Continuum shell-model description of weakly bound/unbound nuclear systems

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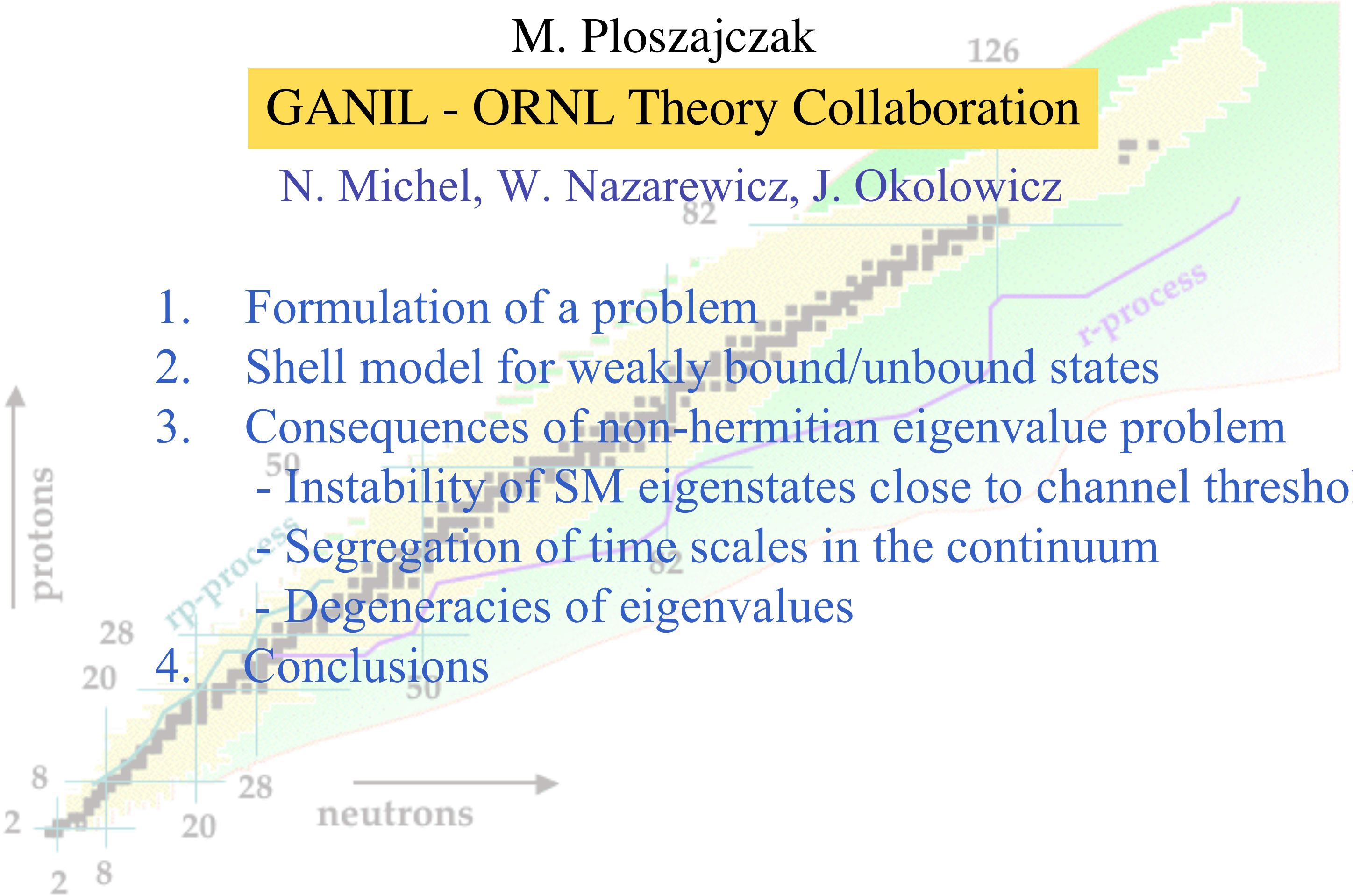
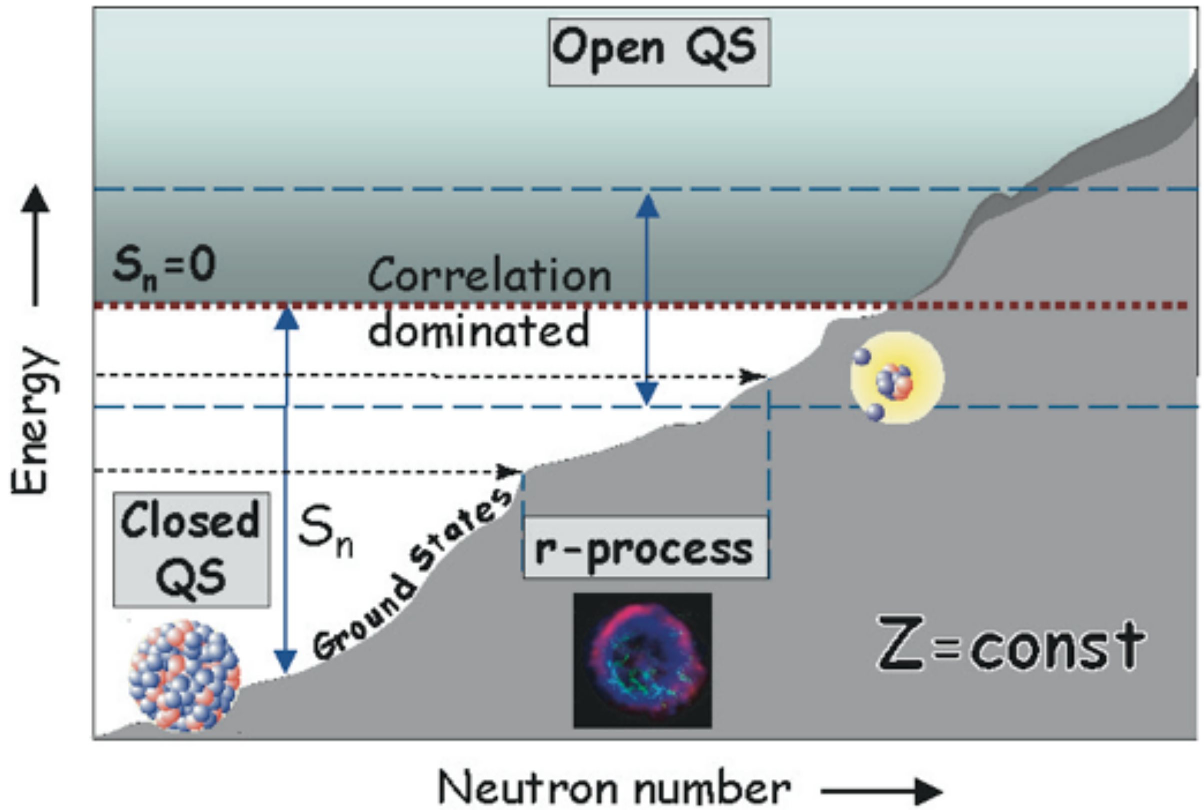
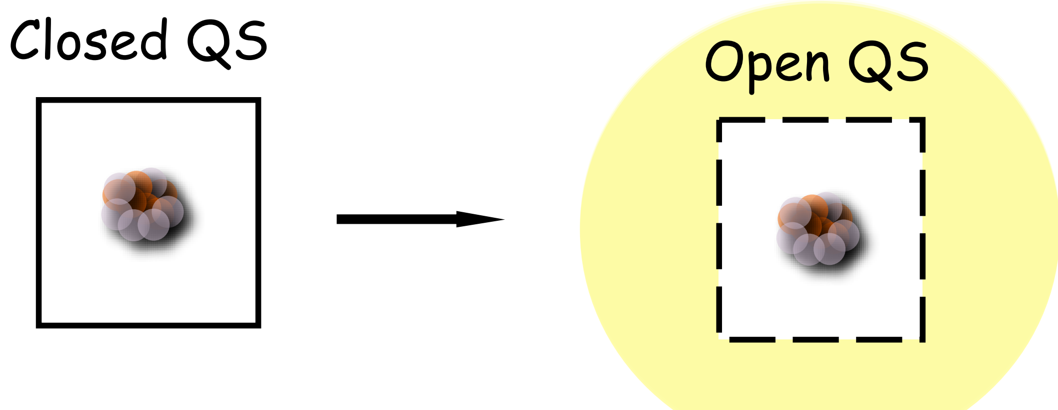
1. Formulation of a problem
2. Shell model for weakly bound/unbound states
3. Consequences of non-hermitian eigenvalue problem
   - Instability of SM eigenstates close to channel threshold
   - Segregation of time scales in the continuum
   - Degeneracies of eigenvalues
4. Conclusions







