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Microscopic Optical Potentials for Helium-6 Scattering off Protons

Abstract The differential cross section and the analyzing power are calculated for elastic scattering of ^{6}He from a proton target using a microscopic folding optical potential, in which the ^{6}He nucleus is described in terms of a ^{4}He-core with two additional neutrons in the valence p-shell. In contrast to previous work of that nature, all contributions from the interaction of the valence neutrons with the target protons are taken into account.

1 Introduction

Recently the helium isotopes have been extensively studied, both experimentally and theoretically. Specifically, elastic scattering of ^{6}He from a polarized proton target has been measured for the first time at an energy of 71 MeV/nucleon [1]. The experiment finds that the analyzing power becomes negative around 50°, which is not predicted by simple folding models for the optical potential [2,3], but which nevertheless describe the differential cross section at this energy reasonably well. This apparent 'A_y problem' leads to the conclusion that folding models which are adequate in describing p–A scattering from stable, closed shell nuclei need to be revisited and extended when applying them to unstable as well as open-shell nuclei.

2 First Order Optical Potential Including Valence Neutrons

The theoretical approach to elastic scattering of a nucleon from a nucleus pioneered by Watson [4,5] has been applied very successfully over the years to a wide range of closed shell nuclei. Here a spectator expansion is constructed within a multiple scattering theory predicated upon the idea that the two-body interactions

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between the projectile and the nucleons in the target dominate the reaction. Generally, only the first order in this expansion is considered, leading to a Watson optical potential for single scattering, which is of the form

$$\langle \mathbf{k}'|\langle \phi_A|PUP|\phi_A\rangle \mathbf{k}\rangle \equiv U_{el}(\mathbf{k}', \mathbf{k}) = \sum_{i=N,P} \langle \mathbf{k}'|\langle \phi_A|\hat{\tau}_{0i}(\mathcal{E})|\phi_A\rangle \mathbf{k}\rangle, \tag{1}$$

where P is a projector on the ground state of the nucleus (we assume here ^{6}He to have 2 neutrons and protons in the s-shell, and 2 neutrons being in the $p_{3/2}$-shell), and $\hat{\tau}_{0i}(\mathcal{E})$ represents the nucleon-nucleon (NN) t-matrix evaluated at the energy $\mathcal{E}$ of the system. The summation over i indicates that one has to sum over N neutrons and Z protons. The structure of Eq. (1) is graphically shown in Fig. 1. The vectors $\mathbf{k}$ and $\mathbf{k}$' are the initial and final momenta of the projectile, while $\mathbf{p}$ and $\mathbf{p}$' are the initial and final momenta of the struck nucleon in the nucleus. Changing variables to $\mathbf{q} = \mathbf{k}' - \mathbf{k} = \mathbf{p} - \mathbf{p}'$, $\mathbf{K} = \frac{1}{2}(\mathbf{k}' + \mathbf{k})$, and $\mathbf{P} = \frac{1}{2}(\mathbf{p}' + \mathbf{p})$ as well as neglecting the recoil of the target nucleus leads to

$$U(\mathbf{q}, \mathbf{K}) = \sum_{i=N,P} \int d^3\mathbf{P}\, \hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}(\mathbf{K} - \mathbf{P}); \mathcal{E}\right) \rho_i\left(\mathbf{P} - \frac{\mathbf{q}}{2}, \mathbf{P} - \frac{\mathbf{q}}{2}\right). \tag{2}$$

The calculation of the first order optical potential relies on two basic input quantities. One is the fully-off-shell NN t-matrix, which represents the current understanding of the nuclear force, and the other is the single particle density matrix of the nucleus under consideration.

