INPP @OHIO UNIVERSITY INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Towards (d,p) Reactions with Heavy Nuclei in a Faddeev Description

Ch. Elster

TORUS Collaboration

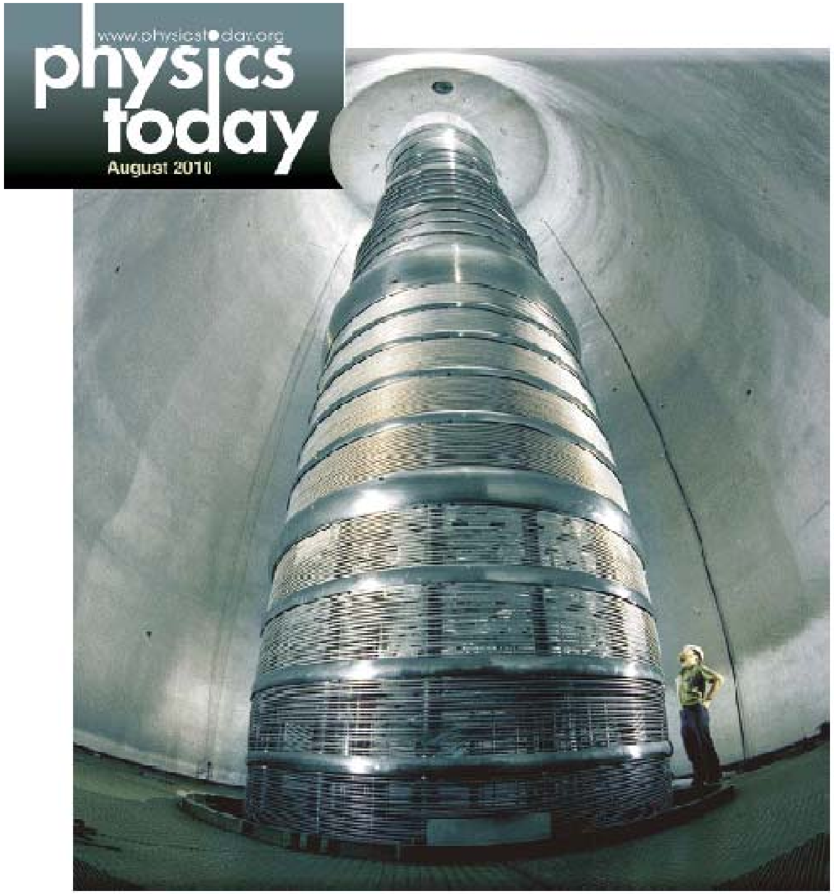
International Workshop on Nuclear Dynamics with Effective Field Theories
Ruhr-Universität Bochum, July 1 - 3, 2013

Dedicated to the memory of late Prof. Walter Glöckle

Department of physics + astronomy

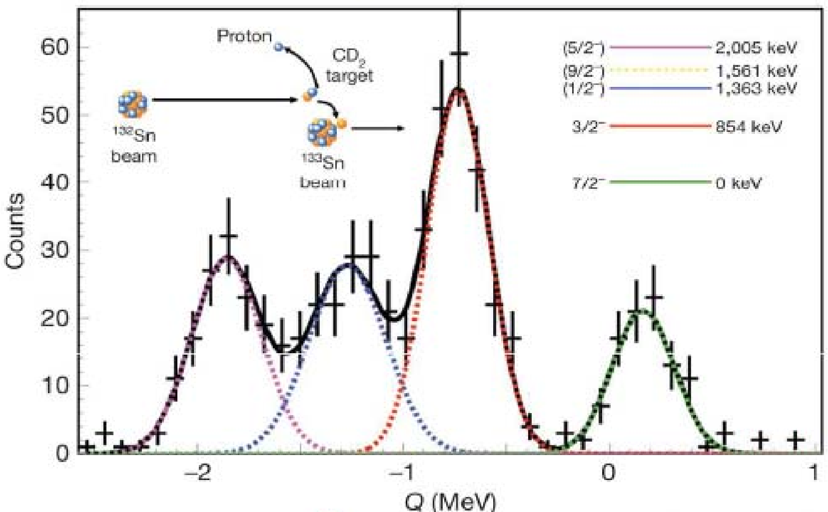
OHIO UNIVERSITY

(d,p) Reactions: Tool to study structure

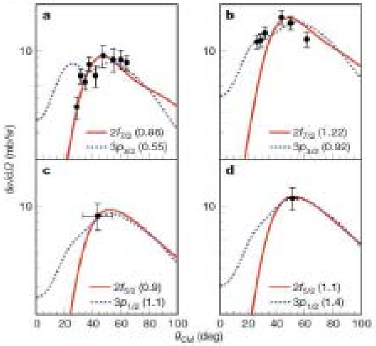


Doubly magic shell game

d(^{132}Sn,^{133}Sn)p@5 MeV/u



connection to r-process ^{132}Sn(n,γ)^{133}Sn



[K. Jones et al, Nature 465 (2010) 454]

What Reactions are we interested in?

$^{3}He(d,p)^{4}He$

Light nuclei

Heavy Nuclei

$^{140}Sn(d,p)^{141}Sn$

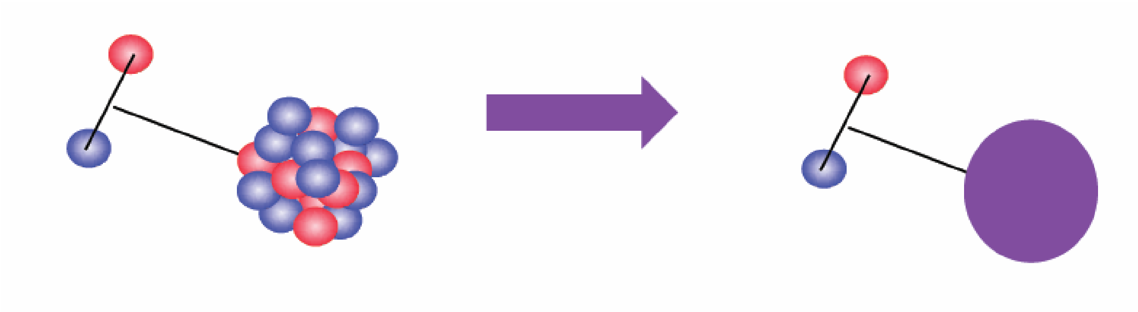
?