Collective exc. ~1 MeV
Pairing ~ 2 MeV

Threshold effects 0.1-3 MeV

The nucleus is a correlated open quantum many-body system

External states (environment): continuum of decay channels

Majority of neutron-rich nuclei are influenced by the continuum coupling even in their ground states

Resonance phenomena are generic (atoms, nanotubes, quantum dots, …) - specific to atomic nuclei are strong correlations

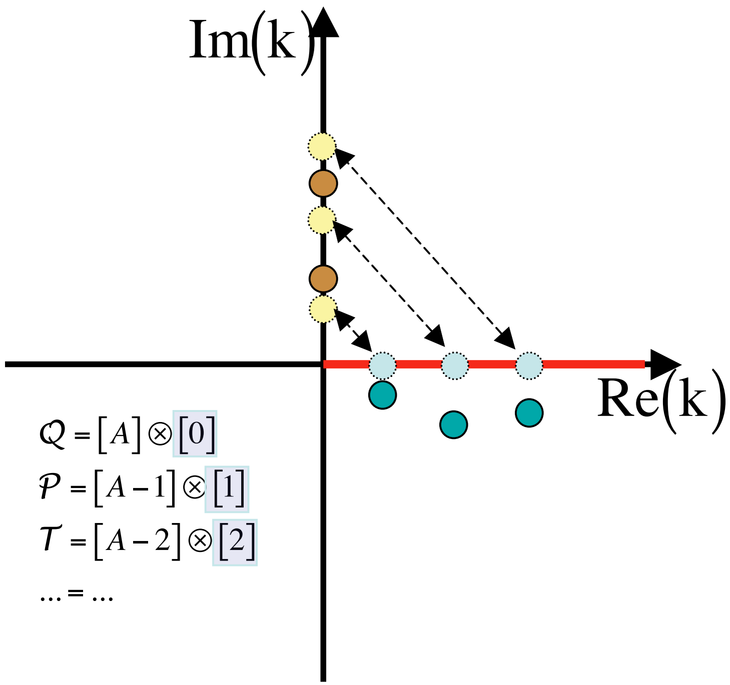
Configuration mixing approach for open quantum systems

Major challenge : unification of structure and reaction aspects of weakly-bound or unbound nuclei

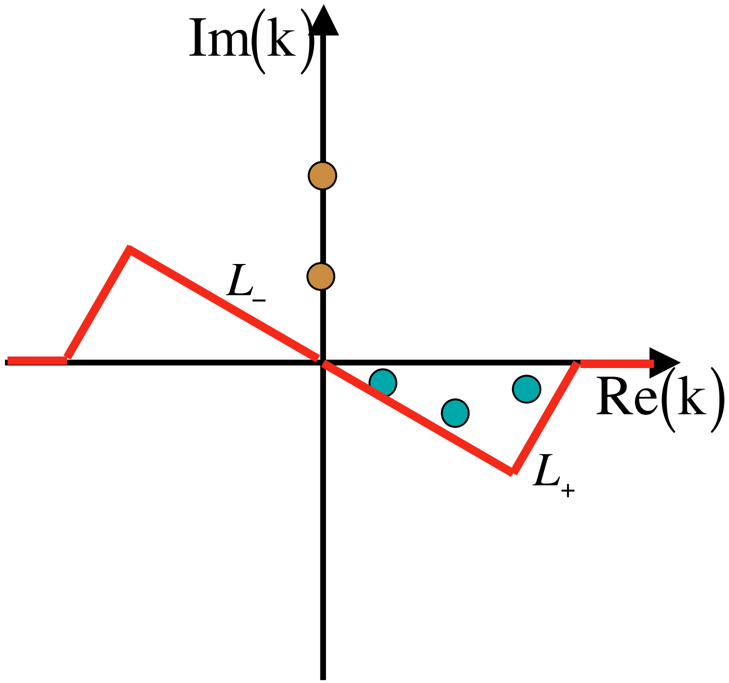
Shell Model Embedded in the Continuum and the Gamow Shell Model