The most general form of any NN t-matrix can be written as linear combination of six spin-momentum operators [6,7] as

$$\begin{aligned}
\hat{\tau}_{0i}(\mathbf{q}, \mathbf{K}, \mathcal{E}) &= A(\mathbf{q}, \mathbf{K}, \mathcal{E})\mathbf{1} + iC(\mathbf{q}, \mathbf{K}, \mathcal{E})(\sigma^{(1)} + \sigma^{(2)}) \cdot \hat{\mathbf{n}} \\
&+ M(\mathbf{q}, \mathbf{K}, \mathcal{E})(\sigma^{(1)} \cdot \hat{\mathbf{n}})(\sigma^{(2)} \cdot \hat{\mathbf{n}}) + \left[G(\mathbf{q}, \mathbf{K}, \mathcal{E}) - H(\mathbf{q}, \mathbf{K}, \mathcal{E})\right] (\sigma^{(1)} \cdot \hat{\mathbf{q}})(\sigma^{(2)} \cdot \hat{\mathbf{q}}) \\
&+ \left[G(\mathbf{q}, \mathbf{K}, \mathcal{E}) + H(\mathbf{q}, \mathbf{K}, \mathcal{E})\right] (\sigma^{(1)} \cdot \hat{\mathbf{K}})(\sigma^{(2)} \cdot \hat{\mathbf{K}}) \\
&+ D(\mathbf{q}, \mathbf{K}, \mathcal{E}) \left[(\sigma^{(1)} \cdot \hat{\mathbf{q}})(\sigma^{(2)} \cdot \hat{\mathbf{K}}) + (\sigma^{(1)} \cdot \hat{\mathbf{K}})(\sigma^{(2)} \cdot \hat{\mathbf{q}})\right],
\end{aligned} \tag{3}$$

where the amplitude $D(\mathbf{q}, \mathbf{K}, \mathcal{E})$ is zero on the energy shell.

Since our goal is to test the effect of the two valence neutrons on the scattering observables rather than to make a detailed comparison with data, we approximate the density matrix for ^{6}He by two harmonic oscillator terms, as is e.g. used for the COSMA densities [8], namely a fully occupied s-shell and two neutrons in the $p_{3/2}$-shell. The oscillator constants are $\nu_s = 0.355$ fm^{-2} corresponding to a ^{6}He charge radius of 2.054 fm [9] and $\nu_p = 0.3225$ fm^{-2} corresponding to a matter radius of 2.32 fm [10]. Explicitly, the radial wave functions are given as

$$\psi_s^m(\mathbf{p}) = (2\pi)^{3/2}\left(\frac{4}{\sqrt{\pi}\nu_s^{3/2}}\right)^{1/2} \frac{1}{4\pi} e^{-p^2/2\nu_s} \mathcal{Y}_{0m}^{\frac{1}{2}}(\hat{p})$$

$$\psi_p^m(\mathbf{p}) = (2\pi)^{3/2}\left(\frac{4}{\sqrt{\pi}\nu_p^{3/2}}\right)^{1/2} \sqrt{\frac{2}{3}} \frac{p}{\sqrt{\nu_p}} e^{-p^2/2\nu_p} \mathcal{Y}_{1m}^{\frac{3}{2}}(\hat{p}), \tag{4}$$

from which the s-shell and p-shell single particle densities are obtained. Specifically, the p-shell projected on the $j = 3/2$ state is given as

$$\psi_{p3/2}(p) := f_{p3/2}(p)\frac{1}{\sqrt{4}}\left(\mathcal{Y}_{1\frac{3}{2}}^{\frac{3}{2}}(\hat{p}) - \mathcal{Y}_{1\frac{1}{2}}^{\frac{3}{2}}(\hat{p}) + \mathcal{Y}_{1-\frac{1}{2}}^{\frac{3}{2}}(\hat{p}) - \mathcal{Y}_{1-\frac{3}{2}}^{\frac{3}{2}}(\hat{p})\right), \tag{5}$$
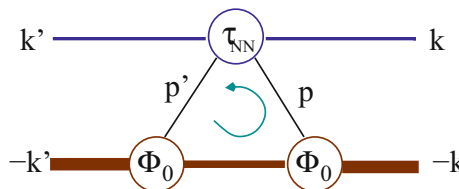

