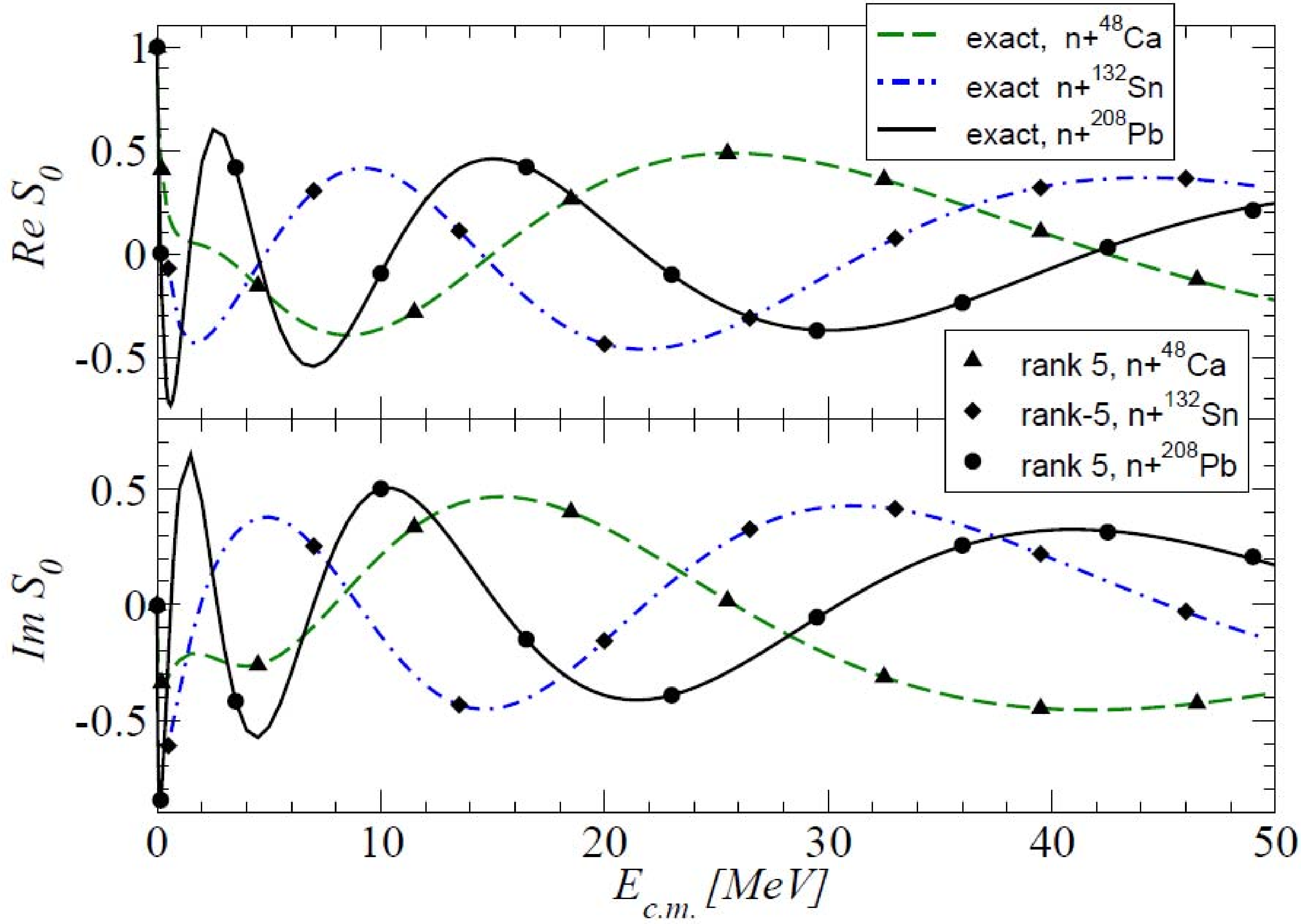


Very smooth s-matrix elements→ low rank separable sufficient

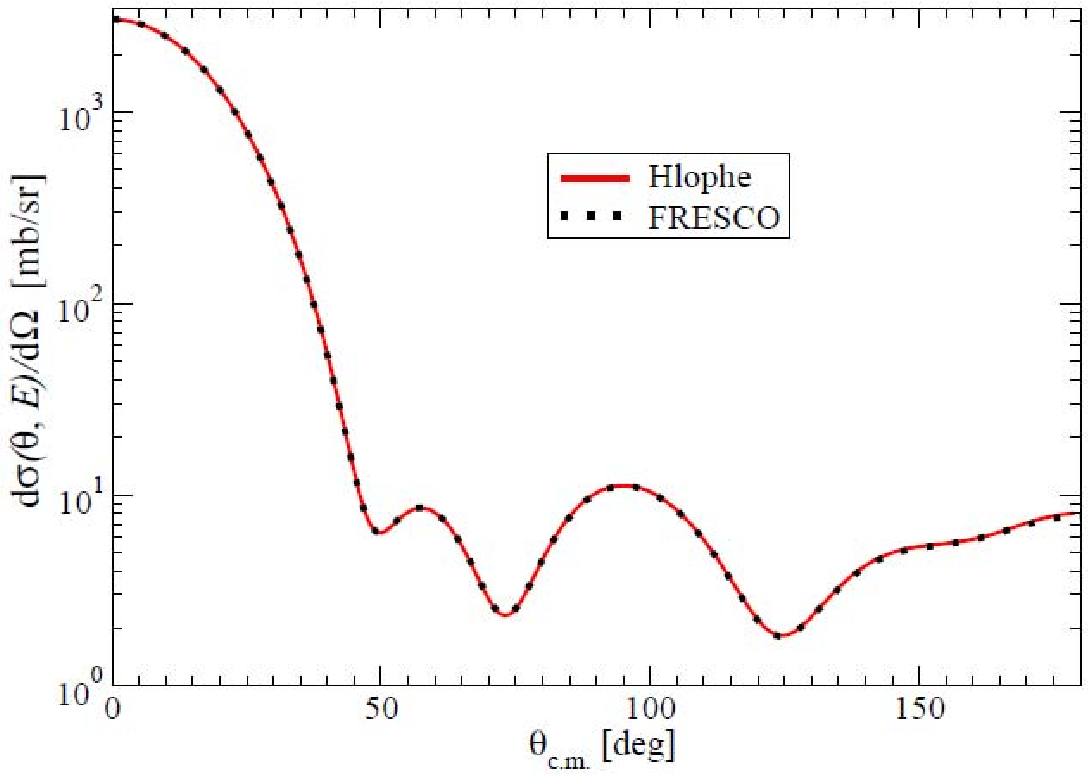
Guideline for Rank of Separable Representation