Reactions: Elastic Scattering, Breakup, Transfer

Department of physics + astronomy

OHIO UNIVERSITY

Reduce Many-Body to Few-Body Problem



Task:

- **Isolate important degrees of freedom in a reaction**
- **Keep track of important channels**
- **Connect back to the many-body problem**

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

np interaction

Optical potentials p+A and n+A

Three-Body Problem

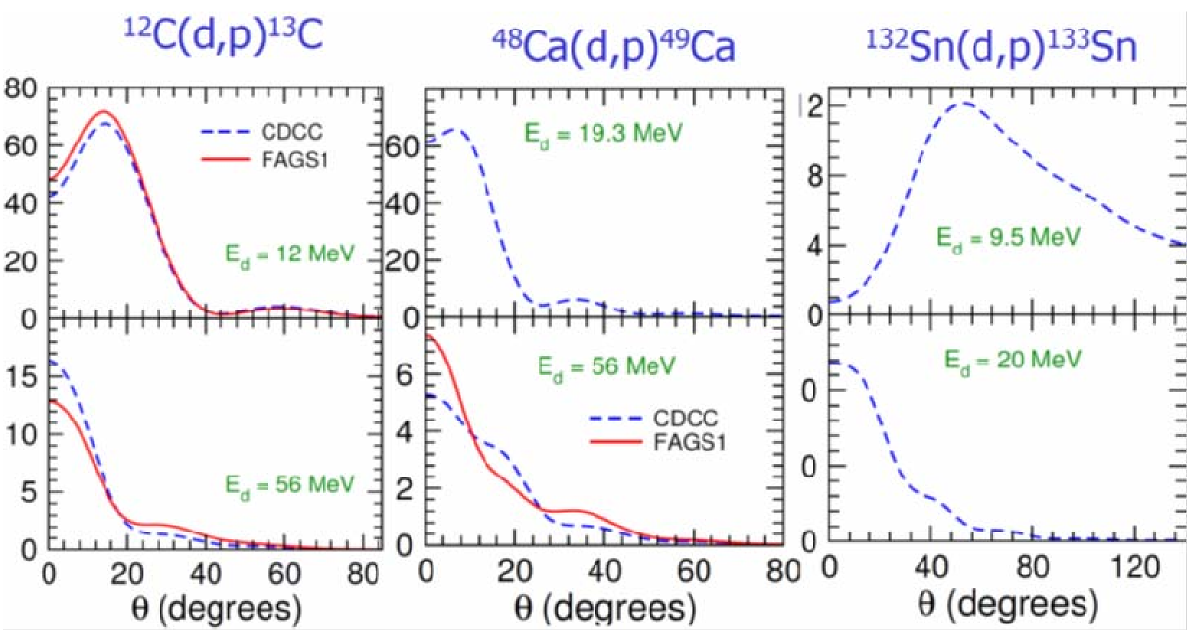
Department of physics + astronomy

OHIO UNIVERSITY 1804

(d,p) Reactions as three-body problem

Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issue: current momentum space implementation of Coulomb interaction (shielding) does **not** converge for Z ≥ 20

Courtesy: F.M. Nunes

(d,p) Reactions as three-body problem

Deltuva and Fonseca, Phys. Rev. C**79**, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)

Issue: traditional Faddeev formulation does **not** contain target excitations

Probably especially important for reactions with exotic nuclei

Proposed Plan to address issues:

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov, Phys.Rev. C86 (2012) 034001

TORUS Collaboration

www.reactiontheory.org

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov, Phys.Rev. C86 (2012) 034001

Generalized Faddeev formulation of (d,p) reactions with

(a) explicit inclusion of target excitations
(b) explicit inclusion of the Coulomb interaction

Faddeev formulation → momentum space

Suggestions:

- **Target excitations:**
  Including specific excited states → Formulation with separable interactions
- **Explicit inclusion of Coulomb interaction**:
  Formulation of Faddeev equations in Coulomb basis instead of plane wave basis

Both need prep work!

Hamiltonian: $H = H_0 + V_{np} + V_{nA} + V_{pA}$

V_{np} : NN interaction – separable representations exist ✔

V_{nA} : Optical potential

Phenomenological optical potentials fitted to data from ^{12}C to ^{208}Pb
given in coordinate-space
parameterized in terms of Woods-Saxon functions

$$U_{nucl}(r) = V(r) + i\left[W(r) + W_s(r)\right] + V_{ls}(r)\,\mathbf{l}\cdot\boldsymbol{\sigma}$$

$$\begin{aligned} V(r) &= -V_r f_{ws}(r, R_0, a_0) \\ W(r) &= -W_v f_{ws}(r, R_w, a_w) \\ W_s(r) &= -W_s(-4a_w) f'_{ws}(r, R_w, a_w) \\ V_{ls}(r) &= -(V_{so} + iW_{so})(-2) g_{ws}(r, R_{so}, a_{so}) \end{aligned}$$

$$\begin{aligned} f_{ws}(r, R, a) &= \frac{1}{1 + \exp(\frac{r-R}{a})} \\ f'_{ws}(r, R, a) &= \frac{d}{dr} f_{ws}(r, R, a) \\ g_{ws}(r, R, a) &= f'(r, R, a)/r \end{aligned}$$

Not useful in this form

Department of physics + astronomy

OHIO UNIVERSITY

Woods-Saxon functions have a semi-analytic Fourier transform:

(fast converging series expansion)

Central term:

$$\overline{V}(\mathbf{q}) = \frac{V_r}{\pi^2}\left\{ \frac{\pi a_0 e^{-\pi a_0 q}}{q\left(1-e^{-2\pi a_0 q}\right)^2}\left[R_0\left(1-e^{-2\pi a_0 q}\right)\cos\left(qR_0\right) - \pi a_0\left(1+e^{-2\pi a_0 q}\right)\sin\left(qR_0\right)\right] - a_0^3 e^{-\frac{R_0}{a_0}}\left[\frac{1}{(1+a_0^2q^2)^2} - \frac{2e^{-\frac{R_0}{a_0}}}{(4+a_0^2q^2)^2}\right]\right\}$$