Resonances are genuine intrinsic properties of quantum systems but they do not belong to the Hilbert space

Hilbert space



rigged Hilbert space



$$\mathcal{H}_{QQ} \to [\mathcal{H}_{QQ}]_{ij} = [\mathcal{H}_{QQ}]_{ji} \quad \longleftrightarrow \quad H \to [H]_{ij} = [H]_{ji}$$

Hilbert space

$$\sum_n |\bar{u}_n\rangle\langle\bar{u}_n| + \int_{R_+} |\bar{u}_k\rangle\langle\bar{u}_k| dk = 1\,;\quad \langle\bar{u}_i|\bar{u}_j\rangle = \delta_{ij}$$

bound states
res. anamneses

non-resonant
continuum

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle$$

$$\Downarrow$$

$$\sum_j |SD_j\rangle\langle SD_j| \cong 1$$

rigged Hilbert space

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1\,;\quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

bound states
resonances

non-resonant
continuum

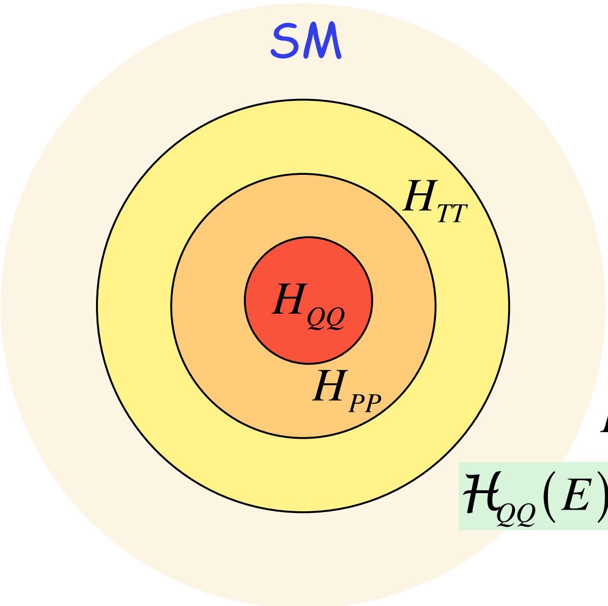
$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle$$

$$\Downarrow$$

$$\sum_k |SD_k\rangle\langle SD_k| \cong 1$$

Hilbert space formulation : Shell Model Embedded in the Continuum

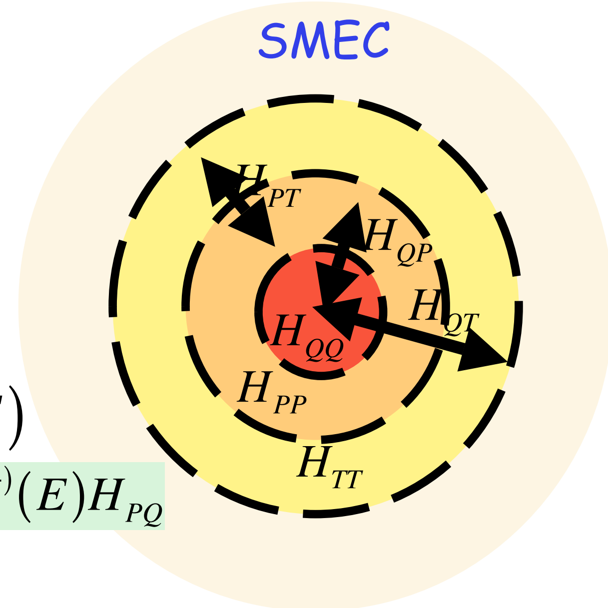
K. Bennaceur et al. Nucl. Phys. A671 (2000) 203



$$Q = [A]$$
$$P = [A-1] \otimes [1]$$
$$T = [A-2] \otimes [2]$$
$$\ldots = \ldots$$

$$H_{QQ} \to \mathcal{H}_{QQ}(E)$$

$$\mathcal{H}_{QQ}(E) = H_{QQ} + H_{QP} G_P^{(+)}(E) H_{PQ}$$



$$\Psi_E^c = \varsigma_E^c + \sum_{i,k} \left(\Phi_i^A + \omega_i^{(+)}(E)\right) \langle \Phi_i^A | \left(E - \mathcal{H}_{QQ}(E)\right)^{-1} | \Phi_k^A \rangle \langle \Phi_k^A | H_{QP} | \varsigma_E^c \rangle$$

non-resonant part

resonant part

Discrete states: $\left[\langle \Phi_i^A | \mathcal{H}_{QQ} | \Phi_j^A \rangle\right]$

Scattering states: $\Psi_E^c = \varsigma_E^c + \sum_i \tilde{\Omega}_i \left[E - \tilde{E}_i + (i/2)\tilde{\Gamma}_i\right]^{-1} \langle \tilde{\Phi}_i | H_{QP} | \varsigma_E^c \rangle$