system	partial wave(s)	rank	EST support point(s) [MeV]
n+^{48}Ca	$l \geq 10$	1	40
	$l \geq 8$	2	29, 47
	$l \geq 6$	3	16, 36, 47
	$l \geq 0$	4	6, 15, 36, 47
n+^{132}Sn and n+^{208}Pb	$l \geq 16$	1	40
	$l \geq 13$	2	35, 48
	$l \geq 11$	3	24, 39, 48
	$l \geq 6$	4	11, 21, 36, 45
	$\mathbf{l \geq 0}$	**5**	**5, 11, 21, 36, 47**

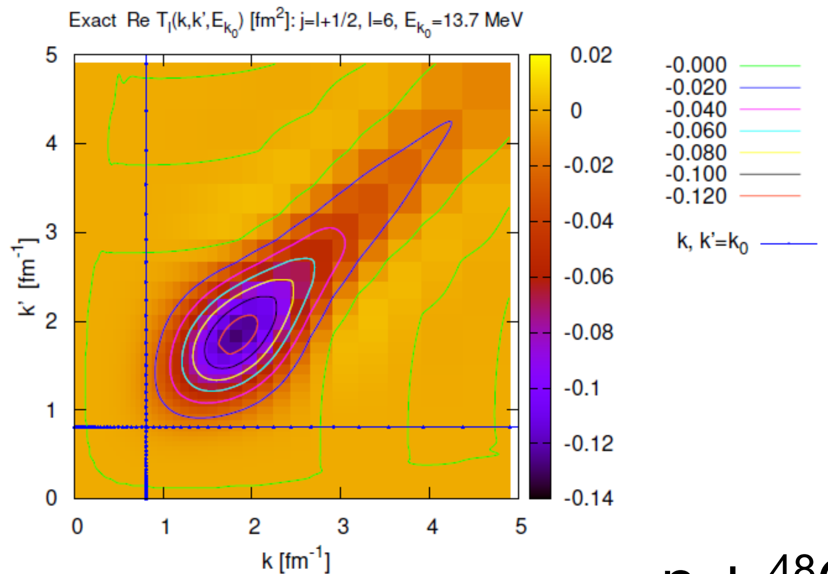
l=0 s-matrix elements:

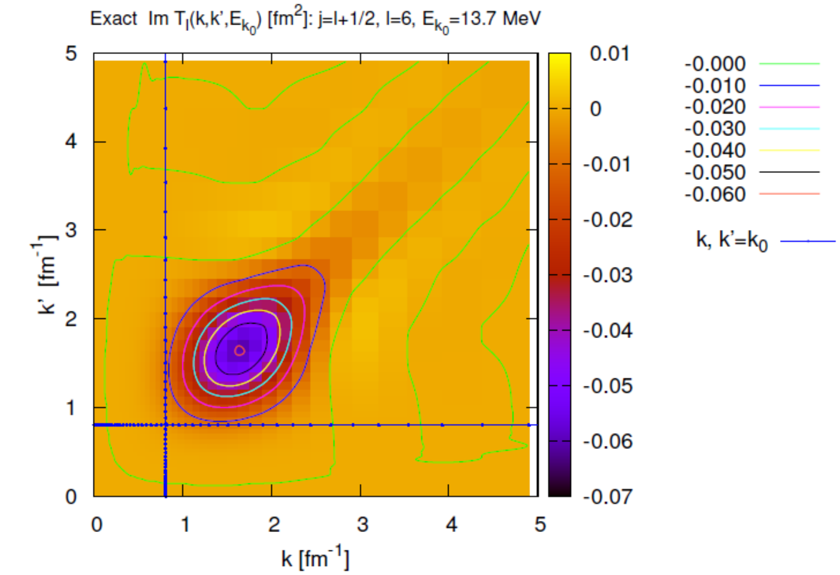


Comparison with r-space calculation: n+^{48}Ca @ 12 MeV

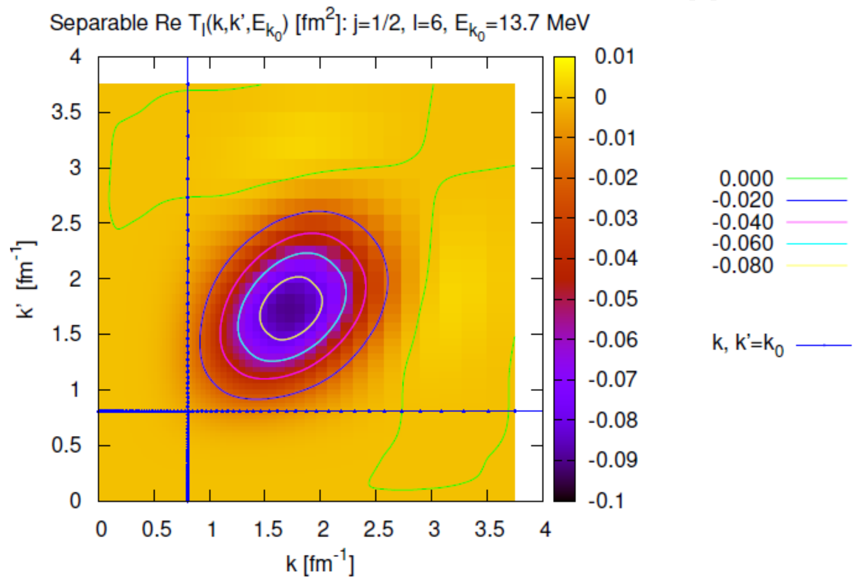


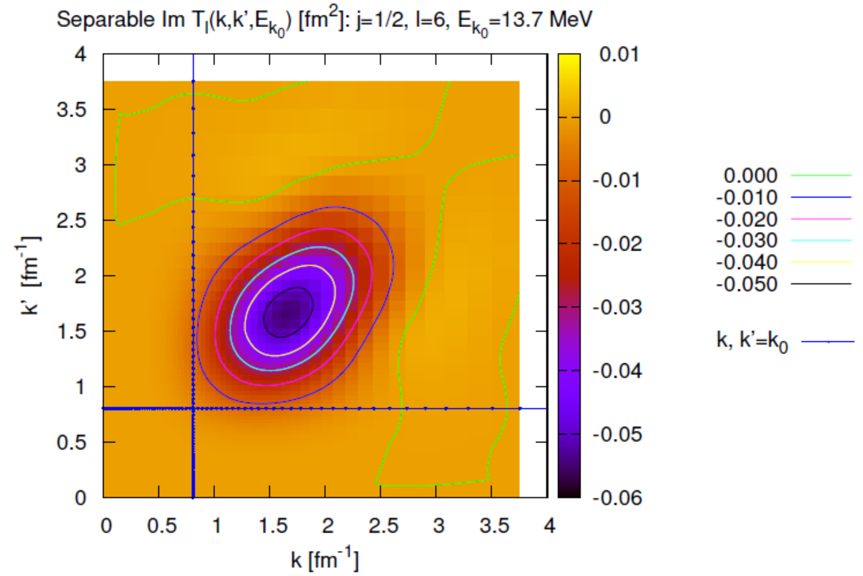
Off-Shell t-matrix elements: $t_l\,(k',k;\,E_{k0})$