Fig. 1 Diagram for the optical potential matrix element in the single-scattering approximation

where the spin-angular momentum functions are denoted by $\mathcal{Y}_{lm}^{j}$ and the radial part by $f_{p_{3/2}}(p)$. When calculating the optical potential of Eq. (1), expectation values of the spin-momentum operators $\sigma^{(2)} \cdot \hat{\mathbf{k}}_i$, with $\mathbf{k}_i \equiv \hat{\mathbf{n}}, \hat{\mathbf{q}}, \hat{\mathbf{K}}$, must be taken with all nuclear single-particle wave functions. If the expectation value is taken with wave functions describing a closed shell, the sum over all nucleons in that shell adds to zero. Thus, for closed shell nuclei only the terms corresponding to the Wolfenstein amplitudes A and C in Eq. (3) contribute to the folding optical potential. For ^{16}O such a calculation is given in Ref. [11]. For an open-shell nucleus as ^{6}He, those expectation values contribute to the optical potential. We find by explicit calculation

$$\psi_{p_{3/2}}(p')\, \sigma^{(2)} \cdot \hat{\mathbf{n}}\, \psi_{p_{3/2}}(p) = -i \frac{2pp'}{9\pi^{3/2}v_p^{5/2}}\, e^{-\frac{1}{2v_p}(p^2+p'^2)} \sin\alpha_{pp'} \cos\beta$$

$$\psi_{p_{3/2}}(p')\, \sigma^{(2)} \cdot \hat{\mathbf{K}}\, \psi_{p_{3/2}}(p) = -i \frac{2pp'}{9\pi^{3/2}v_p^{5/2}}\, e^{-\frac{1}{2v_p}(p^2+p'^2)} \sin\alpha_{pp'} \cos\delta$$

$$\psi_{p_{3/2}}(p')\, \sigma^{(2)} \cdot \hat{\mathbf{q}}\, \psi_{p_{3/2}}(p) = 0, \tag{6}$$

where $\cos\beta$ is the angle between the normal vectors in the nucleon-nucleus (NA) and NN frame, $\alpha_{pp'}$ is the angle between $\mathbf{p}$ and $\mathbf{p'}$, and $\cos\delta$ is the angle between the total momentum in the NN frame and the normal in the NA frame. This means that all amplitudes from the NN t-matrix, Eq. (3), contribute to the optical potential when integrated with the single-particle density matrix of the $p_{3/2}$-shell neutrons. The term proportional to $iC(\mathbf{q}, \mathbf{K}, \mathcal{E})\, \sigma^{(2)} \cdot \hat{\mathbf{n}}$ has no dependence on the spin of the projectile, $\sigma^{(1)}$, and thus will give an additional contribution to the central part of the optical potential. All other terms contain a scalar product of $\sigma^{(1)}$ with a momentum. The operator for the amplitude $M(\mathbf{q}, \mathbf{K}, \mathcal{E})$ has the typical structure of a spin-orbit operator and only contributes to the spin-orbit potential, while the terms proportional to $(\sigma^{(1)} \cdot \hat{\mathbf{K}})$ and $(\sigma^{(1)} \cdot \hat{\mathbf{q}})$ contribute to the central as well as to the spin-orbit potential. The explicit integration over the single-particle wave functions for the valence neutrons and the full NN t-matrix of Eq. (3) reveals that the integrals involving the amplitudes $(G + H)$ and D give a zero contribution to the optical potential of the valence neutrons. Therefore, the final expression for the optical potential for ^{6}He scattering off protons has the following structure

$$U_{^6He}(\mathbf{q}, \mathbf{K}) = \sum_{i=N,P} U_{core}(\mathbf{q}, \mathbf{K}) + U_{val}(\mathbf{q}, \mathbf{K}), \tag{7}$$

where U_{core} is the usual optical potential for a closed shell nucleus. The optical potential due to the valence neutrons acquires additional terms in its central as well as its spin-orbit part,

$$U_{val_{central}} = U_A(\mathbf{q}, \mathbf{K}) + U_A^C(\mathbf{q}, \mathbf{K})$$
$$U_{val_{spin-orbit}} = U_C(\mathbf{q}, \mathbf{K}) + U^M(\mathbf{q}, \mathbf{K}). \tag{8}$$