$$\tilde{\Phi}_i + \tilde{\omega}_i$$

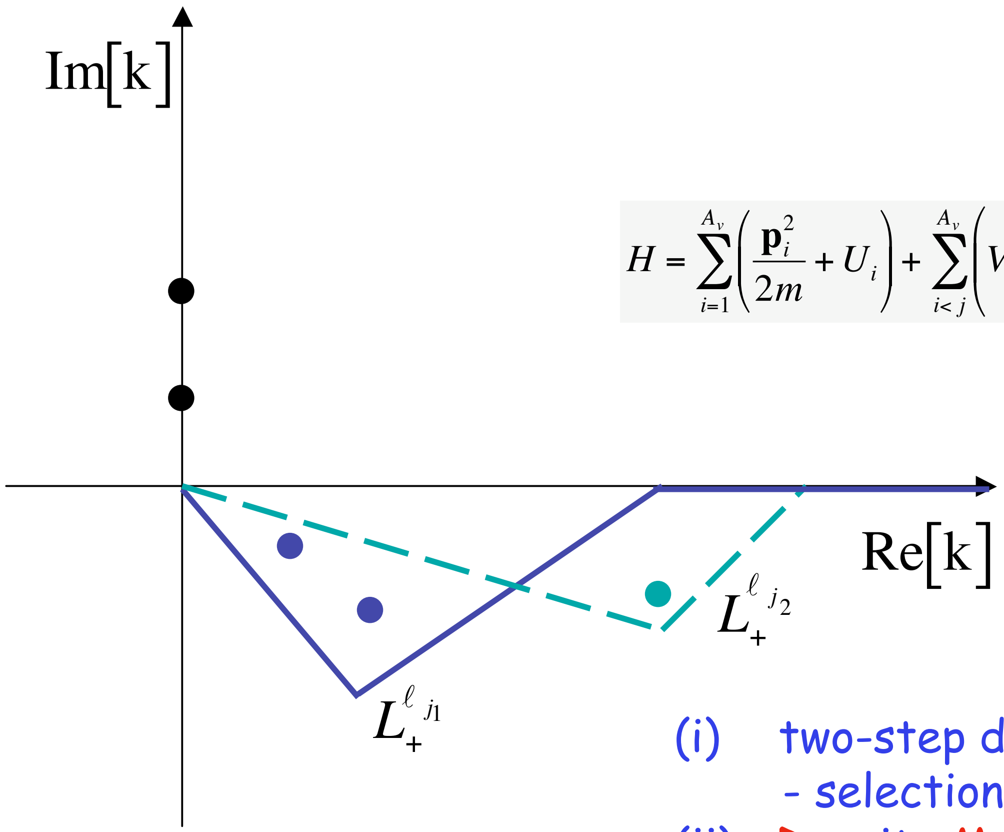
2p radioactivity: J. Rotureau et al., PRL 95 (2005) 042503; NP A767 (2006) 13

Shell Model in the Complex Energy Plane : Gamow Shell Model (Rigged Hilbert space formulation)

$$\hat{H}\Psi = (e - \boxed{i\frac{\Gamma}{2}})\Psi$$

$$\Psi(0,k) = 0 \;, \begin{cases} \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} O_l(kr) \longleftarrow \text{outgoing solution} \\ \Psi(\vec{r},k) \underset{r\to\infty}{\longrightarrow} I_l(kr) + O_l(kr) \end{cases}$$

$$k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$$ (poles of the S-matrix)



$$H = \sum_{i=1}^{A_v}\left(\frac{\mathbf{p}_i^2}{2m} + U_i\right) + \sum_{i<j}^{A_v}\left(V_{ij} + \frac{\mathbf{p}_i\mathbf{p}_j}{(A_c+1)m}\right)$$

(i) two-step diagnalization: $|\Psi_0^{(J)}\rangle \longrightarrow \{|\Psi^{(J)}\rangle\}$

- selection of states: $\max\{|\langle\Psi_i^{(J)}|\Psi_0^{(J)}\rangle|\}$

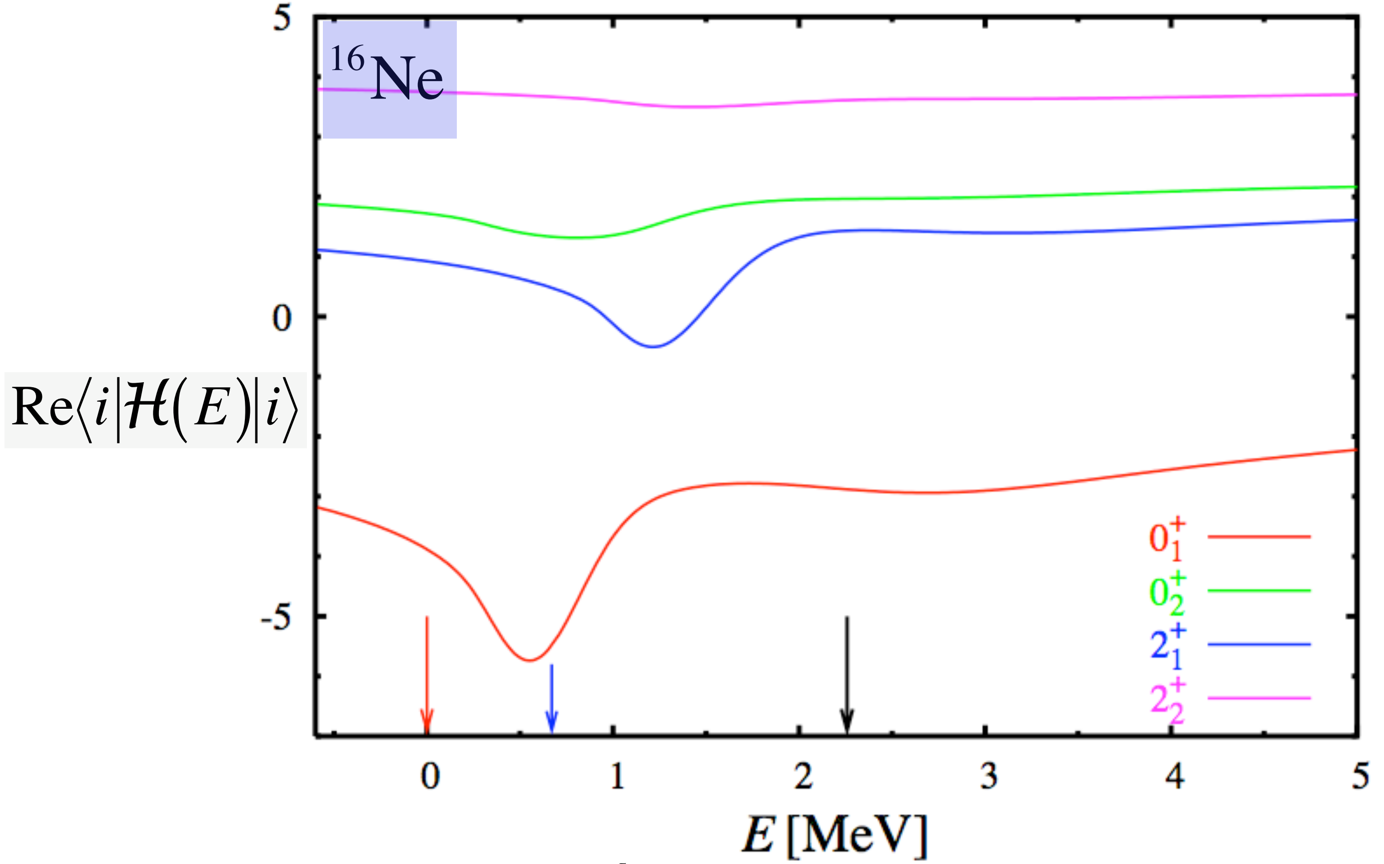
(ii) Density Matrix Renormalization Group method

Applications of the Continuum Shell Model (SMEC, GSM):

- spectroscopy for bound/unbound states: eigenvalues, nuclear moments, EM transitions, spectrocopic factors
- decay spectroscopy: 1p/1n-decays, 2p/2n-radioactivity, first-forbidden β-decay
- reactions in low-energy correlated continuum: (p,p'), (n,n'), (p,γ), (n,γ), Coulomb dissociation, ...