n + ^{48}Ca

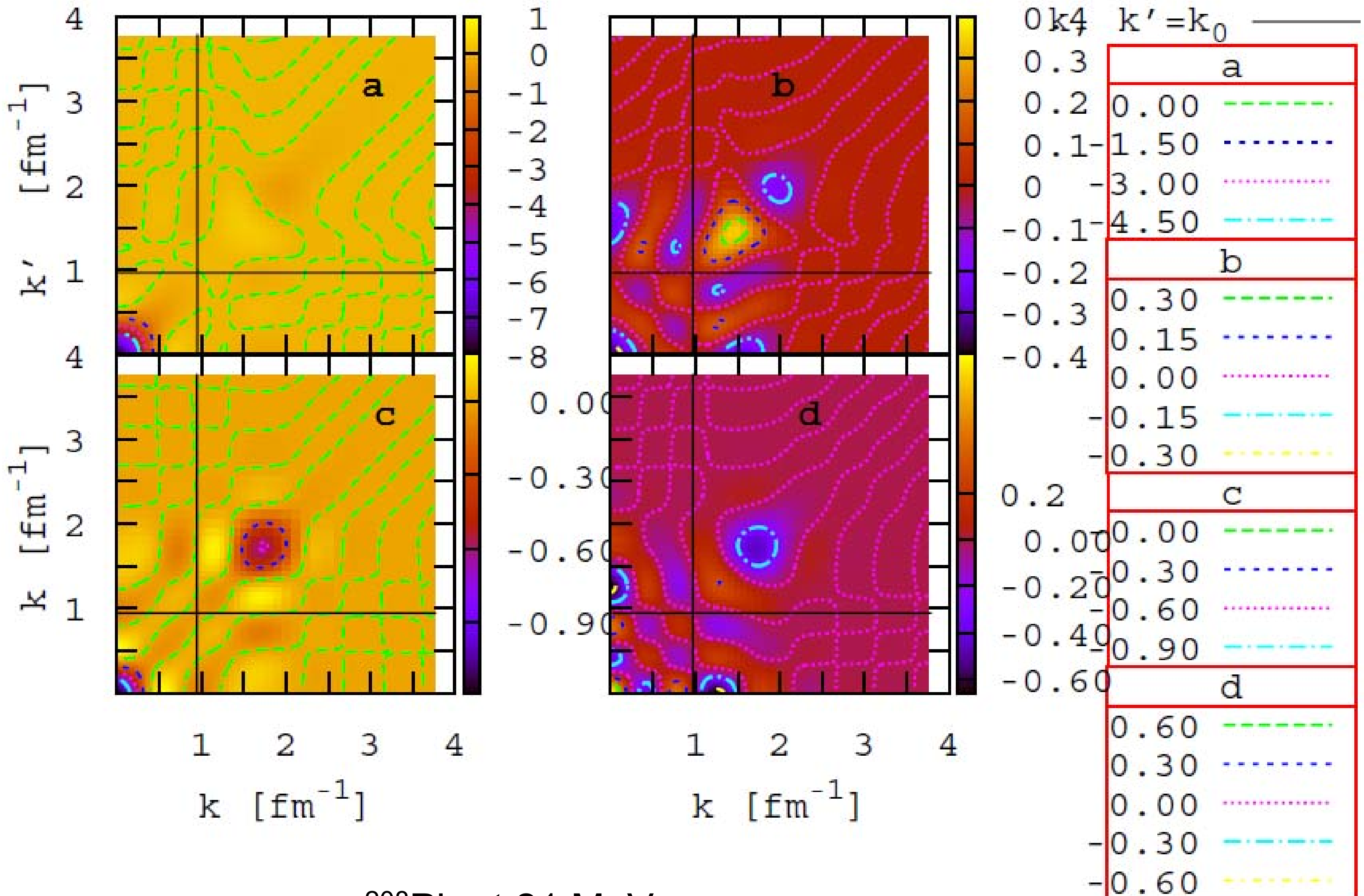




Department of physics + astronomy

OHIO UNIVERSITY

Off-shell t-matrix elements $t_l\,(k',k;E_{k0})$ $l=0$



n+^{208}Pb at 21 MeV

physics + astronomy

OHIO UNIVERSITY

Separable Representation of n+Nucleus Optical Potentials

Method of Ernst-Shakin-Thaler

- In momentum space

Generalized to non-Hermitian potentials

- Universal Rank-5 representation: ^{12}C to ^{208}Pb
- Allows to use all phenomenological Woods-Saxon based optical potentials in momentum-space
- Excellent representation of s-matrices and cross sections in the energy regime 10-50 MeV
- EST projects out high-momentum off-shell components.

Separable Potentials for p+Nucleus Scattering

First step: separate potential $W = V^c + V^s$

into long-range point Coulomb potential V^c

and short-range piece $V^s = V^N + (V^{cd} - V^c)$

Short-range Coulomb contribution

$$(V^{cd} - V^c)(r) = \alpha_e Z_p Z_t \left[\frac{1}{2R_0}\left(3 - \frac{r^2}{R_0^2}\right) - \frac{1}{r} \right]$$

Charged sphere with radius R_0

Fourier transform:

$$(V^{cd} - V^c)(\mathbf{q}', \mathbf{q}) =$$

$$-\frac{\alpha_e Z_p Z_t}{2\pi^2 k^2} \frac{1}{(kR_0)^3} \left[(kR_0)^3 + 3kR_0 \cos(kR_0) - 3\sin(kR_0) \right]$$

$k \to 0$ $\quad (V^{cd} - V^c)(\mathbf{q}', \mathbf{q}) \to -\frac{\alpha_e Z_p Z_t}{20\pi^2} R_0^2$

M. Rodriguez-Gallardo, et al. PRC 78, 034602 (2008)

Separation of Contributions:

$$T = \hat{T}^c + \hat{\Omega}^{c(-)} V^s \Omega^{(+)}$$

In partial wave form:

$$\langle k_0 | T_l(E) | k_0 \rangle = \langle k_0 | \hat{T}_l^c(E) | k_0 \rangle + \langle \hat{\Phi}_l^{c(-)}(k_0) | V^s | \Psi_l^{c(+)}(k_0) \rangle$$

with $\langle \hat{\Phi}_l^{c(-)}(k_0) | = e^{2i\sigma_l(E)} \langle \hat{\Phi}_l^{c(+)}(k_0) |$

$$\langle k_0 | T_l(E) | k_0 \rangle = \langle k_0 | \hat{T}_l^c(E) | k_0 \rangle + e^{2i\sigma_l(E)} \langle \hat{\Phi}_l^{c(+)}(k_0) | V^s | \Psi_l^{c(+)}(k_0) \rangle$$

Full scattering state in Coulomb basis

Multiply with $\langle \Phi_l^{c(+)}(k_0) | V^s$

$$| \Psi_l^{c(+)}(k_0) \rangle = | \hat{\Phi}_l^{c(+)}(k_0) \rangle + \hat{g}_c(E) V^s | \Psi_l^{c(+)}(k_0) \rangle$$

Channel resolvent: $\hat{g}_c(E) = (E - \hat{H}^c + i\varepsilon)^{-1}$

$$\hat{H}^c = H_0 + V^c.$$

Department of physics + astronomy

OHIO UNIVERSITY 1804

LS equation for matrix elements

Elster, Liu, Thaler

J. Phys. G **19**, 2123 (1993)

$$\langle \Phi_l^{c(+)}(k_0)|V^s|\Psi_l^{c(+)}(k_0)\rangle = \langle \Phi_l^{c(+)}(k_0)|V^s|\hat{\Phi}_l^{c(+)}(k_0)\rangle + \langle \Phi_l^{c(+)}(k_0)|V^s \hat{g}_c(E) V^s|\Psi_l^{c(+)}(k_0)\rangle$$

define
$$\langle \Phi_l^{c(+)}(k')|V^s|\hat{\Phi}_l^{c(+)}(k)\rangle \equiv \langle k'|u_l|k'\rangle$$
$$\langle \Phi_l^{c(+)}(k_0)|V^s|\Psi_l^{c(+)}(k_0)\rangle \equiv \langle k'|\tau_l|k'\rangle$$