3 Discussion and Outlook

Our calculations of the optical potential for ^{6}He scattering off protons are carried out with the harmonic oscillator density given in Sect. 2 and a NN t-matrix derived from the CD-Bonn potential [12]. The Wolfenstein amplitudes for neutron-proton (np) scattering at 71 MeV laboratory kinetic energy are shown in Fig. 2 as function of the np c.m. angle. The amplitude A gives the major contribution to the central term, while the imaginary part of C is responsible for the real part of the spin-orbit term. In addition, the valence neutrons contribute through the M amplitude to the spin-orbit term, as well as to the central term through C. Since the expectation values of the spin-momentum operators for the neutron in the $p_{3/2}$-shell of ^{6}He contain a factor $\sin\alpha_{pp'}$, the forward direction of the amplitudes does not contribute to the optical potential.

The calculations of the angular distributions of the differential cross section and the analyzing power A_y at 71 MeV/nucleon and 200 MeV/nucleon are shown in Fig. 3. The dash-dotted lines show a 'traditional' calculation with an optical potential as it would be derived for a closed shell nucleus [11]. The dashed line gives the contribution of the valence neutrons to the central part of the optical potential, while for the solid line their contributions to the cental and the spin-orbit term are added. All optical potentials are calculated neglecting the target recoil, similar to the calculations in Ref. [11]. Overall, the additional contributions of the valence neutrons to the optical potential are small. However, they show energy dependence. At 71 MeV/nucleon, the contribution to the central term is negligible, only the contribution from the M amplitude to the spin orbit
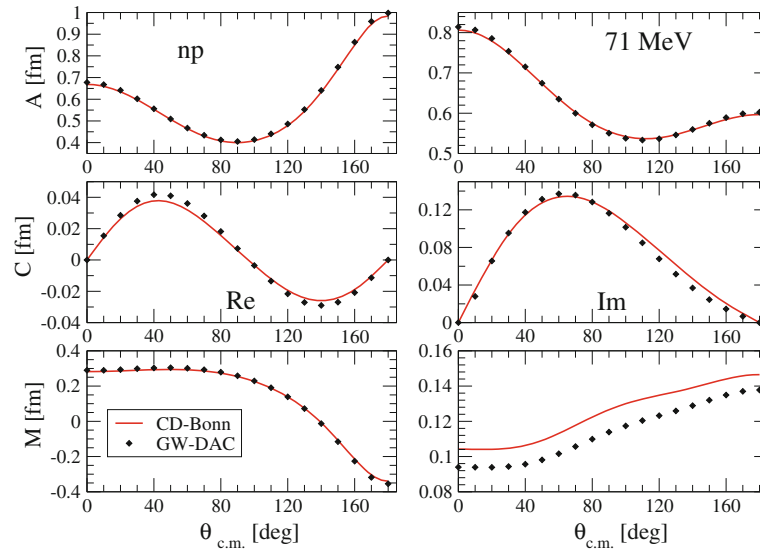
Fig. 2 The Wolfenstein amplitudes for neutron–proton scattering at 71 MeV laboratory NN kinetic energy based on the CD-Bonn potential [12], which contribute to the optical potential for ^{6}He. The *points* show the extracted values from the GW-DAC current analysis [13,14]
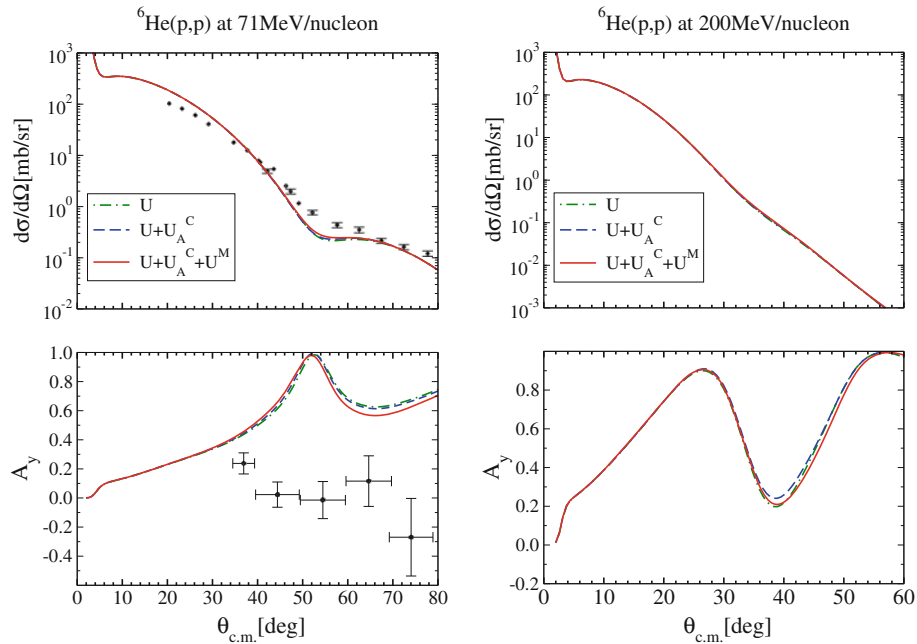

