FEW-BODY CALCULATIONS AND THEIR APPLICATION TO DIRECT NUCLEAR REACTIONS

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1. FOUR-NUCLEON REACTIONS

The 4N scattering problem gives rise to the simplest set of nuclear reactions that shows the complexity of heavier systems

$$\left\{\begin{array}{l} n^3\mathrm{H} \rightarrow n^3\mathrm{H} \\ \qquad\qquad\qquad \text{dominated by isospin } \mathcal{T}=1 \\ p^3\mathrm{He} \rightarrow p^3\mathrm{He} \end{array}\right.$$

$$\left\{\begin{array}{ll} dd \rightarrow dd & \text{dominated by isospin } \mathcal{T}=0 \\ \quad\ \rightarrow n^3\mathrm{He} & \multirow{2}{*}{$\Big\}\ \mathcal{T}=0+\mathcal{T}=1$} \\ \quad\ \rightarrow p^3\mathrm{H} & \end{array}\right.$$

$$\left\{\begin{array}{ll} n^3\mathrm{He} \rightarrow n^3\mathrm{He} & \\ \qquad\quad\ \rightarrow p^3\mathrm{H} & \text{mixed isospin } \mathcal{T}=0 \text{ and } \mathcal{T}=1 \\ \qquad\quad\ \rightarrow dd & \end{array}\right.$$

The Equations

We solve the Alt, Grassberger and Sandhas (AGS) equations for transition operators (same as Yakubovsky eq. for the wave function components).

In the symmetrized form they involve two coupled equations $(1 \equiv 3+1 \qquad 2 \equiv 2+2)$

$$\mathcal{U}_{(R)}^{11} = -(G_0\, t^{(R)} G_0)^{-1}\, P_{34} - P_{34} U^{(R)}\, G_0\, t^{(R)} G_0\, \mathcal{U}_{(R)}^{11} + \tilde{U}^{(R)}\, G_0\, t^{(R)} G_0\, \mathcal{U}_{(R)}^{21}$$

$$\mathcal{U}_{(R)}^{21} = (G_0\, t^{(R)}\, G_0)^{-1}\;(1 - P_{34}) + (1 - P_{34}) U^{(R)}\, G_0\, t^{(R)}\, G_0\, \mathcal{U}_{(R)}^{11}$$

for $1+3 \longrightarrow 1+3$ and $1+3 \longrightarrow 2+2$.

Likewise

$$\mathcal{U}^{12}_{(R)} = (G_0\, t^{(R)} G_0)^{-1} - P_{34}\, U^{(R)}\, G_0\, t^{(R)} G_0\, \mathcal{U}^{12}_{(R)} + \tilde{U}^{(R)} G_0\, t^{(R)} G_0\, \mathcal{U}^{22}_{(R)},$$

$$\mathcal{U}^{22}_{(R)} = (1 - P_{34}) U^{(R)}\, G_0\, t^{(R)} G_0\, \mathcal{U}^{12}_{(R)}$$

for $2 + 2 \longrightarrow 2 + 2$ and $2 + 2 \longrightarrow 1 + 3$.

$$U^{(R)} = P\, G_0^{-1} + P\, t^{(R)}\, G_0\, U^{(R)} \qquad (1 + 3)$$

$$\tilde{U}^{(R)} = \tilde{P}\, G_0^{-1} + \tilde{P}\, t^{(R)}\, G_0\, \tilde{U}^{(R)} \qquad (2 + 2)$$

$$P = P_{12}\, P_{23} + P_{13}\, P_{23}$$

$$\tilde{P} = P_{13}\, P_{24}$$

are permutation operators.

R is the screening radius for the pp screened Coulomb potential.

Defining the initial/final (1+3) and (2+2) states

$$|\chi_1^{(R)}> = G_0\, t^{(R)}\, P|\chi_1^{(R)}>$$

$$|\chi_2^{(R)}> = G_0\, t^{(R)}\, \tilde{P}|\chi_2^{(R)}>$$

The transition amplitudes are (α, $\beta = 1, 2$)

$$\langle \vec{p}_f\, |T^{\alpha\beta}_{(R)}|\, \vec{p}_i\rangle = S_{\alpha\beta}\langle \chi_\alpha^{(R)}(\vec{p}_f)|\mathcal{U}^{\alpha\beta}_{(R)}|\chi_\beta^{(R)}(p_i)\rangle$$

$$\mathrm{S}_{11} = 3;\ \mathrm{S}_{21} = \sqrt{3},\ \mathrm{S}_{22} = 2,\ \mathrm{S}_{12} = 2\sqrt{3}$$

The R $\rightarrow \infty$ limit is taken in the following way

$$\langle \vec{p}_f\, |T^{\beta\alpha}|\, \vec{p}_i\rangle = \delta_{\beta\alpha}\, \langle \vec{p}_f\, |T^{\mathrm{c.m.}}_{\alpha C}|\, \vec{p}_i\rangle + \lim_{R\rightarrow\infty} \times \left\{ \mathcal{Z}^{-\frac{1}{2}}_{\beta R}(p_f)\langle \vec{p}_f\, |[T^{\beta\alpha}_{(R)} - \delta_{\beta\alpha}\, T^{\mathrm{c.m.}}_{\alpha R}]|\, \vec{p}_i\rangle \mathcal{Z}^{-\frac{1}{2}}_{\alpha R}(p_i) \right\}$$

with $[\mathcal{Z}^{\alpha}_{R}]^{-\frac{1}{2}}$ and $[\mathcal{Z}^{\beta}_{R}]^{-\frac{1}{2}}$ being the renormalization factors.

“Complexity Digest”

These are three-variable integral equations:

– Triple partial wave expansion;

– Triple discretization of Jacobi momenta;

– Gaussian integration;

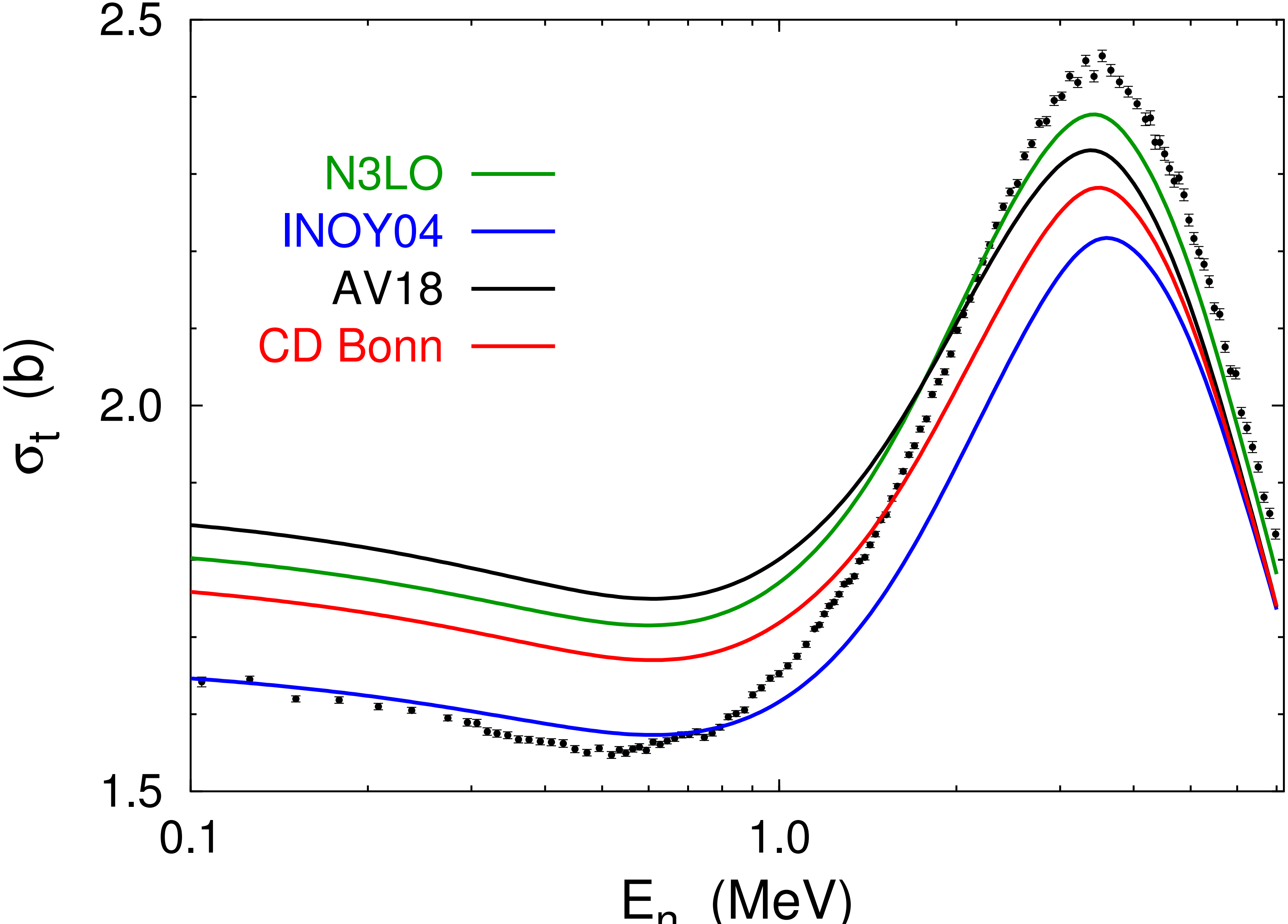
– Spline interpolation;

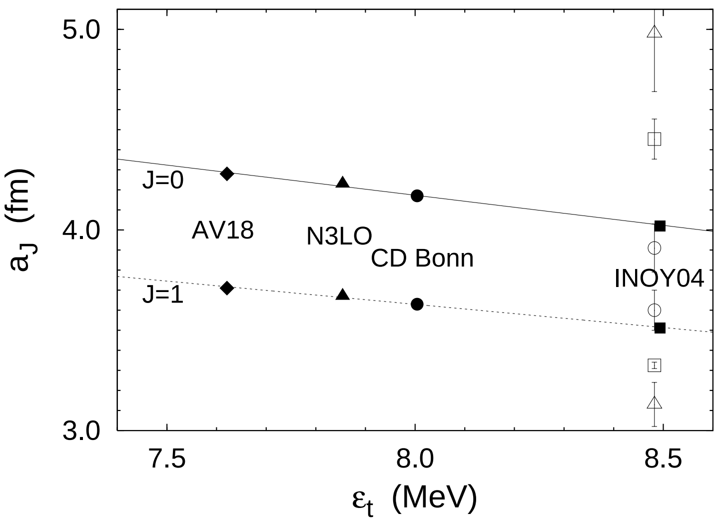
– Include up to 15000 partial waves (combined 2N, 3N, 4N);

– System of $> 10^8$ linear equations (size of the kernel $\approx 10^8$ GB);

– Summing up the Neumann series by Padé method.