LS equation:
$$\langle k|\tau_l|k_0\rangle = \langle k|u_l|k_0\rangle + \int \langle k|u_l|k'\rangle \frac{4\pi k'^2 dk'}{E - E' + i\varepsilon} \langle k'\tau_l|k_0\rangle$$

Following the EST scheme in the Coulomb basis (rank-1)

$$t(E) \equiv |h_k\rangle \tau(E) \langle h_k| \qquad \tau(E) = \left[\frac{1}{\lambda} - \langle h_k|\hat{g}_c^{(+)}(E)|h_k\rangle\right]^{-1}$$

$$\langle h_k|\hat{g}_c^{(+)}(E)|h_k\rangle = \int dk' k'^2 \frac{\langle h_k|\Phi^c(k')\rangle\langle\Phi^c(k')|h_k\rangle}{E - E' + i\varepsilon}$$

Similar suggestion by Cattapan, Pisent, Vanzani, NPA 241, 204 (1975)

For generalized EST scheme: $\mathrm{U} = \frac{u|\Psi^{(+)}(k_0)\rangle\langle\Psi^{(-)}(k_0)|u}{\langle\Psi^{(-)}(k_0)|u|\Psi^{(+)}(k_0)\rangle}$

Separable t-matrix in Coulomb basis

$$\langle\Phi^c(p')|t(E)|\Phi^c(p)\rangle = \frac{\langle\Phi^c(p')|u|\Psi^{(+)}_{k_E}\rangle\langle\Psi^{(-)}_{k_E}|u\Phi^c(p)\rangle}{\langle\Psi^{(-)}_{k_E}(p)|u - ug_c(E)u|\Psi^{(+)}_{k_E}\rangle}$$

$\langle\Phi^c(p)|u|\Psi^{(+)}_{k_E}\rangle$ are Coulomb distorted form factors

and e.g. $\langle\Psi^{(-)}_{k_E}|ug_c(E)u|\Psi^{(+)}_{k_E}\rangle = \int dpp^2 \frac{\langle\Psi^{(-)}_{k_E}u|\Phi^c(p)\rangle\langle\Phi^c(p)|u\Psi^{(+)}_{k_E}\rangle}{E - \frac{p^2}{2\mu} + i\varepsilon}$

Matrix elements:

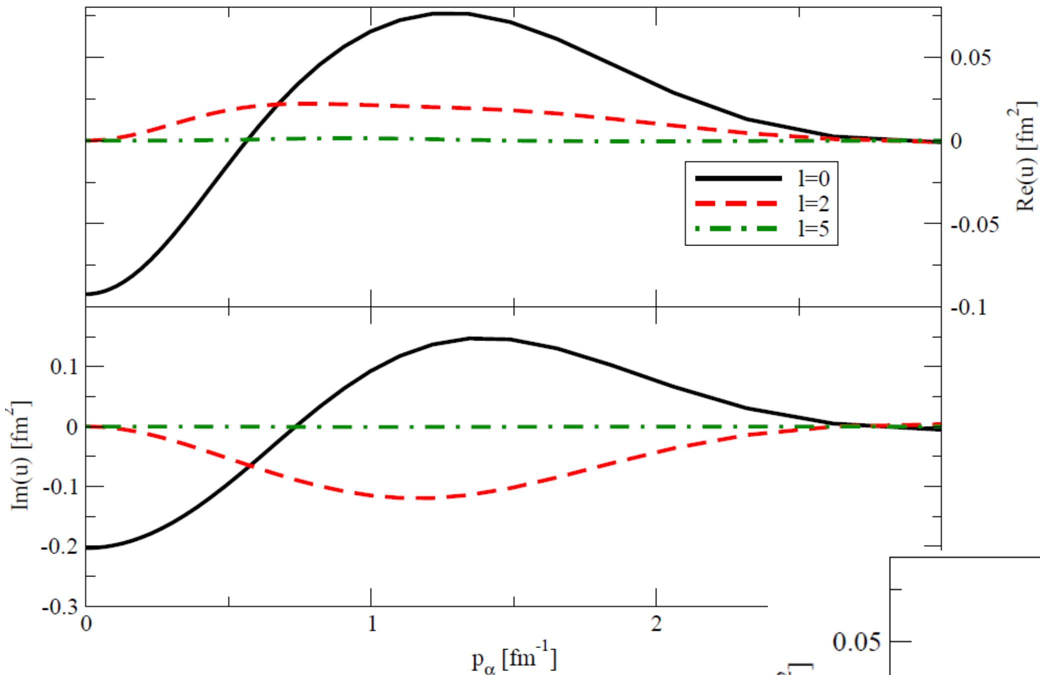
$$\langle\Psi^{(-)}_{k_E}u|\Phi^c(p)\rangle = \int dp''p''^2 t(p'', k_E; E_k)\Phi^c(p'') \equiv t^c(p, k_E; E_{k_E})$$

Then all calculations of the separable t-matrix should be the same

needs to be tested!