Fig. 3 The angular distribution of the differential cross section (*top panels*) and the analyzing power (A_y) (*bottom panels*) for elastic scattering of ^{6}He from protons at projectile laboratory energies of 71 MeV/nucleon (*left*) and 200 MeV/nucleon (*right*). The calculations show a first order optical potential based on the CD-Bonn potential [12]. The *dash-dotted line* represents a calculation with a traditional optical potential derived by using the NN amplitudes A and C only, while for the *dashed line* and *solid lines* the contributions of the valance neutrons to the central and spin-orbit terms are successively added. All calculations do not include the target recoil. The experimental data are taken from Ref. [1]

term is visible. At 200 MeV/nucleon both contributions are of the same size, but act in opposite directions so that the net contribution vanishes. This changes slightly when adding the effect of recoil. Our preliminary calculations show that when including recoil, the effect on the spin-orbit term through the M amplitude is slightly enhanced, while the effect due to the additional central term slightly decreases. This will be discussed in a future manuscript.

In previous work some of the authors investigated the optical potential for ^{6}He derived in a cluster description [15], in which the internal dynamics of ^{6}He as alpha-core plus two neutrons was taken into account explicitly. In this work it was found that the description of the analyzing power at 71 MeV was sensitive to the cluster dynamics. However, it was also found, that the quality of the optical potential for the alpha-core was important. In principle, the additional terms due to the open shell structure of the $p_{3/2}$-valence neutrons would also have to be included for the two neutrons in the cluster description.

Our present work mainly concentrates on taking into account effects related to the dynamics of the ^{6}He nucleus as well as the interaction between the nucleons. In contrast to that our description of the single-particle density matrix is relatively simple. In a recent work [16] a somewhat opposite approach was taken by deriving an optical potential in the Glauber approximation, however employing highly sophisticated wave functions for the ^{6}He nucleus. It appears that a microscopic understanding of the reaction ^{6}He(p,p)^{6}He requires considering the dynamics of the reaction as well as the structure of ^{6}He at an equal level of sophistication.

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