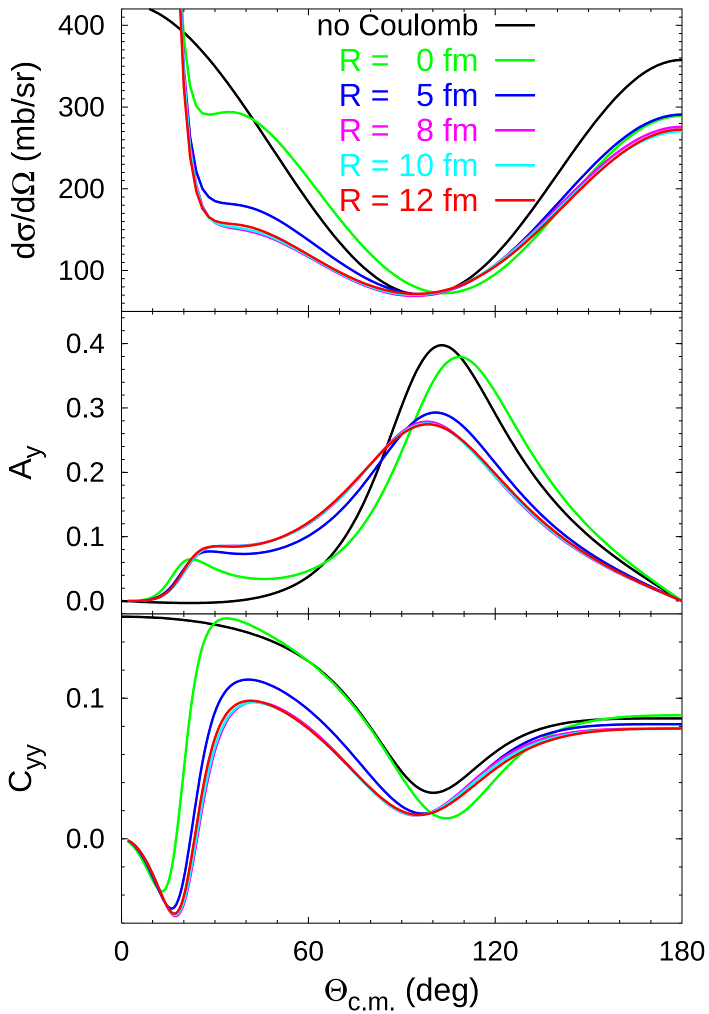
n-^{3}H total cross section



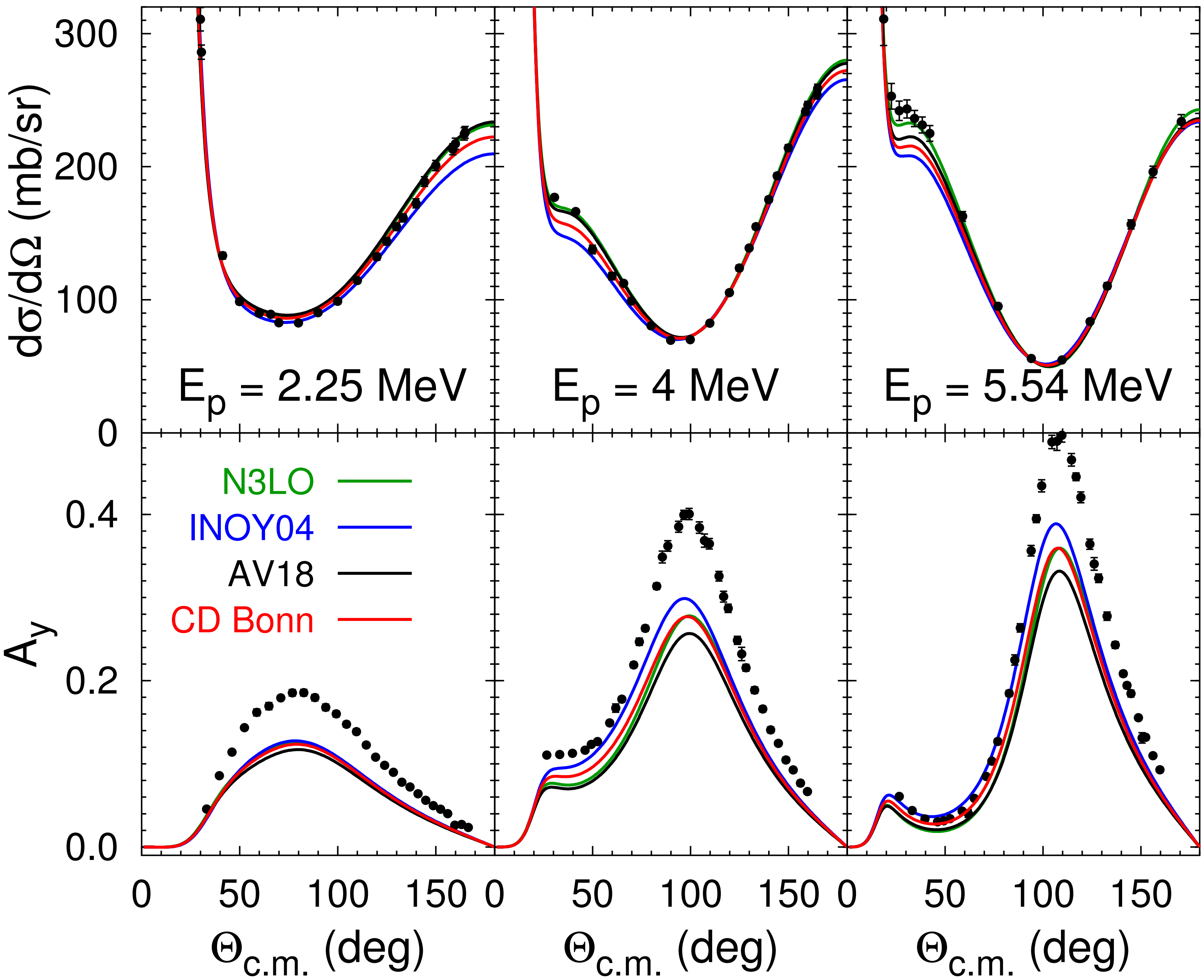


	ε_t	ε_α	a_0	a_1	σ_t (0)	σ_t (3.5)
AV18	**7.621**	**24.24**	**4.28**	**3.71**	**1.88**	**2.33**
Nijmegen II	**7.653**	**24.50**	**4.27**	**3.71**	**1.87**	**2.31**
Nijmegen I	**7.734**	**24.94**	**4.25**	**3.69**	**1.85**	**2.30**
N3LO	**7.854**	**25.38**	**4.23**	**3.67**	**1.83**	**2.38**
CD Bonn	**7.998**	**26.11**	**4.17**	**3.63**	**1.79**	**2.28**
INOY04	**8.493**	**29.11**	**4.02**	**3.51**	**1.67**	**2.22**