Surface term:

$$\overline{W}_s(\mathbf{q}) = -4a_w\,\frac{W_s}{\pi^2}\left\{\frac{\pi a_w e^{-\pi a_w q}}{\left(1-e^{-2\pi a_w q}\right)^2}\left[\left(\pi a_w\left(1+e^{-2\pi a_w q}\right) - \frac{1}{q}\left(1-e^{-2\pi a_w q}\right)\right)\cos(qR_w) + R_w\left(1-e^{-2\pi a_w q}\right)\sin(qR_w)\right] + a^2 e^{-R_w/a_w}\left[\frac{1}{(1+a_w^2q^2)^2} - \frac{4e^{-R_w/a_w}}{(4+a_w^2q^2)^2}\right]\right\}.$$

Department of physics + astronomy

OHIO UNIVERSITY 1804

Separabilization: Method of Ernst-Shakin-Thaler

BUT: *Needs to be generalized for complex potentials*

Example: Rank-1 separable potential:

$$\mathbf{V} = \frac{v|\Psi^{(-)}_{k_E}\rangle\langle\Psi^{(+)}_{k_E}|v}{\langle\Psi^{(-)}_{k_E}|v|\Psi^{(+)}_{k_E}\rangle},$$

Definition with In-state necessary to fulfill reciprocity theorem

$$\mathcal{K}V\mathcal{K}^{-1} = V^{\dagger}$$

$\mathcal{K}$ is the time-reversal operator.

$$t(p',p,E) = \frac{t(p',k_E,E)\,t(p,k_E,E)}{\langle\Psi^{(-)}_{k_E}|v(1-g_0(E)v)|\Psi^{(+)}_{k_E}\rangle} \equiv t(p',k_E,E)\,\tau(E)\,t(p,k_E,E)$$

$$\begin{aligned}
\tau(E)^{-1} &= t(k_E,k_E,E_{k_E}) \\
&+2\mu\left[\mathcal{P}\int dp p^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_E^2-p^2} - \mathcal{P}\int dp p^2\frac{t(p,k_E,E_{k_E})t(p,k_E,E_{k_E})}{k_0^2-p^2}\right] \\
&+i\pi\mu\left[k_0 t(k_0,k_E,E_{k_E})t(k_0,k_E,E_{k_E}) - k_E t(k_E,k_E,E_{k_E})t(k_E,k_E,E_{k_E})\right]
\end{aligned}$$

Generalization to arbitrary rank

$$\mathbf{V} = \sum_{i,j} v|\Psi_i^{(-)}\rangle\langle\Psi_i^{(-)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|v$$

with $$\delta_{ik} = \sum_j \langle\Psi_i^{(-)}|M|\Psi_j^{(+)}\rangle\langle\Psi_j^{(+)}|v|\Psi_k^{(-)}\rangle = \sum_j \langle\Psi_i^{(+)}|v|\Psi_j^{(-)}\rangle\langle\Psi_j^{(-)}|M|\Psi_k^{(+)}\rangle$$

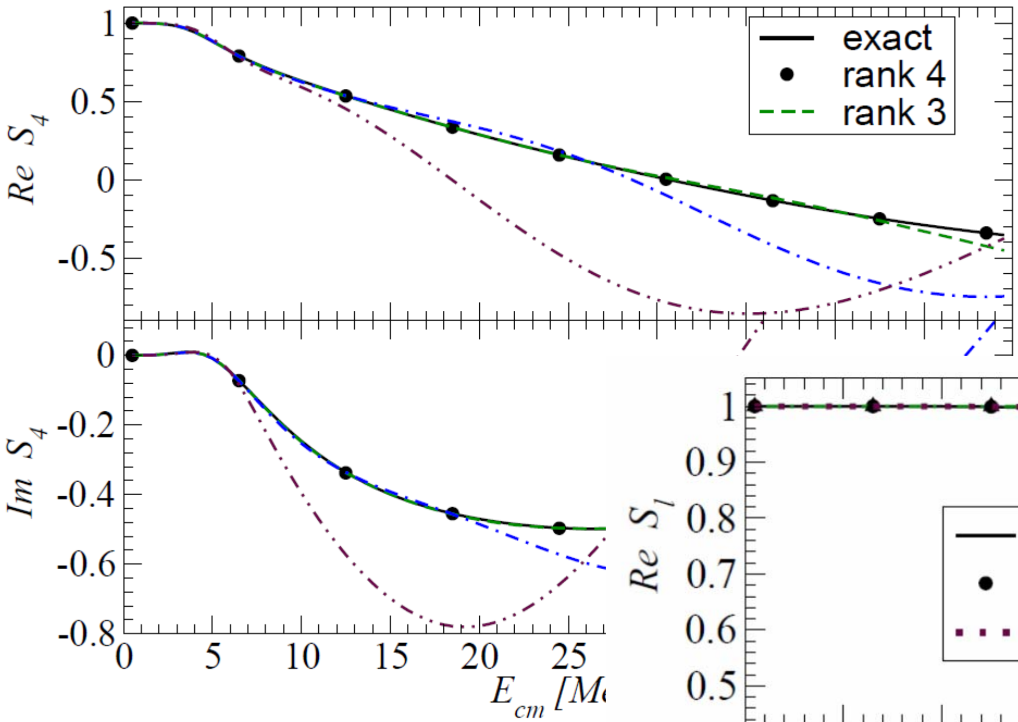
t-matrix

$$\hat{t}(E) = \sum_{i,j} v|\Psi_i^{(-)}\rangle\tau_{ij}(E)\langle\Psi_j^{(+)}|v$$