Consequences of non-hermitian eigenvalue problem

- Instability of SM eigenstates close to channel thresholds

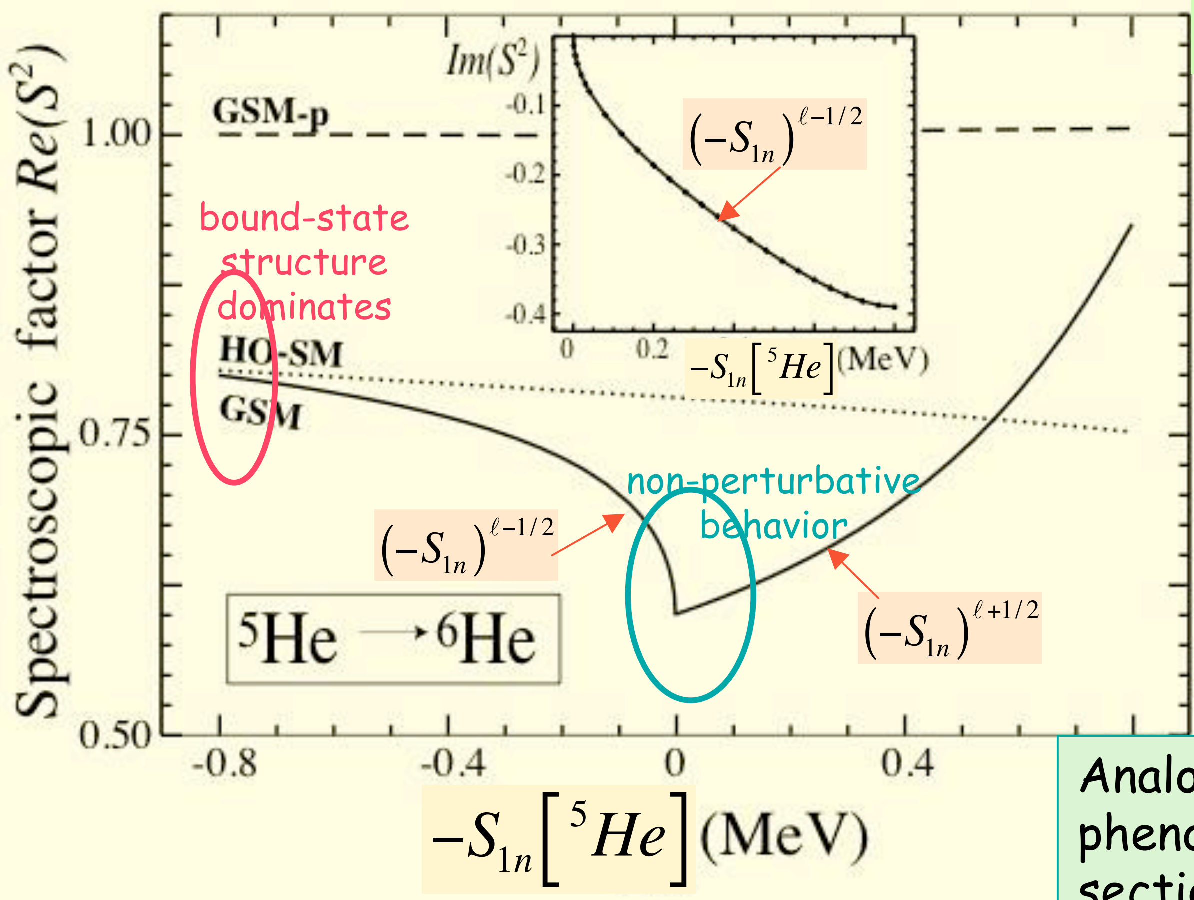


- $\left[{}^{15}\mathrm{F}\left(1/2_1^+\right)\oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$
- $\left[{}^{15}\mathrm{F}\left(5/2_1^+\right)\oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$
- $\left[{}^{15}\mathrm{F}\left(1/2_1^-\right)\oplus \mathrm{p}\left(\ell_j\right)\right]^{J^\pi}$

One-neutron spectroscopic factor

$$S^2 \equiv \int u_{\ell j}^2(r)dr = \sum\!\!\!\!\!\!\int_{\mathcal{B}} \langle \widetilde{\Psi_A^{J_A}} || a_{\ell j}^+(\mathcal{B}) || \Psi_{A-1}^{J_{A-1}} \rangle^2$$

$$\left\langle {}^6\mathrm{He(g.s.)} \middle| \left[{}^5\mathrm{He(g.s.)} \otimes p_{3/2} \right]^{0^+} \right\rangle$$