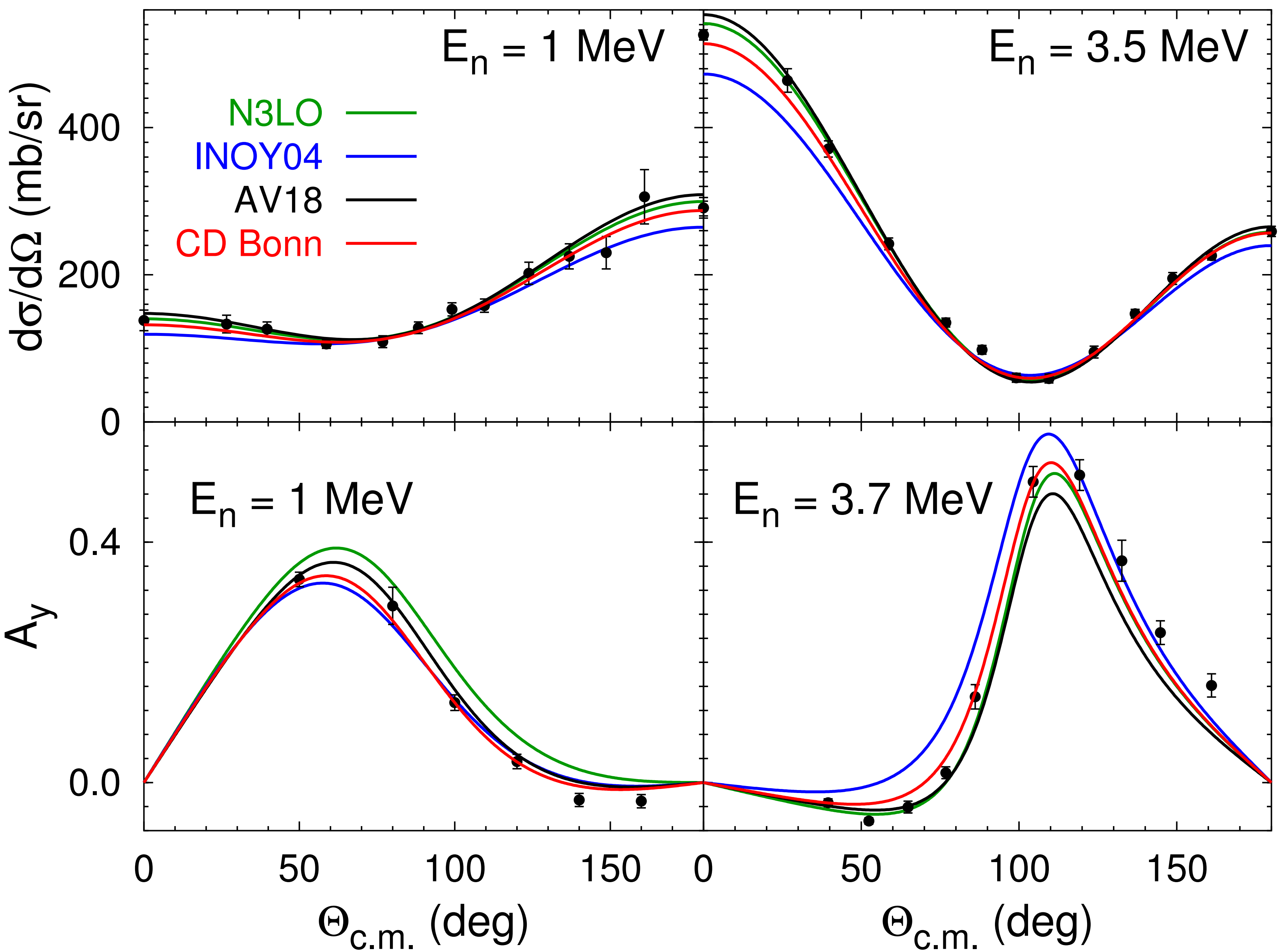
p-^{3}He OBSERVABLES at E_p = 4 MeV



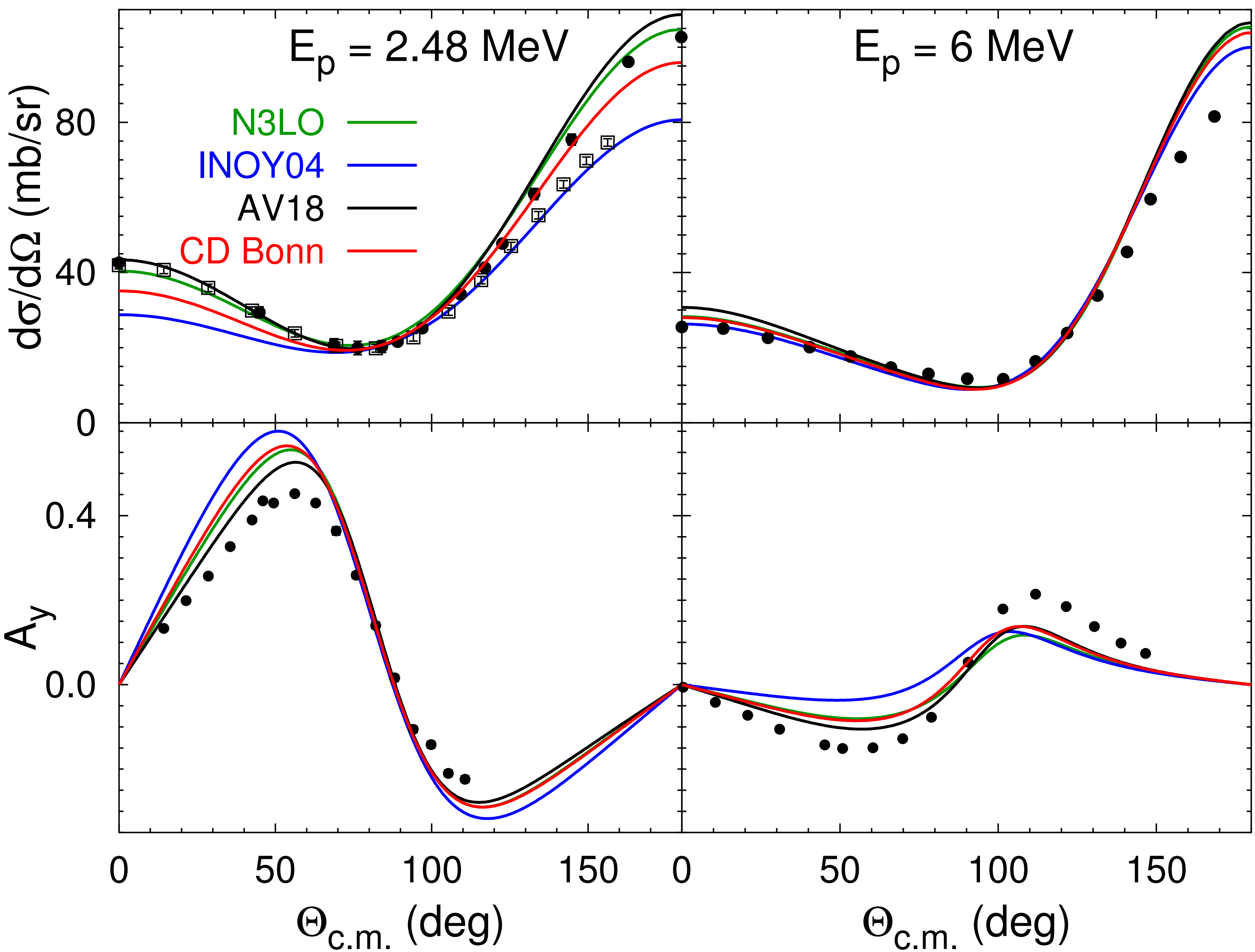
p-^{3}He scattering

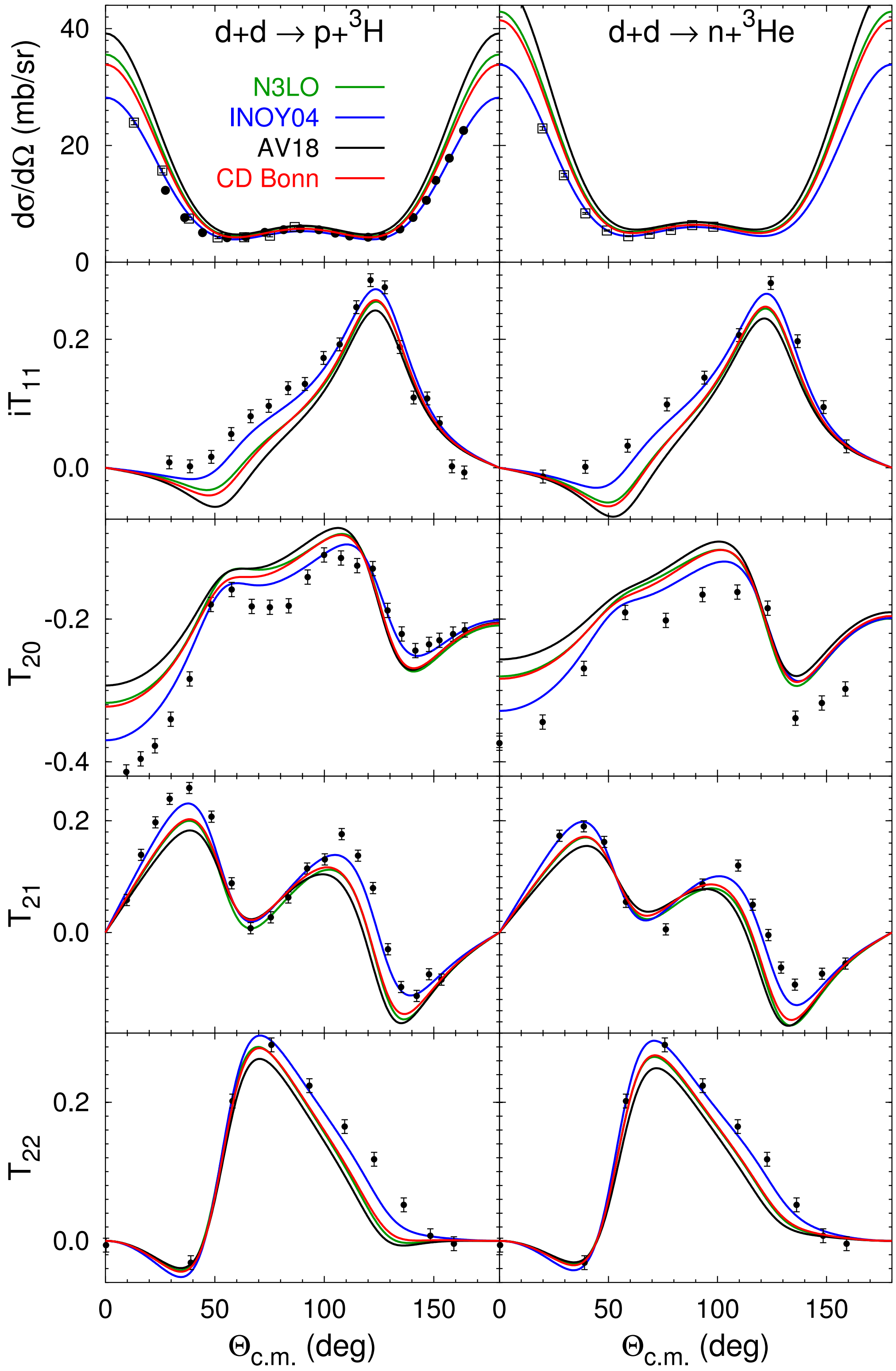


n-^{3}He scattering



$p + {}^3\text{H} \rightarrow n + {}^3\text{He}$ **transfer**





$d + d \rightarrow p + {}^3\mathrm{H}$

and

$d + d \rightarrow n + {}^3\mathrm{He}$

transfer

2. THE FOUR-NUCLEON SYSTEM WITH Δ-ISOBAR EXCITATION