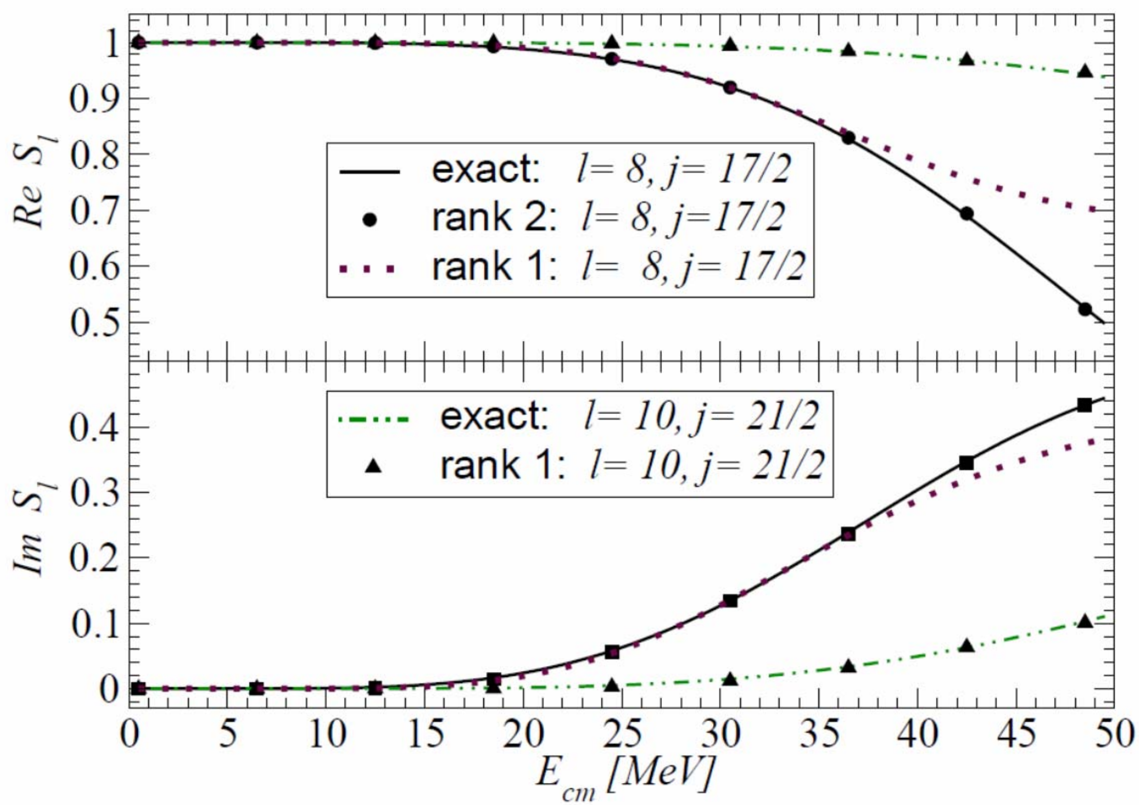
$$\sum_j \tau_{ij}(E)\underbrace{\langle\Psi_j^{(-)}|v - vg_0(E)v|\Psi_k^{(+)}\rangle}_{} = \delta_{ik}$$