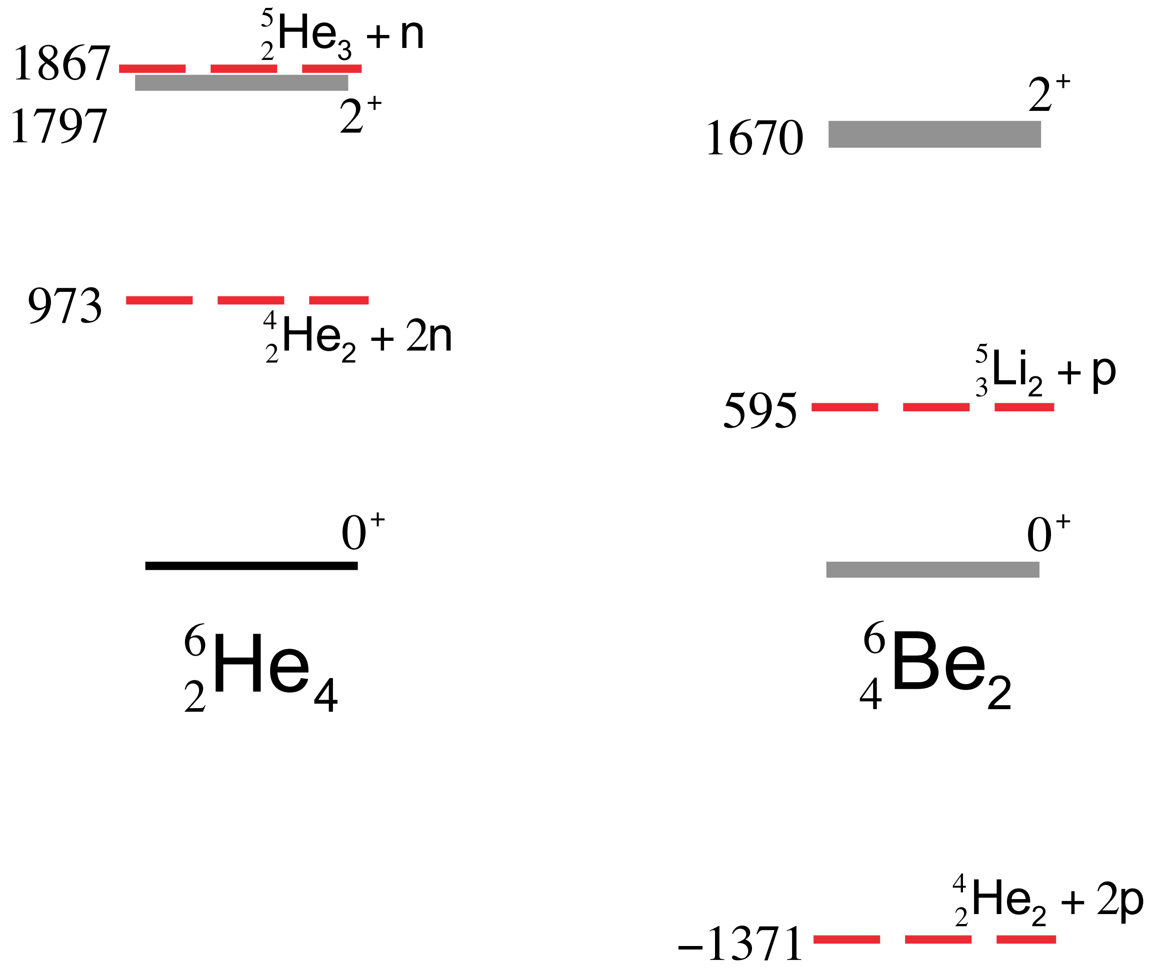
Analogy with Wigner threshold phenomenon for reaction cross-sections

N. Michel, et al., Phys. Rev. C75, 031301(R)

$$Y(b,a)X : \sigma_\ell \sim k^{2\ell-1} \quad \Longleftrightarrow \quad (-S_n)^{\ell-1/2} \ \text{for} \ S_n < 0$$

$$X(a,b)Y : \sigma_\ell \sim k^{2\ell+1} \qquad\qquad (-S_n)^{\ell+1/2} \ \text{for} \ S_n > 0$$

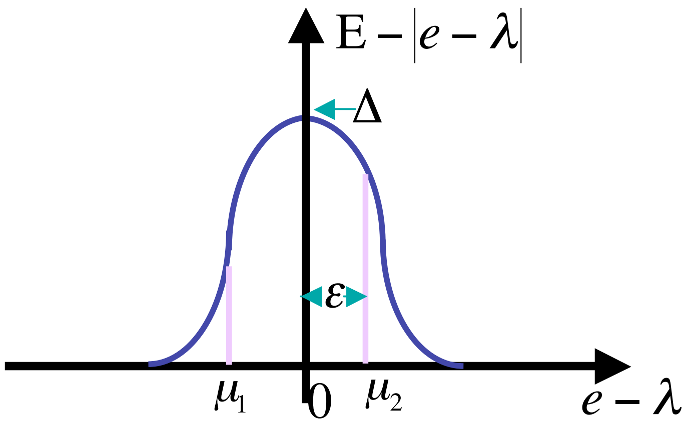
Configuration mixing in mirror nuclei



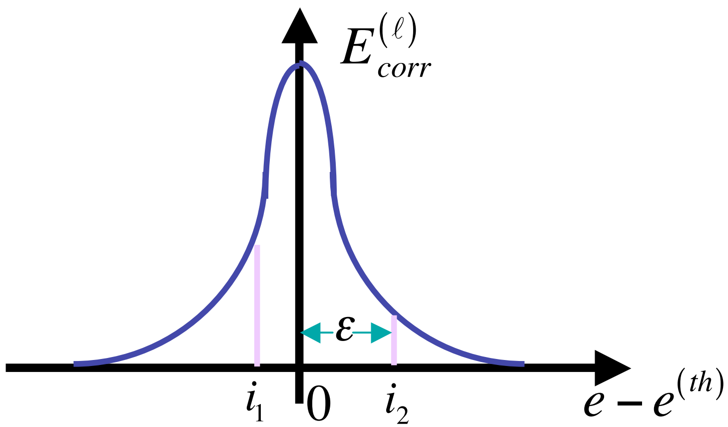
Instability of a single-particle motion in a potential with a pair deformation