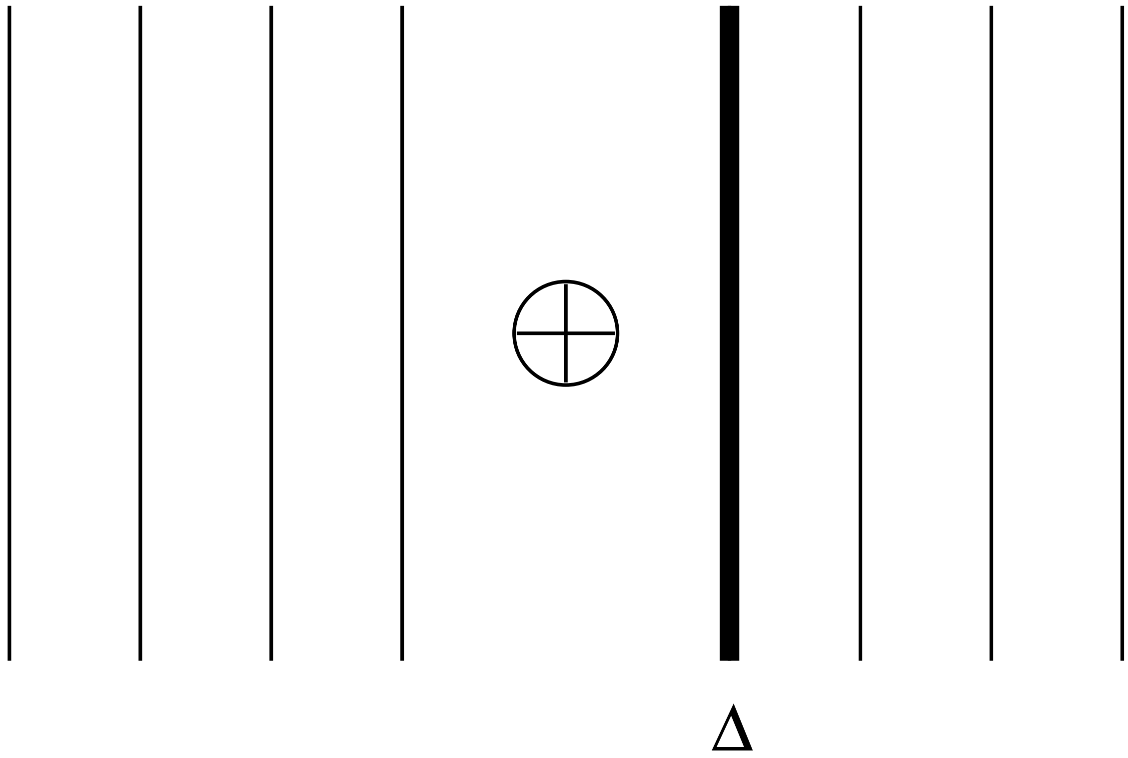
(with Peter Sauer)

- We use the CD-Bonn + Δ to generate effective three- and four-nucleon forces in the 4N system.

- CD-Bonn + Δ is a charge dependent realistic NN interaction based on meson exchange (π, ρ, ω, σ) and fitted to the NN data up to π production threshold with χ^2/datum $\simeq 1$.

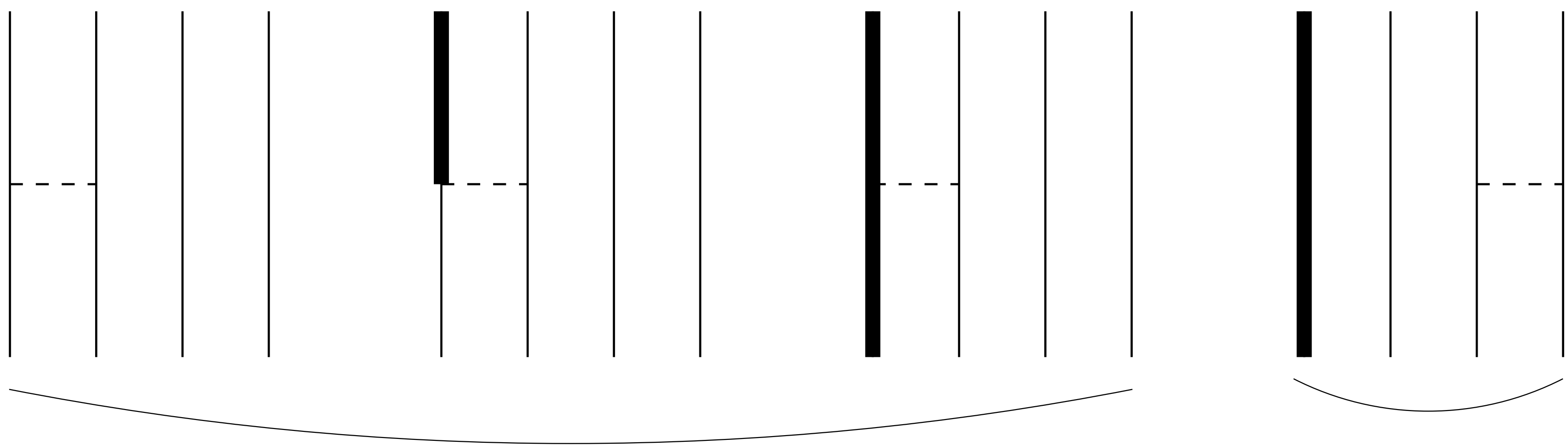
- Two-, three- and four-nucleon forces are consistent with each other.