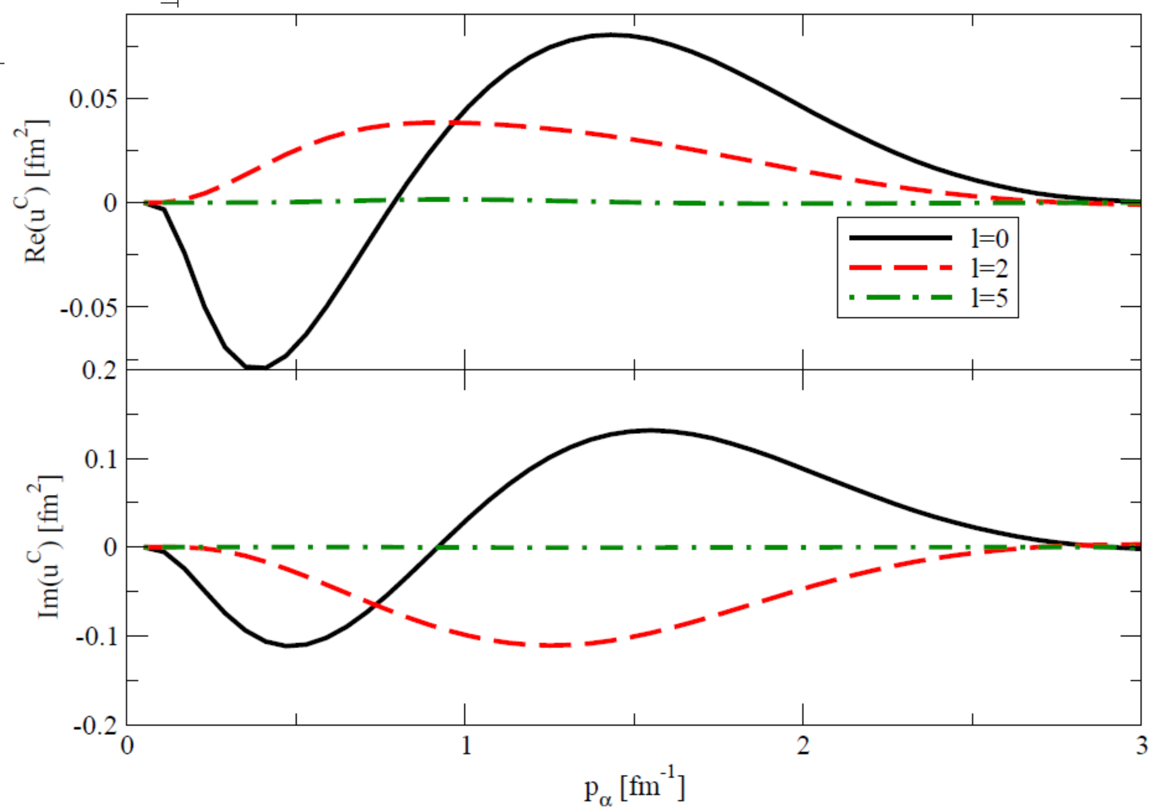
t (p,k_0,E_{k0}) for ^{12}C

j=l-1/2



t^c (p,k_0,E_{k0}) for ^{12}C

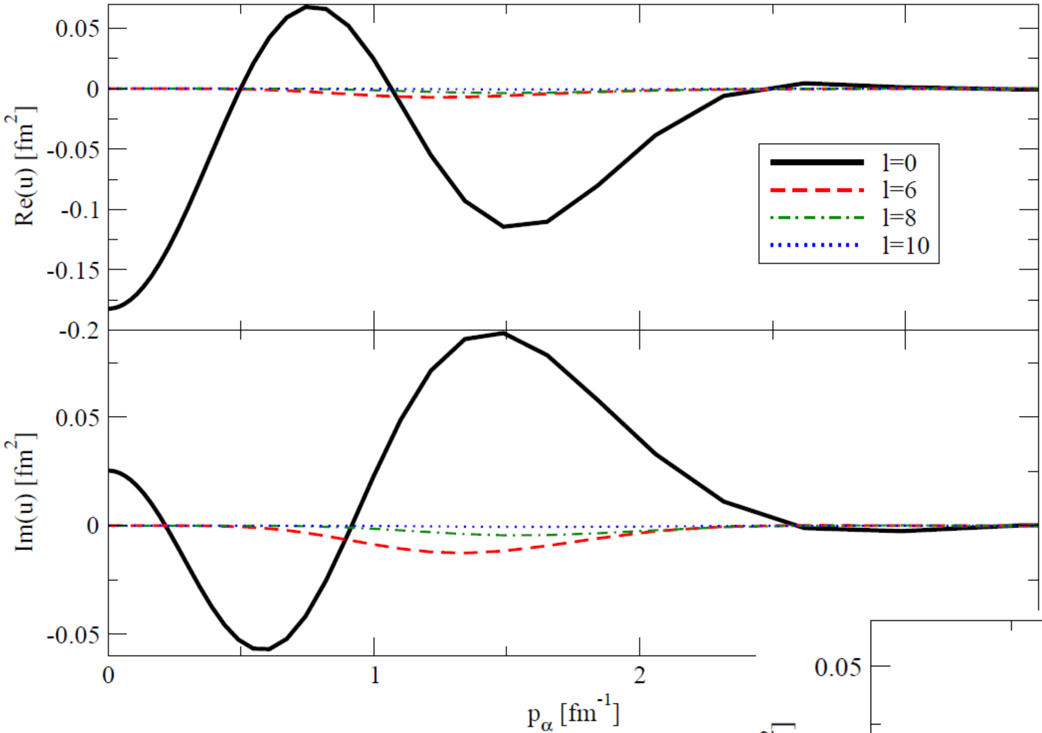
j=l-1/2

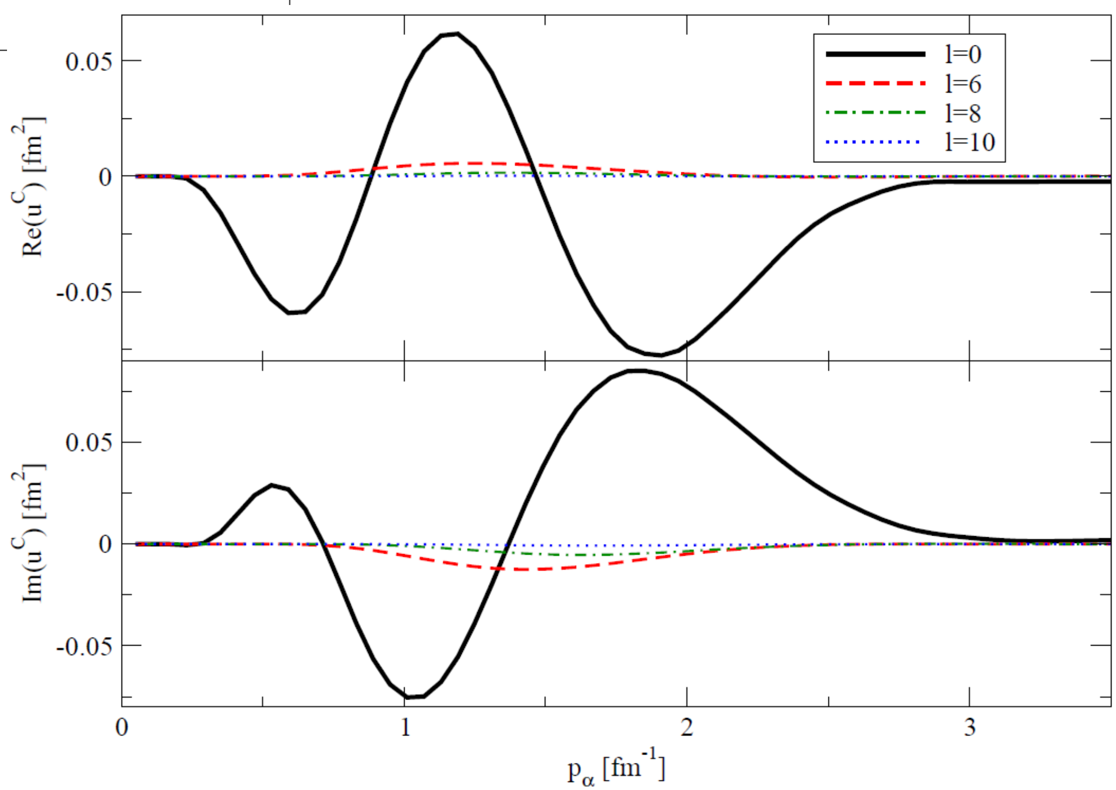


Department of physics + astronomy

$t\ (p, k_0, E_{k0})$ for ^{48}Ca

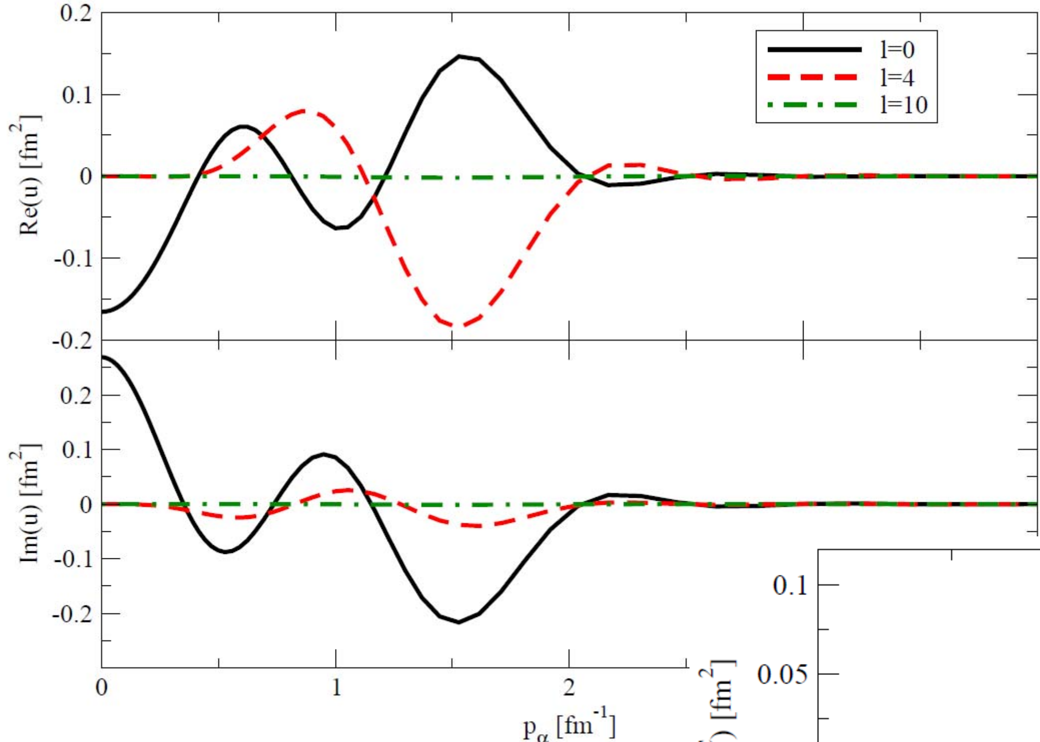
j=l-1/2





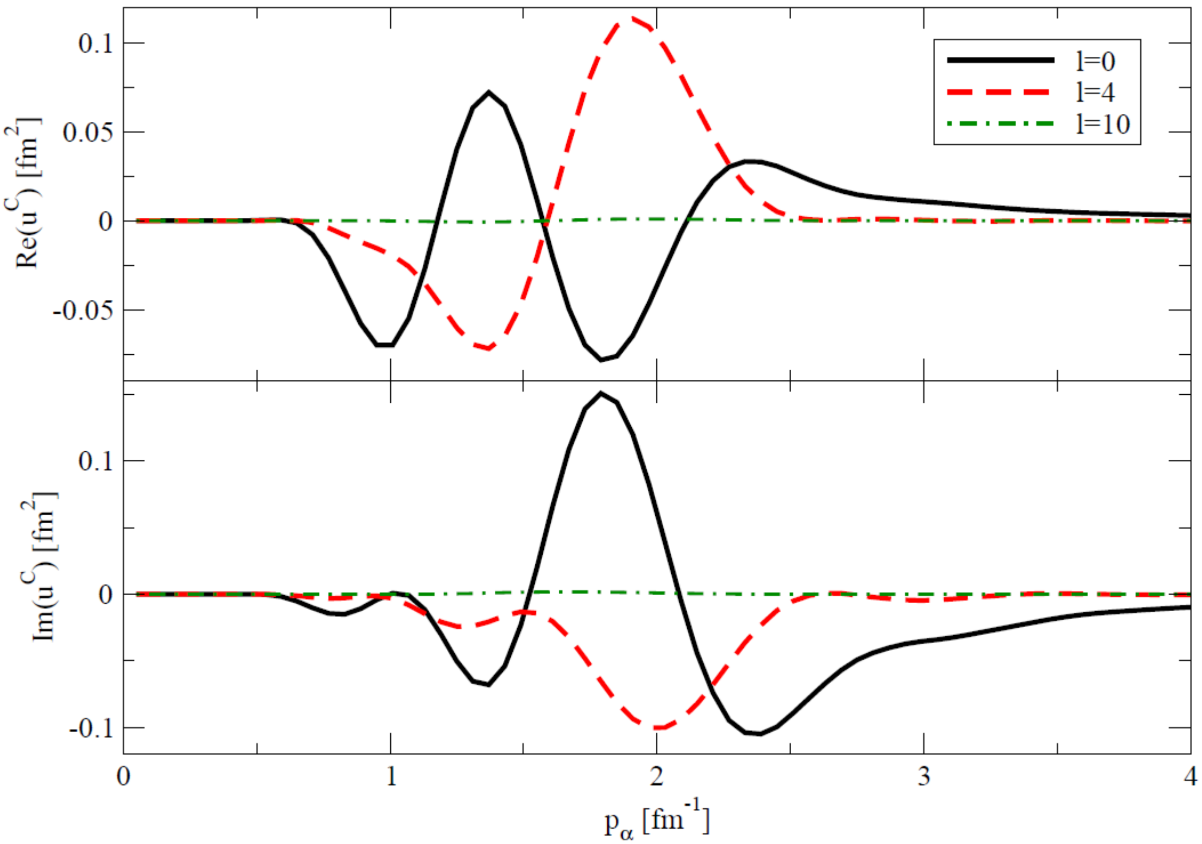
$t^c\ (p, k_0, E_{k0})$ for ^{48}Ca

j=l-1/2



$t\ (p,k_0,E_{k0})$ for ^{208}Pb

$j=l+1/2$



$t^c\ (p,k_0,E_{k0})$ for ^{208}Pb

$j=l+1/2$

Calculation of Coulomb distorted form factors (half-shell t-matrices):

Lengthy and another talk !

Soon to come:

Check if the p+nucleus calculations work

Not necessary for (d,p) reactions,
But nice if they do

Roadmap:

(d,p) Reactions as 3-Body Problem applicable for heavy (and light) nuclei

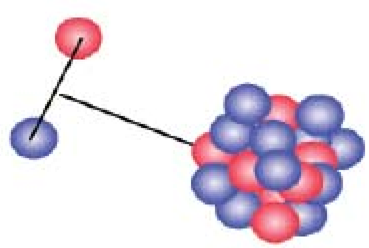
- **Formulation of Faddeev equations in Coulomb basis (no screening):**
  A.M. Mukhamedzanov, V. Eremenko, A.I. Sattarov (PRC 86 (2012) 034001)

- **Construction of separable optical potentials ($n+^{12}C, ^{48}Ca, ^{132}Sn, ^{208}Pb$) :**
  L. Hlophe (Ohio U) and TORUS collaboration (manuscript ready)

- **Formulation of practical implementation of Coulomb distorted nuclear matrix elements with Yamaguchi test potential :**
  N. Uphadyay (MSU / LSU) and TORUS collaboration

- **Numerical implementation with realistic separable nuclear potential :**
  V. Eremenko (OU) and TORUS collaboration

Then…



Department of physics + astronomy

TORUS: Theory of Reactions for Unstable iSotopes

A Topical Collaboration for Nuclear Theory

http://www.reactiontheory.org/

- Ch. Elster, V. Eremenko[†‡], and L. Hlophe[†]: Institute of Nuclear and Particle Physics, and Department of Physics and Astronomy, Ohio University, Athens, OH 45701.
- F. M. Nunes and N. J. Upadhyay[†]: National Superconducting Cyclotron Laboratory and Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824.
- G. Arbanas: Nuclear Science and Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831.
- J. E. Escher and I. J. Thompson: Lawrence Livermore National Laboratory, L-414, Livermore, CA 94551.

[†] Post-Docs or Grad Students.

[‡] M. V. Lomonosov Moscow State University, Moscow, 119991, Russia.