Compute and solve system of linear equations

n + ^{48}Ca : l=4, j=9/2

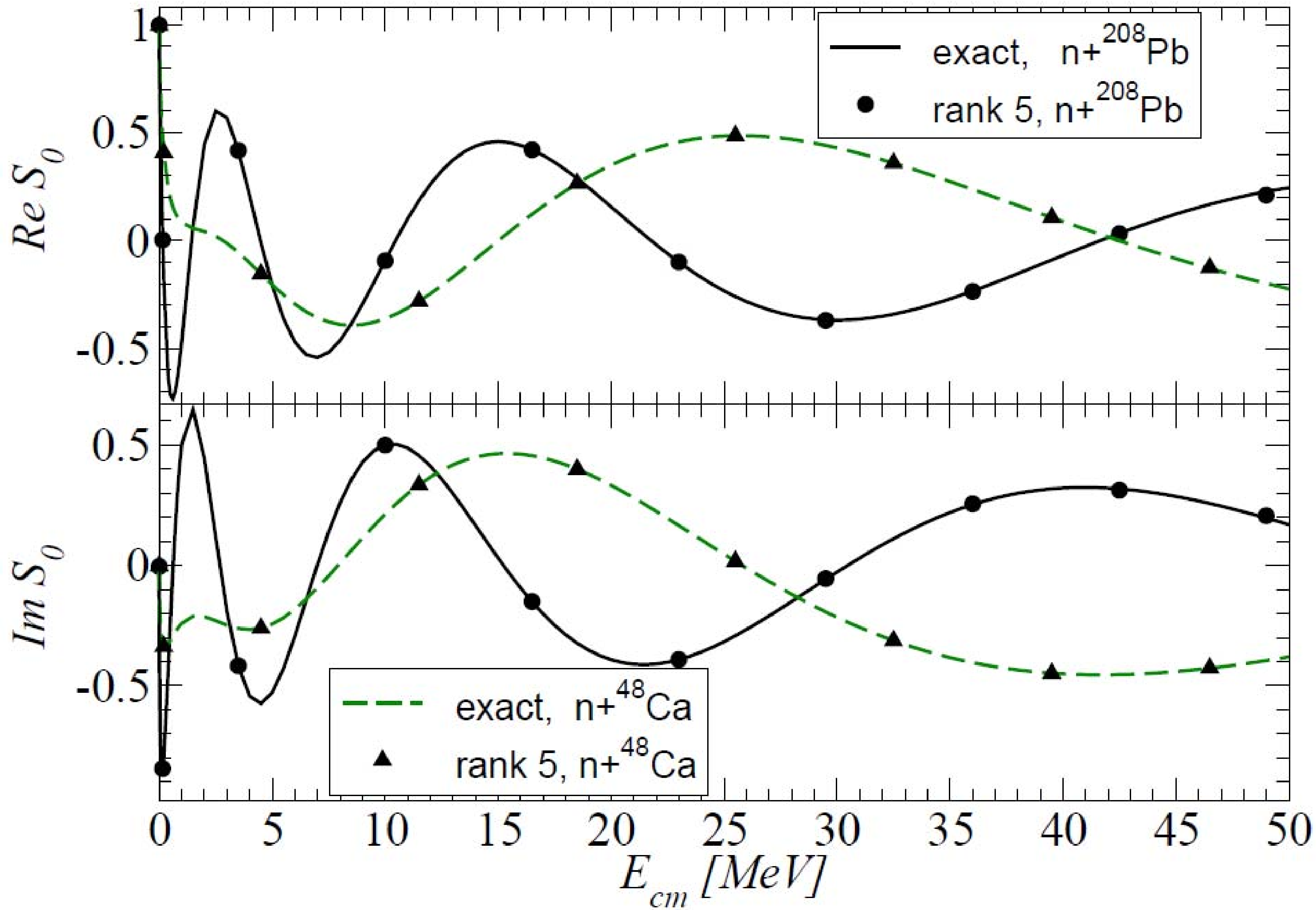


Re S4
Im S4
exact
rank 4
rank 3
E cm [M



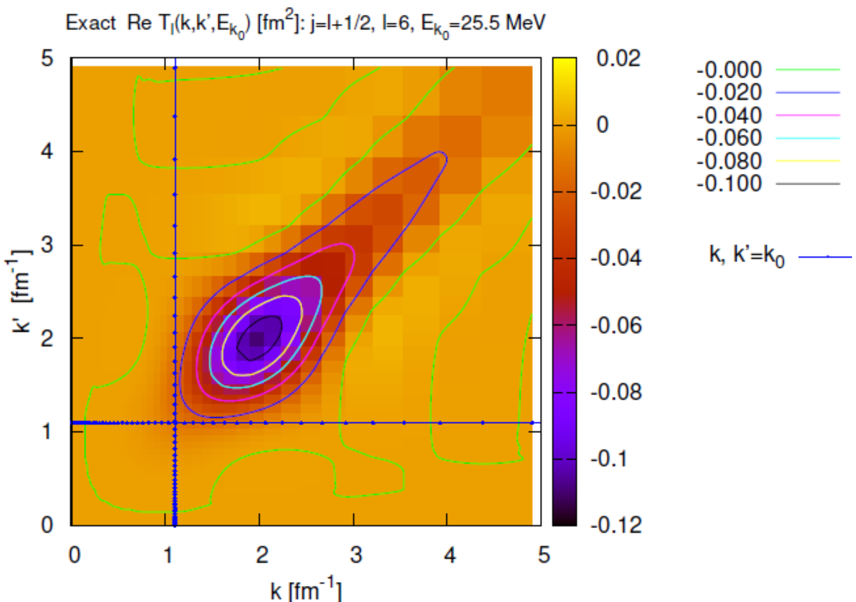
Re Sl
Im Sl
exact: l= 8, j= 17/2
rank 2: l= 8, j=17/2
rank 1: l= 8, j= 17/2
exact: l= 10, j= 21/2
rank 1: l= 10, j= 21/2
E cm [MeV]