Hilbert space



only single Δ excitation:
relation to π production

Hamiltonian



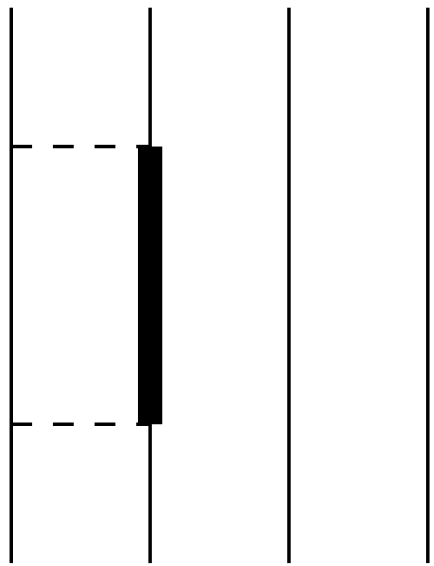
CD Bonn + Δ

χ^2/datum = 1.02 for NN scattering

= 0

undetermined
by NN data

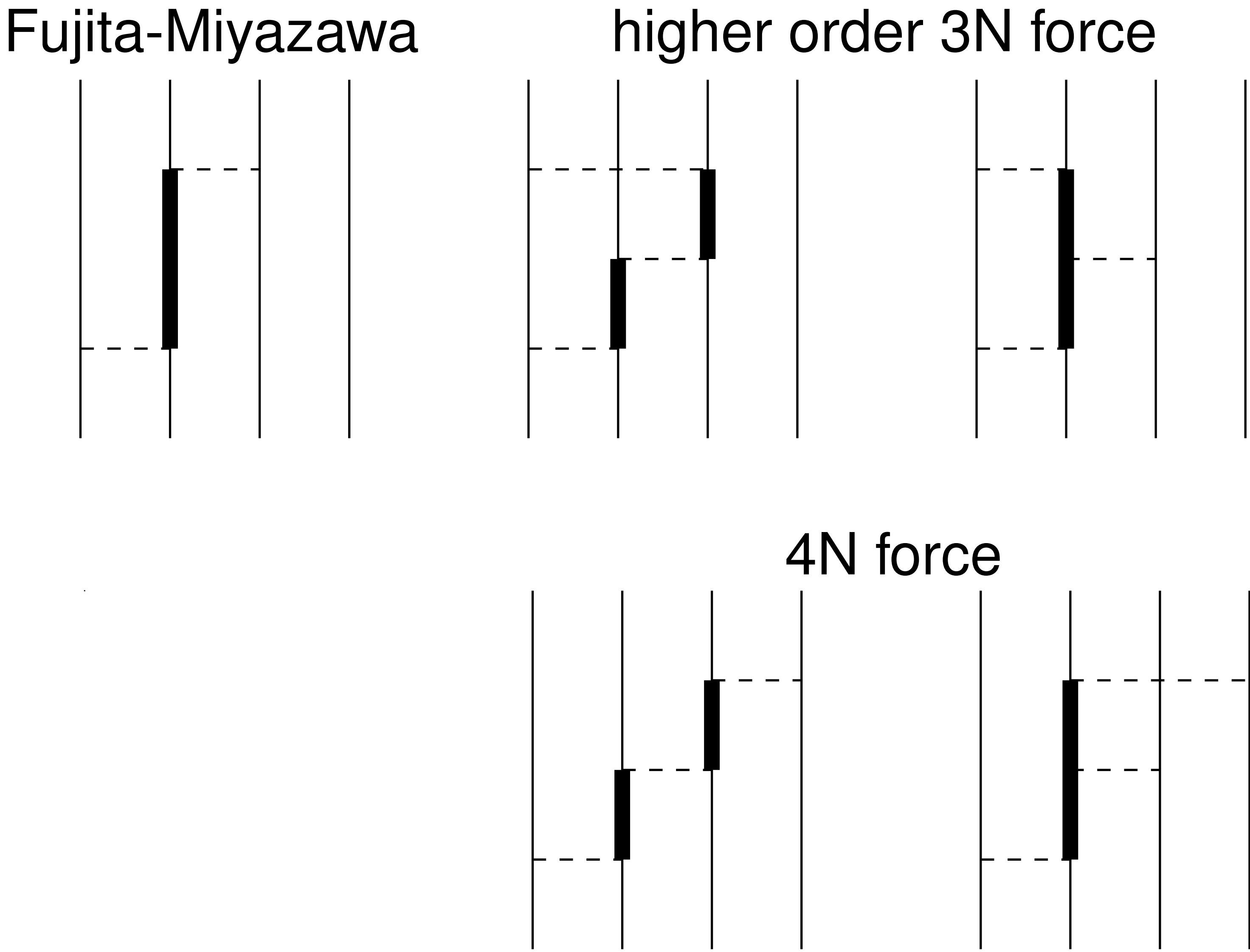
2N dispersive effect



less attractive 2N force in 4N system compared to

3N system: one more nucleon propagated

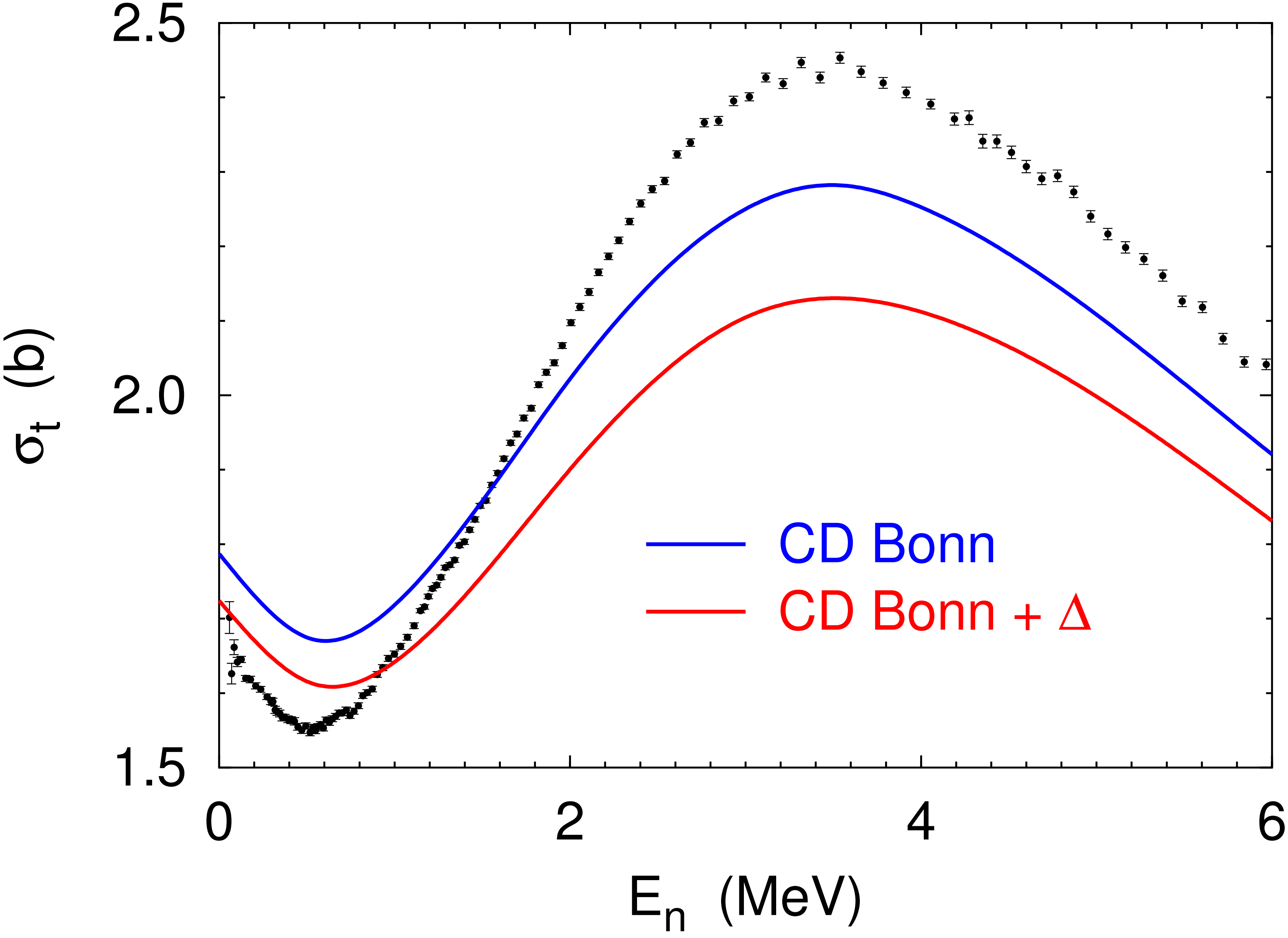
Effective 3N and 4N forces



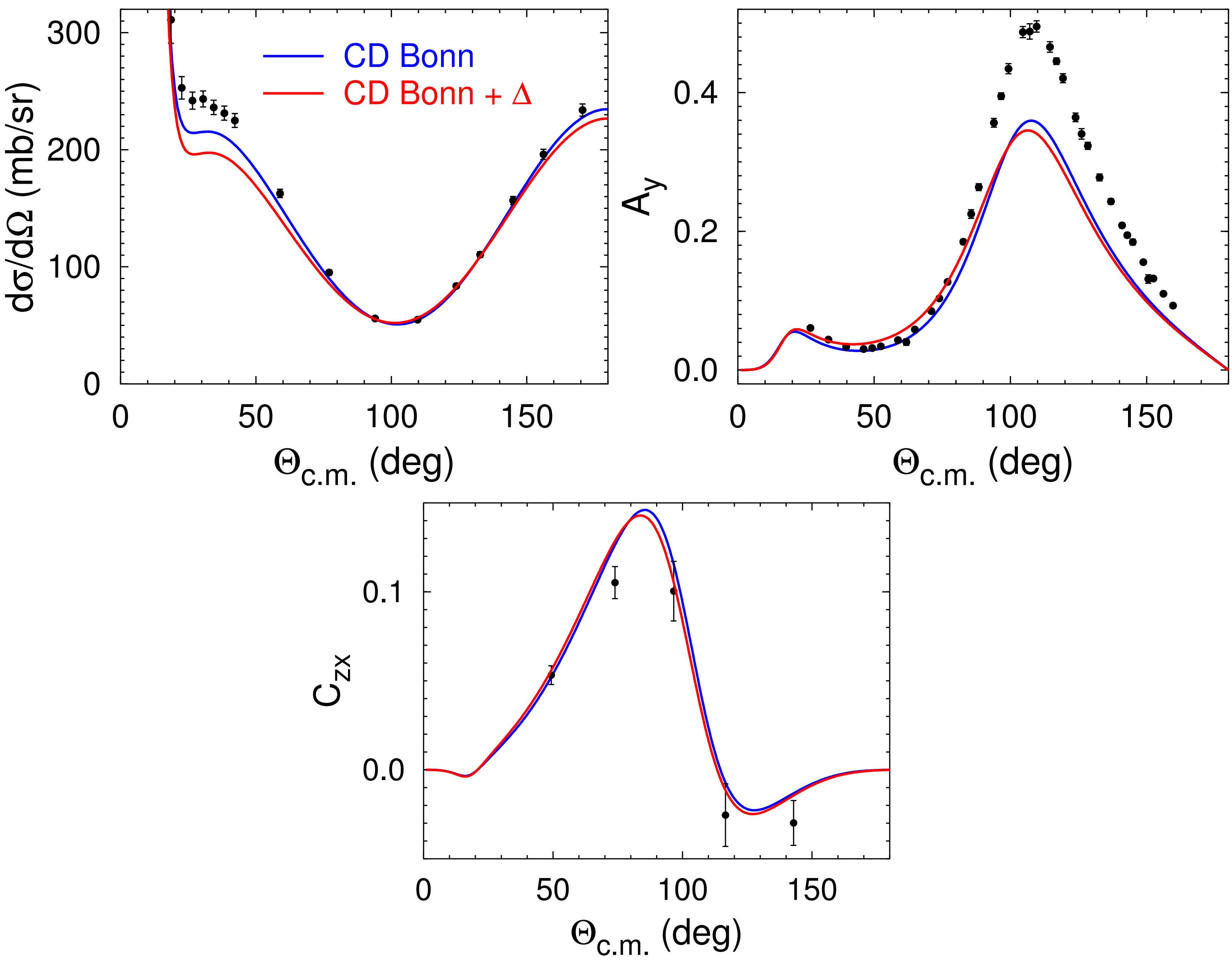
3N and 4N binding energies

	^3H	^3He	^4He
CD Bonn	8.00	7.26	26.18
CD Bonn + Δ	8.28	7.54	27.10
exp	8.48	7.72	28.30
ΔE_2	-0.51	-0.48	-2.80
ΔE_3(FM)	0.50	0.48	2.25
ΔE_3(h.o.)	0.29	0.28	1.30
ΔE_4			0.17