Instability of SM eigenstates at channel threshold

Pairing correction to single-particle eigenstates



Continuum coupling correction to shell model eigenstates



Admixture of single-particle configurations with $e > \lambda$

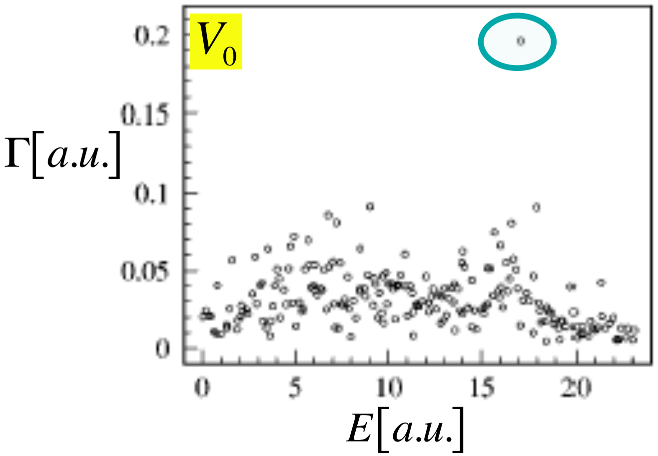
Admixture of many-body continuum states

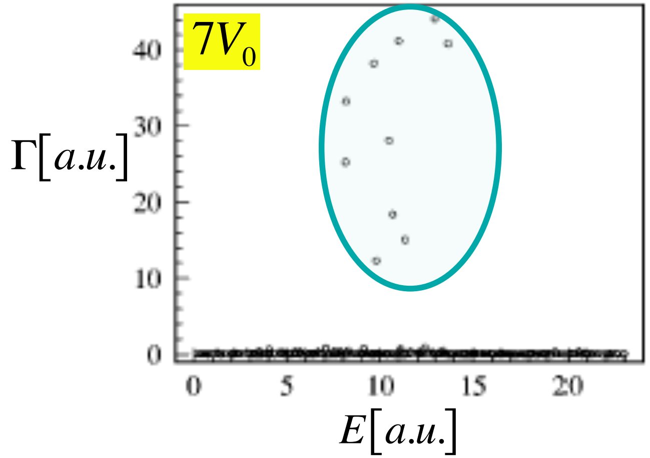
$\Psi^{(SM)}_{[A-1],\mu} \otimes \varphi^{(c)}_{[1],i}$, $\Psi^{(SM)}_{[A-2],\nu} \otimes \varphi^{(c)}_{[2],j}$ with $E > E_{th}$

Consequences of non-hermitian eigenvalue problem

- Segregation of time scales in the continuum

$J^{\pi}=0^{+}, T=0$ states in ^{24}Mg , 10 channels





$$\mathcal{H}(E)=\underbrace{H_{SM}}_{[M\times M]}-\frac{i}{2}\underbrace{V(E)V^{T}(E)}_{[M\times\Lambda]}$$

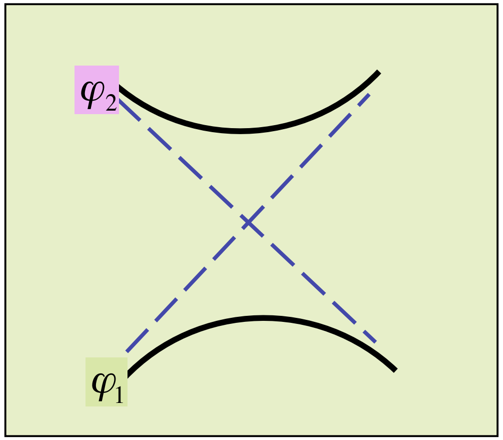
The number of broad states is limited by the number of decay channels

Consequences of non-hermitian eigenvalue problem

- Degeneracies of eigenvalues

Hermitian problems

Level repulsion



Level crossing if $\varphi_1 \neq \varphi_2$

Non-Hermitian problems

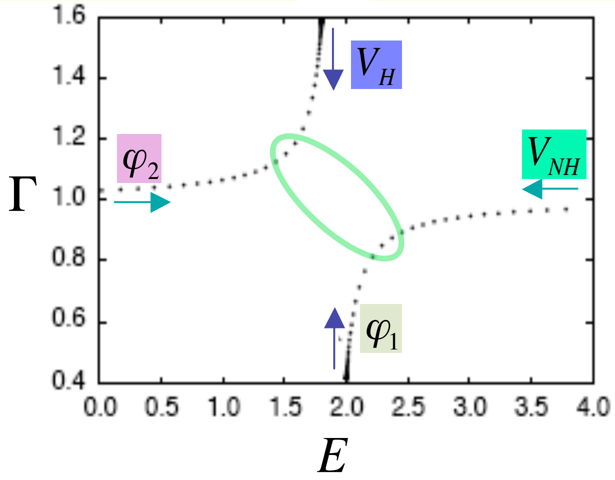
V_H

Level repulsion
Width clustering

vs

V_{NH}

Level clustering
Width repulsion



Exceptional point $\varphi_1 = \varphi_2$

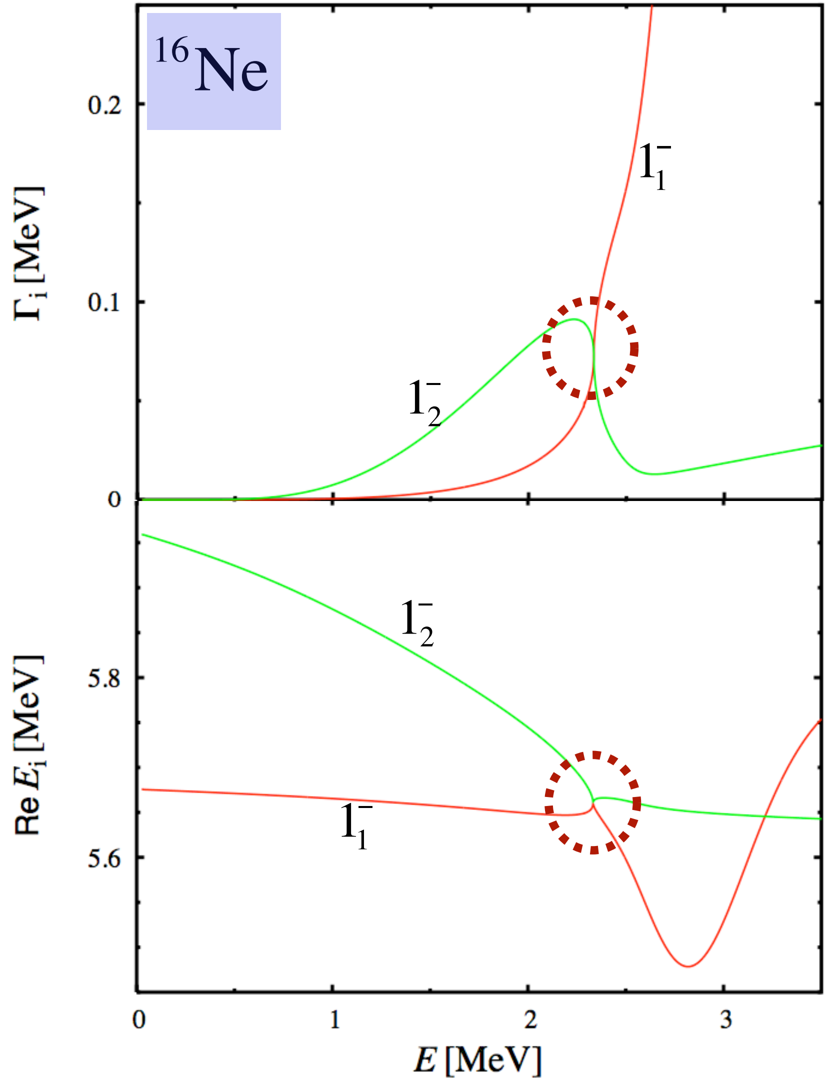
$\varphi_1 = \varphi_1^*$

Exceptional point:

- defect of the vector space: $D_H = n - 1$
- appearance of a topological phase:

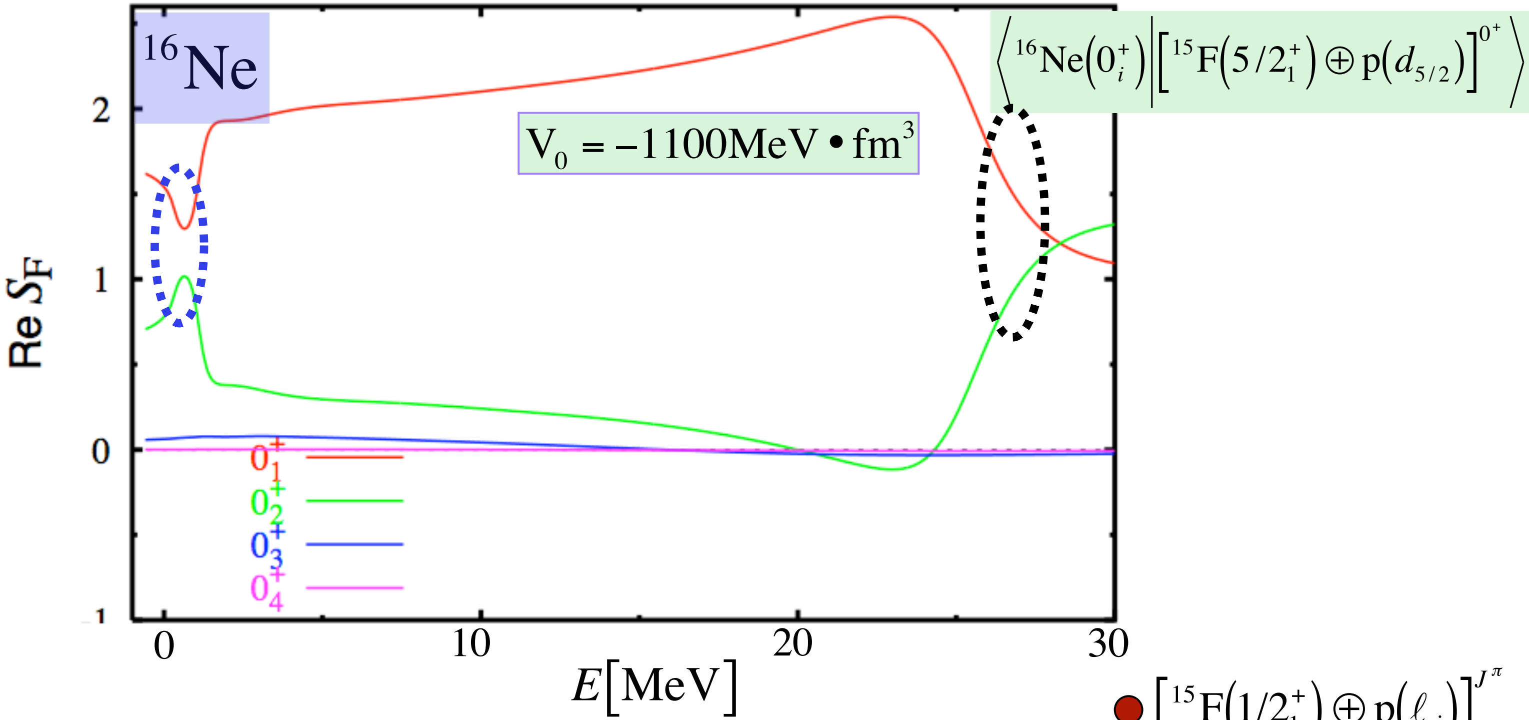
$$\Phi_1 \to i\Phi_2$$
$$\Phi_2 \to -i\Phi_1$$

Exp.:
C. Dembowski et al., PRL 86 (2001) 787
(microwave cavity)
H. Catarius et al., PRL 99 (2007) 173003
(hydrogen atom in external field)



Channels:
$$\left[{}^{15}\mathrm{F}(1/2_1^+) \oplus \mathrm{p}(\ell_j)\right]^{J^\pi}$$
$$\left[{}^{15}\mathrm{F}(5/2_1^+) \oplus \mathrm{p}(\ell_j)\right]^{J^\pi}$$
$$\left[{}^{15}\mathrm{F}(1/2_1^-) \oplus \mathrm{p}(\ell_j)\right]^{J^\pi}$$

ZBM + WB interaction
$V_0 = -1617.4\,\mathrm{MeV} \bullet \mathrm{fm}^3$



$\left[{}^{15}\mathrm{F}(1/2_1^+) \oplus \mathrm{p}(\ell_j) \right]^{J^\pi}$

$\left[{}^{15}\mathrm{F}(5/2_1^+) \oplus \mathrm{p}(\ell_j) \right]^{J^\pi}$

$\left[{}^{15}\mathrm{F}(1/2_1^-) \oplus \mathrm{p}(\ell_j) \right]^{J^\pi}$

ZBM + WB interaction

Effects of EPs also in other observables: EM transitions, multipole (quadrupole) moments, reaction cross-sections,...

Conclusions

The new paradigm: shell-model treatment of open channels

Two formulations: complex-energy shell model in rigged Hilbert space with resonant continuum, and real-energy continuum shell model in Hilbert space with non-resonant continuum

The non-resonant continuum is important for the spectroscopy of weakly bound nuclei (energy shifts and degeneracies, modification of effective NN correlations (SFs), additional binding, manifestation of clustering, exceptional points, segregation of time scales...)

New exotic phenomena in weakly bound/unbound nuclei:

- continuum anti-odd-even staggering effect
- modification of 'magic numbers' and spin-orbit splitting
- halos and correlations, continuum induced anti-halo effect
- correlated continuum in reactions with multiple weakly bound/unstable subsystems
- influence of the poles of S-matrix on spectra and wave functions
- new kinds of natural radioactivity (e.g. 2p radioactivity, etc.)
- .....