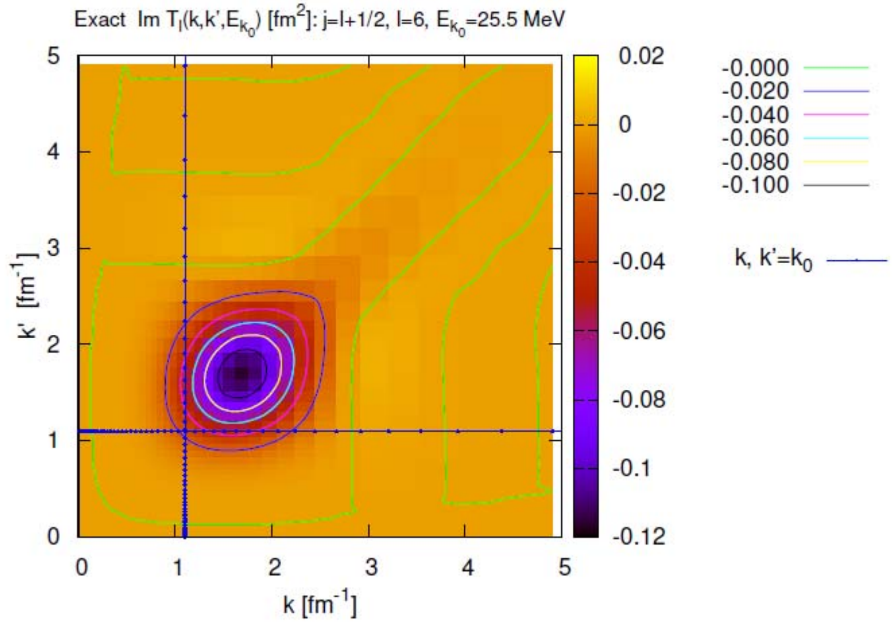
n + ^{48}Ca and n + ^{208}Pb : l=0



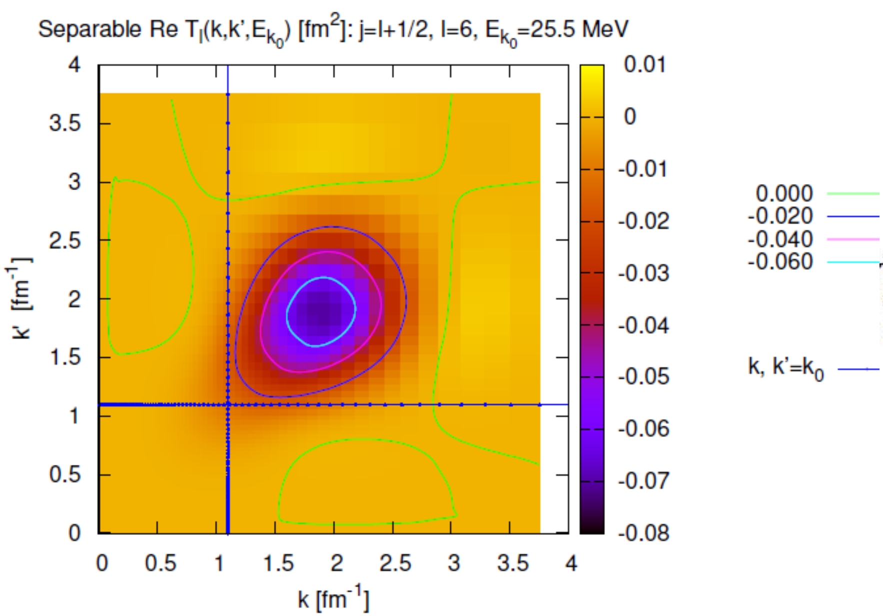
Department of physics + astronomy

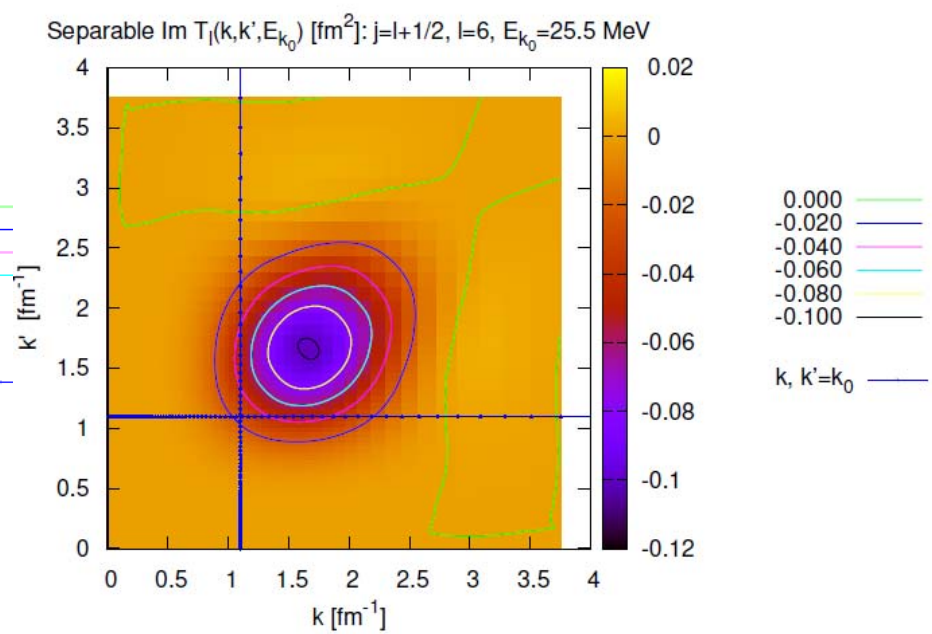
OHIO UNIVERSITY


Exact Re $T_l(k,k',E_{k_0})$ [fm^2]: j=l+1/2, l=6, E_{k_0}=25.5 MeV
-0.000
-0.020
-0.040
-0.060
-0.080
-0.100
k, k'=k_0
k [fm^{-1}]
k' [fm^{-1}]


Exact Im $T_l(k,k',E_{k_0})$ [fm^2]: j=l+1/2, l=6, E_{k_0}=25.5 MeV
-0.000
-0.020
-0.040
-0.060
-0.080
-0.100
k, k'=k_0
k [fm^{-1}]
k' [fm^{-1}]

n + ^{48}Ca


Separable Re $T_l(k,k',E_{k_0})$ [fm^2]: j=l+1/2, l=6, E_{k_0}=25.5 MeV
0.000
-0.020
-0.040
-0.060
k, k'=k_0
k [fm^{-1}]
k' [fm^{-1}]


Separable Im $T_l(k,k',E_{k_0})$ [fm^2]: j=l+1/2, l=6, E_{k_0}=25.5 MeV
0.000
-0.020
-0.040
-0.060
-0.080
-0.100
k, k'=k_0
k [fm^{-1}]
k' [fm^{-1}]

Separabilization of Optical Potentials

Method of Ernst-Shakin-Thaler

•In momentum space

•**Generalized to non-Hermitian potentials**

•Universal Rank-5 representation: ^{12}C to ^{208}Pb

•Allows to use all phenomenological Woods-Saxon based optical potentials in our q-space Faddeev equations

•EST projects out high-momentum off-shell components.

•(spawned project at MSU to compare with SRG methods)

•Next: coupling to excited states

Suggestion: **A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov**, Phys.Rev. C86 (2012) 034001

Solve Faddeev equations in Coulomb basis

This implies calculation of integrals like

$$Z_l^{SC}(p,p_\alpha) = \int \frac{dp'\,p'^2}{2\pi^2} V_l(p,p')\ \psi^C_{p_\alpha l}(p')$$

If $V_l(p,p')$ separable, then integral contains a smooth function and $\psi^C_{p_\alpha l}(p')$

Very nasty!

Coulomb wave function in momentum space and pw decomposition

Ref.: AMM *et al.*, Soviet Journal of Physics 3, 180 (1966).

Pole at $p_\alpha = p'$

Department of physics + astronomy

OHIO UNIVERSITY 1804

Some insights on momentum space Coulomb wave functions:
There are two representations: pole and non-pole regions

$$\psi^C_{p_\alpha l}(p') = \frac{-4\pi\, e^{-\eta_\alpha \pi/2}}{p'} \left(\frac{(p'+p_\alpha)^2+\gamma^2}{4p'p_\alpha}\right)^l \times \Gamma(1+i\eta_\alpha)\, e^{i\alpha_l}$$

$$\times \lim_{\gamma\to+0} \mathrm{Im}\left\{\left[e^{-i\alpha_l}\frac{(p'+p_\alpha+i\gamma)^{i\eta_\alpha-1}}{(p'-p_\alpha+ii\gamma)^{i\eta_\alpha+1}}\right.\right.$$

$$\left.\left.\times\, {}_2F_1\left(-l,-l-i\eta_\alpha;\,1-i\eta_\alpha;\,\frac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}\right)\right] + \gamma\,[\,\ldots\ldots\,]\right\}$$

Switch: $$\frac{4{p'}^2p_\alpha^2}{({p'}^2+p_\alpha^2+\gamma^2)^2} = \frac{(p'-p_\alpha)^2+\gamma^2}{(p'+p_\alpha)^2+\gamma^2}$$

$\psi^C_{p_\alpha l}(p')$ at low & high mc

$$\psi^C_{p_\alpha l}(p') = -2\pi\, e^{-\eta_\alpha\pi/2}\,(p'\,p_\alpha)^l\left[\frac{\Gamma(l+1+i\eta_\alpha)\Gamma(\frac{1}{2})}{\Gamma(l+\frac{3}{2})}\right]$$

$$\times \lim_{\gamma\to+0}\left\{\left[\left(\frac{2({p'}^2-(p_\alpha+i\gamma)^2)^{i\eta_\alpha}}{({p'}^2+p_\alpha^2+\gamma^2)^{l+i\eta_\alpha+1}}\right)\left(\frac{\eta_\alpha(p_\alpha+i\gamma)}{{p'}^2-(p_\alpha+i\gamma)^2}-\frac{\gamma(l+i\eta_\alpha+1)}{{p'}^2+p_\alpha^2+\gamma^2}\right)\right.\right.$$

$$\left.\left.\times\, {}_2F_1\left(\frac{l+i\eta_\alpha+2}{2},\frac{l+i\eta_\alpha+1}{2};\,l+\frac{3}{2};\,\frac{4{p'}^2p_\alpha^2}{({p'}^2+p_\alpha^2+\gamma^2)^2}\right)\right]+\gamma\,[\,\ldots\ldots\,]\right\}$$

Code will eventually be published

Department of physics + astronomy
OHIO UNIVERSITY 1804

Integral is oscillatory singular:

Integral:

$$\int_0^\infty \frac{dp'\,p'^2}{2\pi^2}\,V_l(p_\alpha,p')\,\psi^C_{p_\alpha l}(p')$$

$V_l(p_\alpha,p') \longrightarrow$ well-behaved function

$\psi^C_{p_\alpha l}(p') \longrightarrow$ contains singularity!