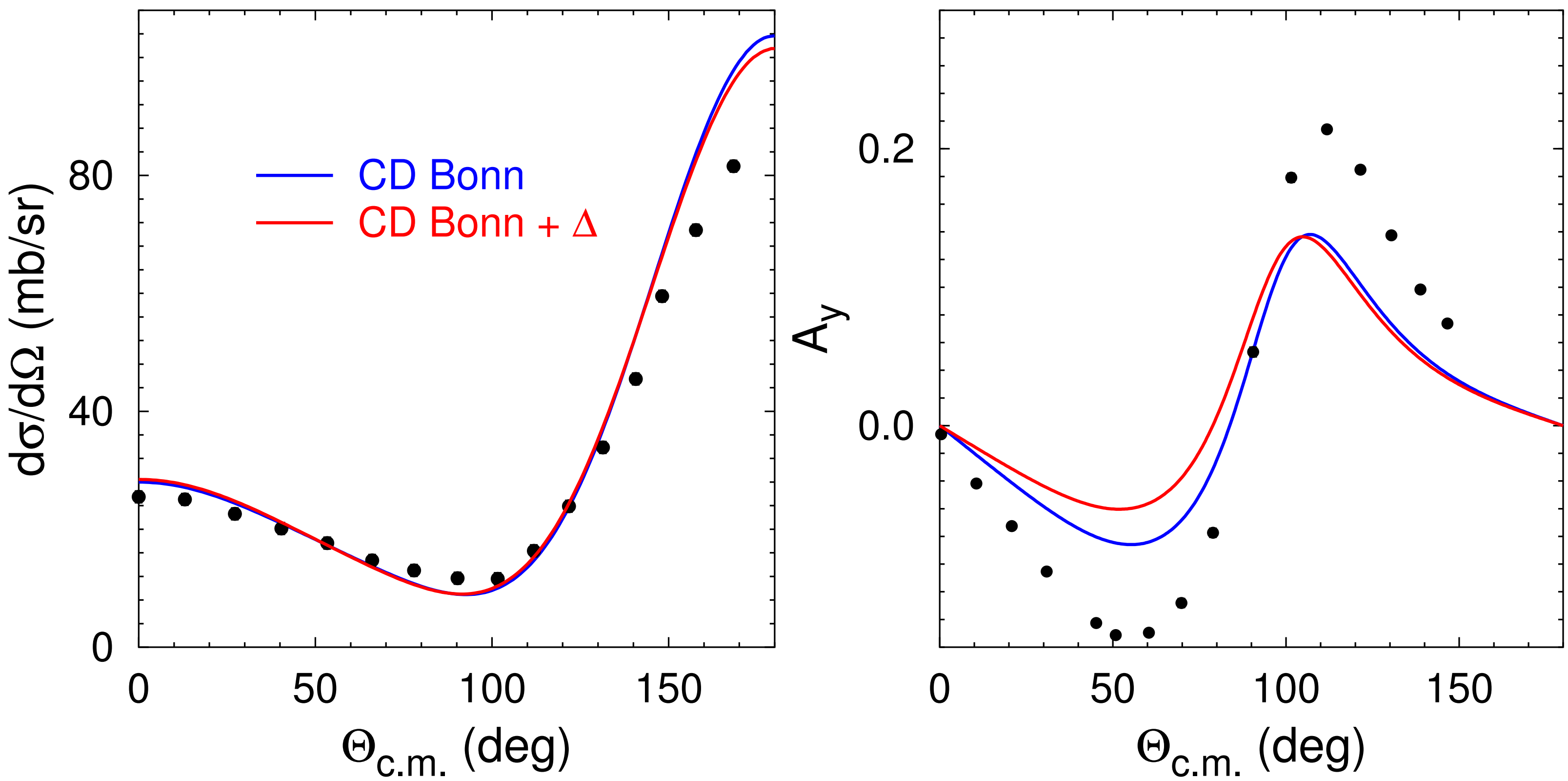
n-^{3}H total cross section



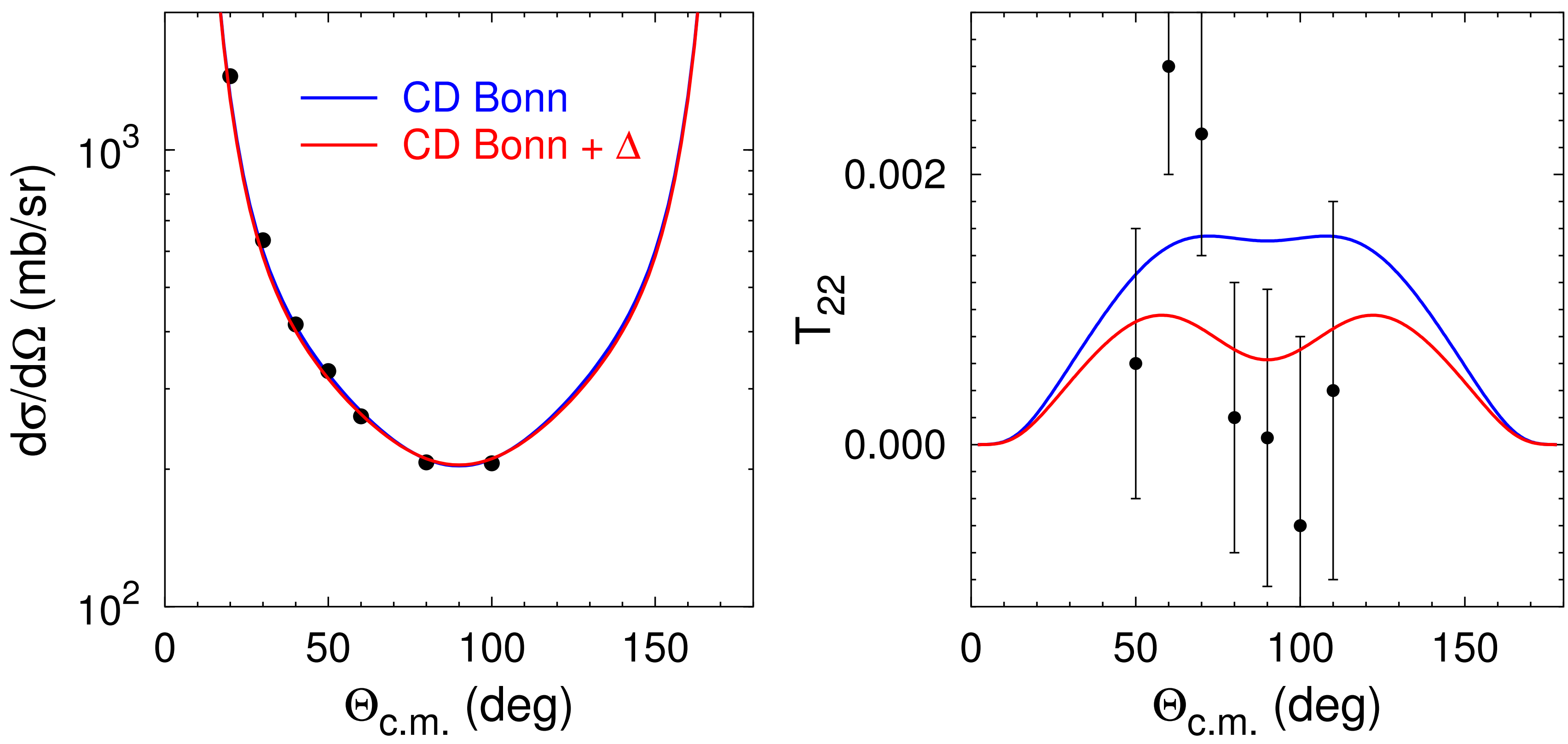
p-^{3}He scattering at $E_p = 5.54$ MeV



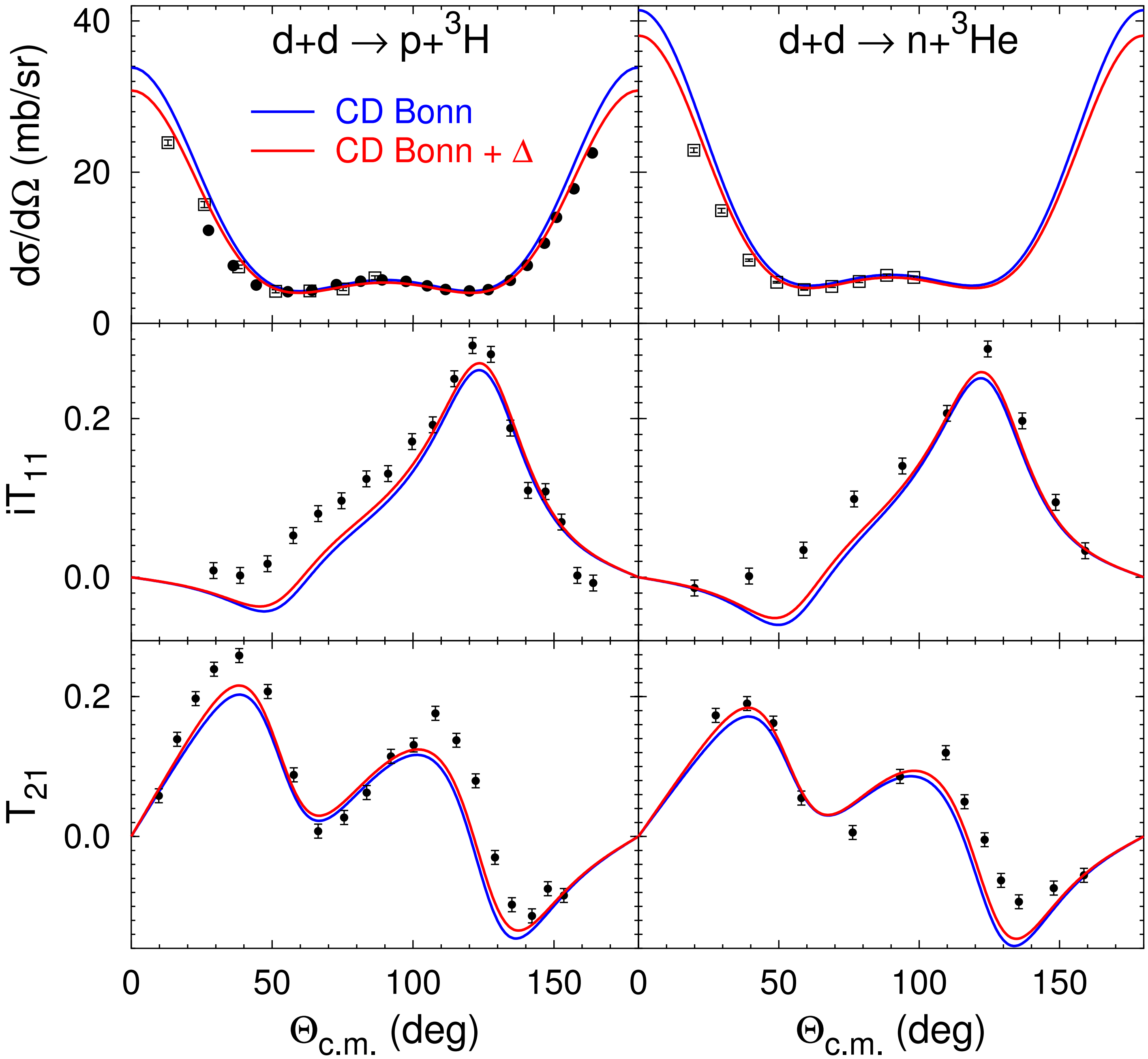
$p + {}^3\mathrm{H} \rightarrow n + {}^3\mathrm{He}$ transfer at $E_p = 6$ MeV