Irregular (containing singularity)

$$\int_0^\infty \frac{dp'\,p'^2}{2\pi^2}\,V_l(p_\alpha,p')\,\psi^C_{p_\alpha l}(p') = \int_0^{p_\alpha-\Delta} \cdots\cdots + \int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \cdots\cdots + \int_{p_\alpha+\Delta}^{\infty} \cdots\cdots$$

Regular

Singularity regularized with Gel'fand-Shilov method

$$\int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \cdots\cdots \longrightarrow \int_{p_\alpha-\Delta}^{p_\alpha+\Delta} \frac{\phi(p'-p_\alpha)}{(p'-p_\alpha+i\gamma)^{i\eta_\alpha+1}}\,dp'$$

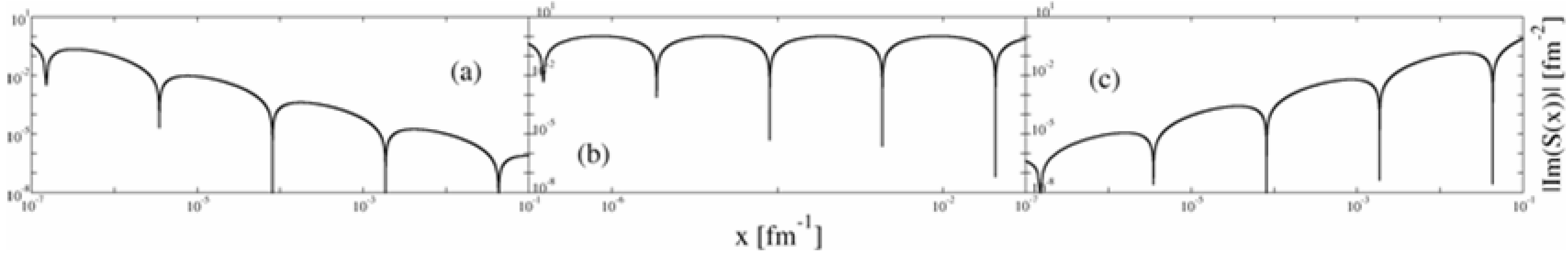
$$= \int_{p_\alpha-\Delta}^{p_\alpha+\Delta} dp' \, \frac{\left[\phi(p'-p_\alpha) - \phi(p_\alpha) - (p'-p_\alpha+i\gamma)\,\phi'(p_\alpha)\right]}{(p'-p_\alpha+i\gamma)^{i\eta_\alpha+1}}$$

$$+ \frac{i\phi(p_\alpha)}{\eta_\alpha}\left[(\Delta+i\gamma)^{-i\eta_\alpha} - (-\Delta+i\gamma)^{-i\eta_\alpha}\right]$$

$$+ \frac{\phi'(p_\alpha)}{(1-i\eta_\alpha)}\left[(\Delta+i\gamma)^{1-i\eta_\alpha} - (-\Delta+i\gamma)^{1-i\eta_\alpha}\right]$$

$\Delta \sim 10^{-6} - 10^{-4}$

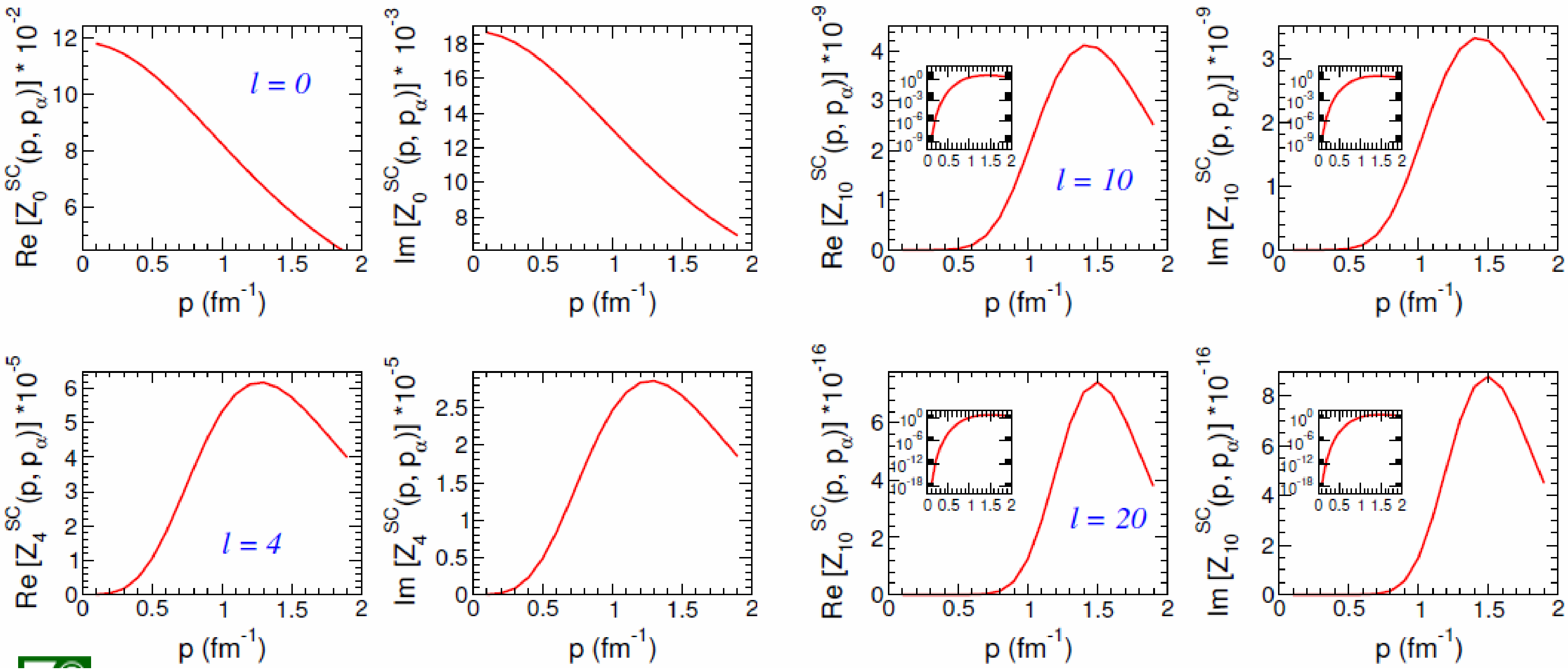
Then contribution of pole region very small



$V_l(p,p') \longrightarrow$ Yamaguchi form

$$Z_l^{SC}(p,p_\alpha) = \int_0^\infty \frac{dp'\,p'^2}{2\pi^2}\, V_l(p,p')\, \psi^C_{p_\alpha l}(p')$$

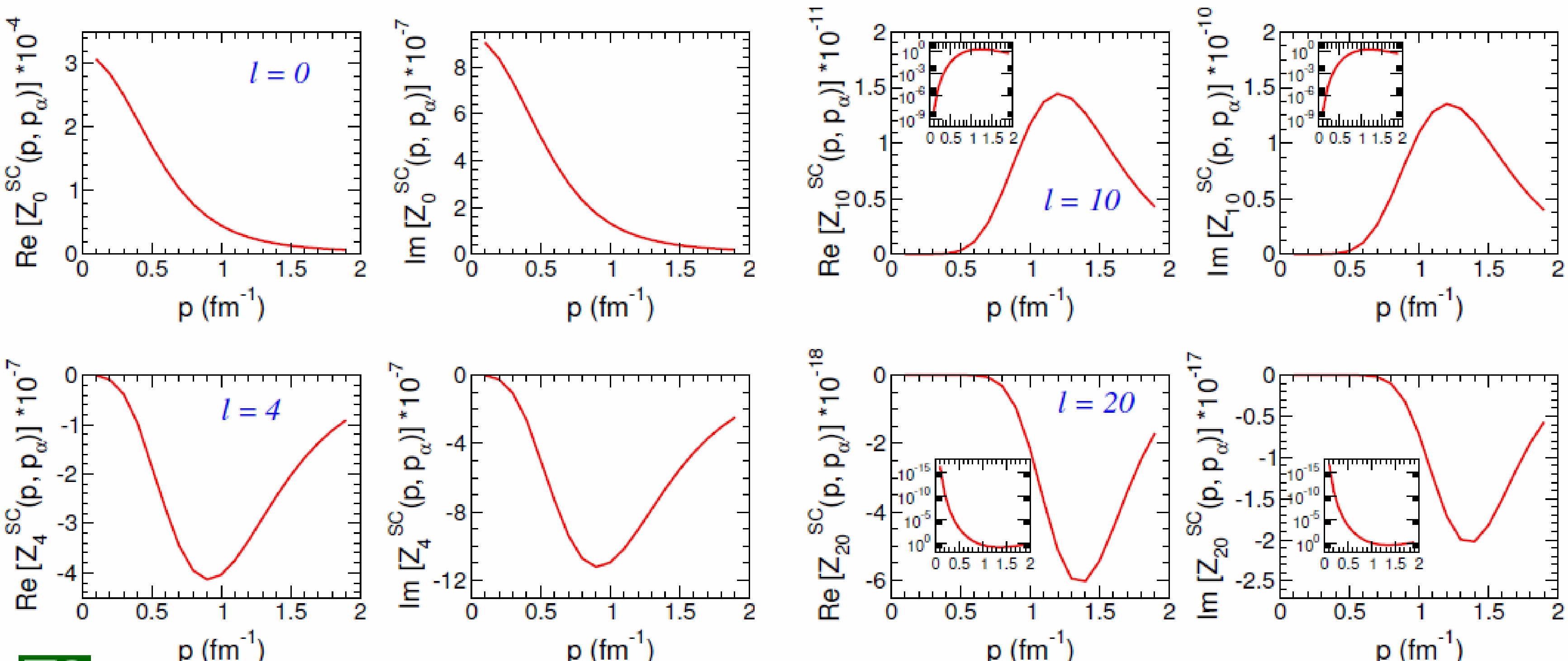
$\mathrm{p}+{}^{12}\mathrm{C}$ @ $\mathrm{E_{cm}} = 10$ MeV:



$V_l(p,p') \longrightarrow$ Yamaguchi form

$$Z_l^{SC}(p,p_\alpha) = \int_0^\infty \frac{dp'\,p'^2}{2\pi^2} V_l(p,p')\,\psi^C_{p_\alpha l}(p')$$

p+^{208}Pb @ E_{cm} = 8 MeV:



Looks promising

Department of physics + astronomy

OHIO UNIVERSITY

Faddeev Equations in Coulomb basis

Developing pw Coulomb wave functions in momentum space
essential to distinguish pole and non-pole regions
by use of different expansions of hypergeometric functions

Regularization of integrals over Coulomb wave function with Gel'fand-Shilov method
tested so far with Yamaguchi type functions
for numerical and Mathematica cross checks

Next calculations with EST form factors

Persistence of Neelam Upadhyay and Vasily Eremenko

TORUS postdocs

TORUS: Theory of Reactions for Ustable iSotopes

A Topical Collaboration for Nuclear Theory

Ian Thompson, Jutta Escher (LLNL)

Filomena Nunes (NSCL MSU)

Akram Mukhamedzhanov (TAMU)

Ch. E. (OU)

Few-Body Collaboration

Goran Arbanas (ORNL)

Post docs: Neelam Upadhyay (NSCL)
Vasily Eremenko (OU)
Grad Student: Linda Hlophe (OU)

Department of physics + astronomy

OHIO UNIVERSITY