d-d elastic scattering at $E_d = 3$ MeV



$d + d \to N + [3N]$ transfer at $E_d = 3$ MeV



CONCLUSIONS

- Ab initio 4N calculations with the inclusion of Coulomb are now as reliable and accurate as 3N calculations without Coulomb.

- Presently known NN force models badly fail to reproduce σ_t in $n - {}^3$H scattering, and 3N forces may not bring a cure. Further investigations are needed.

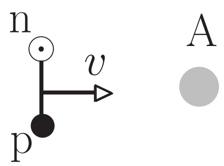
- We have a 4N A_y problem in $p - {}^3$He that seems to be absent in $n - {}^3$He and $p - {}^3$H.

- Reactions driven by $d - d$ look surprisingly good, particularly if the NN interaction reproduces ^{3}H and ^{3}He binding.

- 4N forces included for the first time EVER.

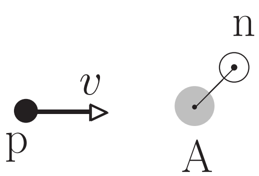
- 4N bound state: Δ-mediated 2N, 3N and 4N forces are insufficient for 4N binding. Inclusion of additional irreducible 3N force is necessary.
- 4N scattering: besides scaling, Δ-mediated 2N and 3N forces increase the discrepancy in N+[3N] total cross section σ_t.
- 4N force effects in 4N system much weaker than 3N force effects.

3. THREE-BODY (LIKE) NUCLEAR REACTIONS



$A = {}^{12}\mathrm{C},\ {}^{16}\mathrm{O}$

Inert Cores



Elastic

Inelastic

Transfer

Charge Exchange

Breakup

GOALS:

a) Use an exact three-body approach (Faddeev/AGS).

b) Choose a given Hamiltonean/Interaction Model.

c) Obtain fully converged results for the observables.

3.1 The Interactions for Model 1 (M1)

$d + A \to$ anything	$n - A$	optical potential $\left(\frac{1}{2} E_d\right)$
	$p - A$	optical potential $\left(\frac{1}{2} E_d\right)$
	$n - p$	CD-Bonn

$p + (An) \to$ anything	$n - A$	real interaction that fits single particle states of (An) nucleus
	$p - A$	optical potential (E_p)
	$n - p$	CD-Bonn

PROBLEMS: The particles are the same but the Hamiltoneans are different.

The Reactions in M1

$$d + A \longrightarrow d + A$$

$n + p + A$

~~$p + (An)$~~ because $n - A$ is complex

~~$n + (Ap)$~~ because $p - A$ is complex

$$p + (An) \longrightarrow p + (An)$$

$d + A$

$n + p + A$

~~$n + (Ap)$~~ because $p - A$ is complex

PROBLEMS: The reaction and its inverse are not related by detailed balance

3.2 The Interactions for Model 2 (M2)

$d + A \longrightarrow$ anything	$n - A$	Energy dependent optical potential
	$p - A$	Energy dependent optical potential
	$n - p$	CD-Bonn

$p + (An) \longrightarrow$ anything	$n - A$	Energy dependent optical potential
	$p - A$	Energy dependent optical potential
	$n - p$	CD-Bonn

PROBLEMS: Non orthogonality.

HOW TO IMPLEMENT:

$$e_{ij} = \mathrm{E} - p_k^2/2M_{k,ij}$$

a) e_{ij} = c.m. pair energy; E = total c.m. three-body energy.

b) The parameters of the optical potential change with e_{ij}.

$$V_R = v_R + 0.4ZA^{-1/3} \pm 27.0(N-Z)/A - 0.3e_{\mathrm{c.m.}}\Theta(e_{\mathrm{c.m.}})$$

$e_{ij} > 0$ complex potential

$e_{ij} < 0$ real potential

supports a number of bound states for (An) and (Ap)

Pauli forbidden states are removed (projected out).

TABLE OF BOUND STATES

	$1s_{1/2}$	$2s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$	$1d_{5/2}$
^{13}C	38.022*	1.857	18.722*	4.946	1.092
^{13}N	33.864*		15.957*	1.944	
^{17}O	37.213*	3.272	19.267*	16.067*	4.143
^{17}F	32.559*	0.105	15.561*	12.348*	0.600

*** projected out**

TABLE OF STRENGTH PARAMETERS

	$v_R(nA)$	$v_R(pA)$	$V_{so}(nA)$	$V_{so}(pA)$
Watson *et al.*	60.00	60.00	5.5	5.5
N-^{12}C (s)	67.50	66.47		
N-^{12}C (p)	61.67	61.50	20.38	20.83
N-^{12}C (d)	66.42	66.42	5.5	5.5
N-^{16}O (s)	61.65	60.94		
N-^{16}O (d)	61.47	60.89	5.4	5.4

The Reactions in M2

$$d + A \longrightarrow d + A$$

$$n + p + A$$

$$p + (An)$$

$$n + (Ap)$$

$$p + (An) \longrightarrow p + (An)$$

$$d + A$$

$$n + p + A$$

$$n + (Ap)$$

The reaction and its inverse are related by detailed balance

3.3 The Interactions for Model 3 (M3)

"Hybrid Model"

$d + A \rightarrow$ anything

$n - A$, $p - A$:

- From M2 in partial waves where (An) and (Ap) have bound states.
- From M1 in all other partial waves.

$p + (An) \rightarrow$ anything

$p - A$:

- From M2 in partial waves where (Ap) have bound states.
- From M1 in all other partial waves.

$n - A$: From M1 in all partial waves.

SOME CONCERNS:

a) **Limited Non orthogonality.**

b) **Partial wave dependent potentials.**

The Reactions in M3

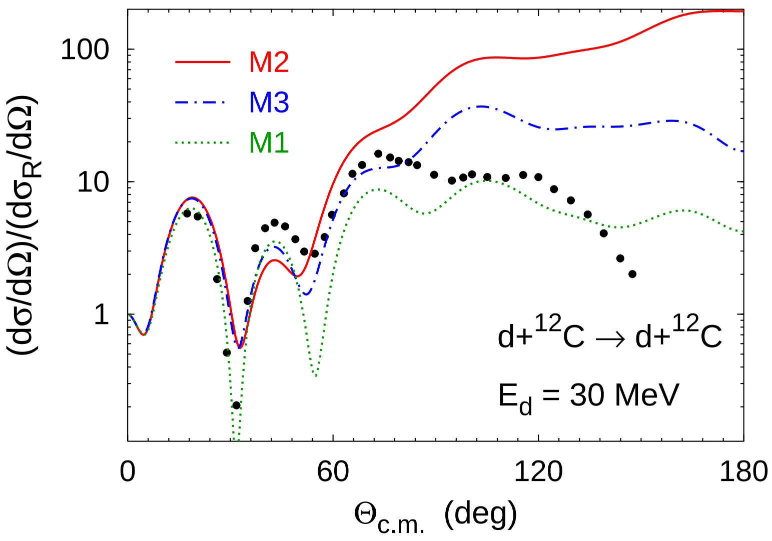
$$
\begin{aligned}
d + A \longrightarrow\; & d + A \\
& n + p + A \\
& p + (An) \\
& n + (Ap)
\end{aligned}
$$

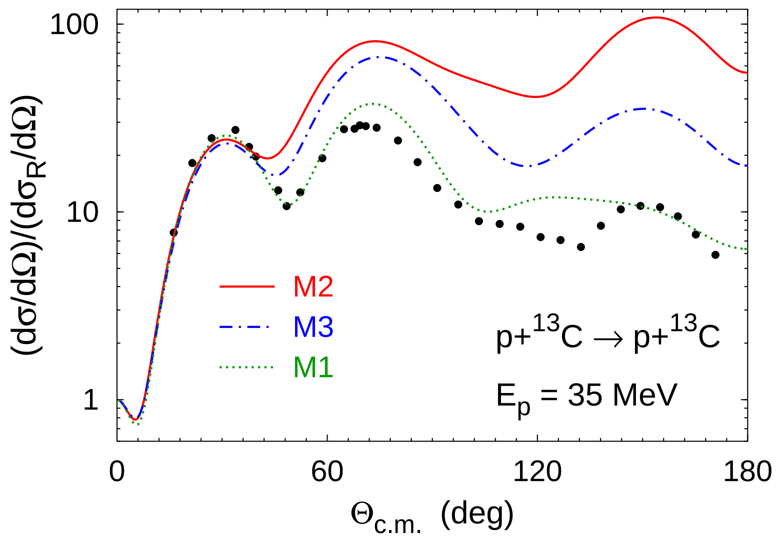
$$
\begin{aligned}
p + (An) \longrightarrow\; & p + (An) \\
& d + A \\
& n + p + A \\
& n + (Ap)
\end{aligned}
$$

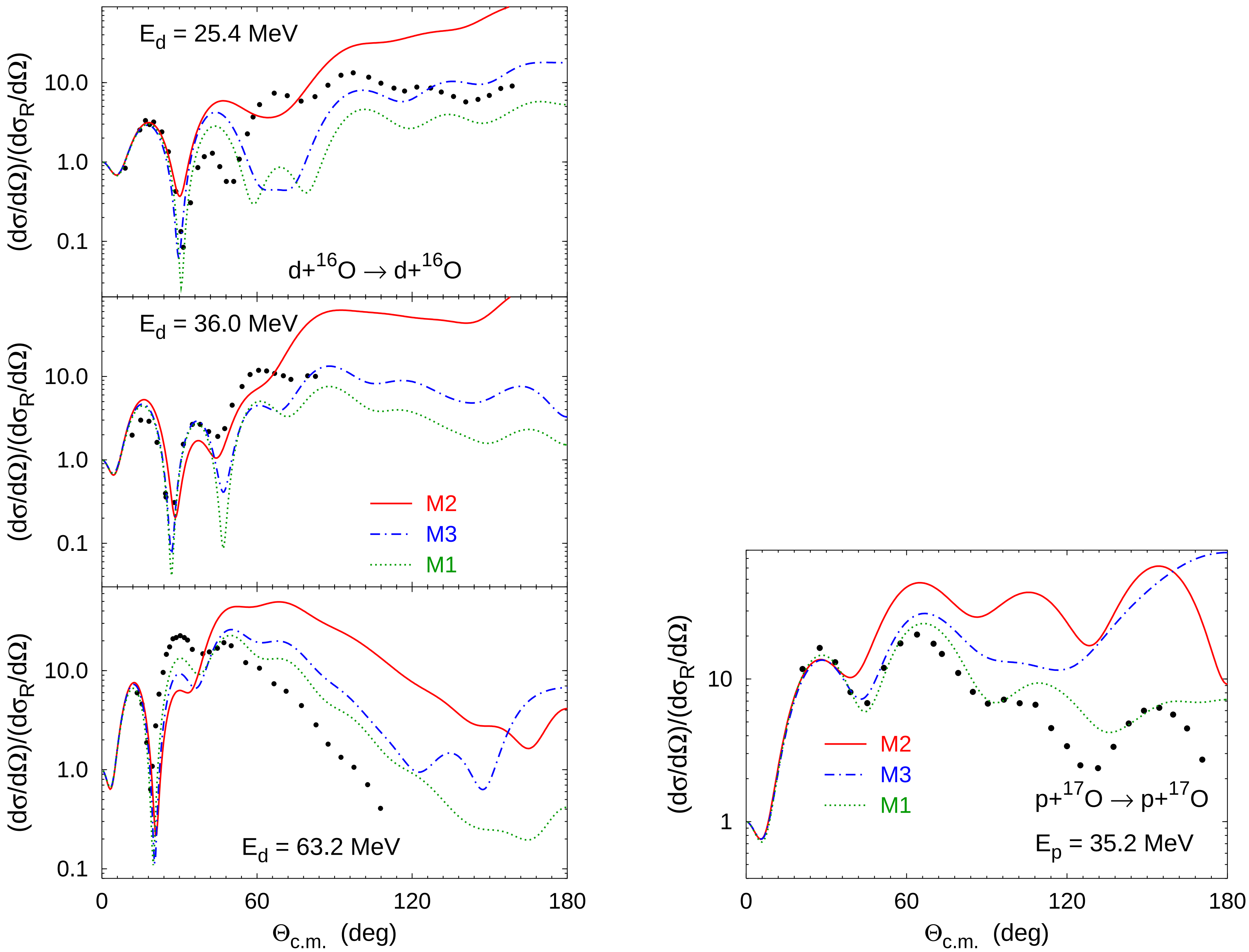
The reaction and its inverse are “almost” related by detailed balance

3.4 Results M1, M2, M3

ELASTIC SCATTERING

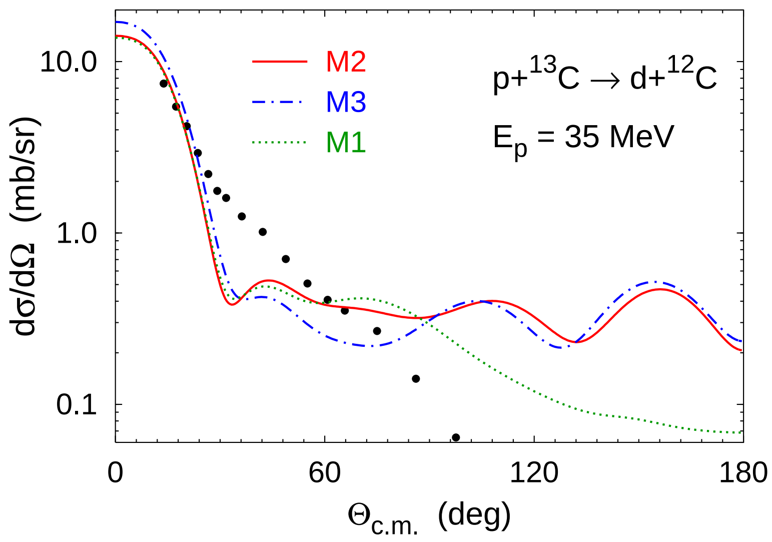


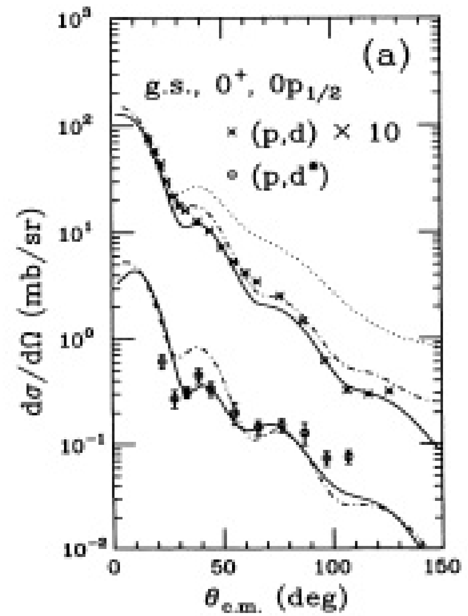




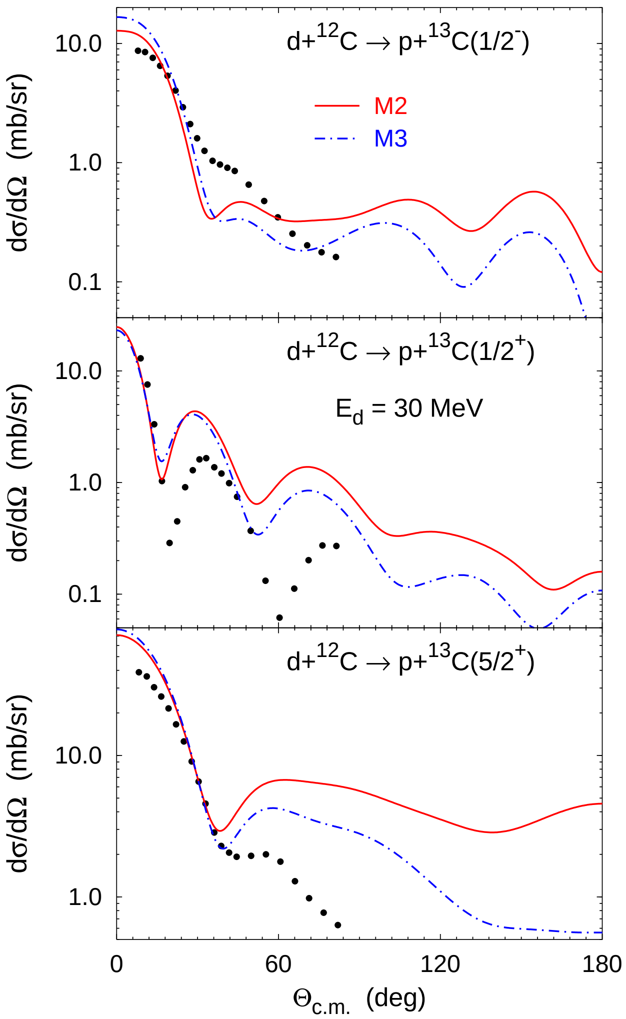
E_d = 25.4 MeV
d+16O → d+16O
E_d = 36.0 MeV
E_d = 63.2 MeV
M2
M3
M1
(dσ/dΩ)/(dσ_R/dΩ)
10.0
1.0
0.1
0
60
120
180
Θ_c.m. (deg)
p+17O → p+17O
E_p = 35.2 MeV
10
1

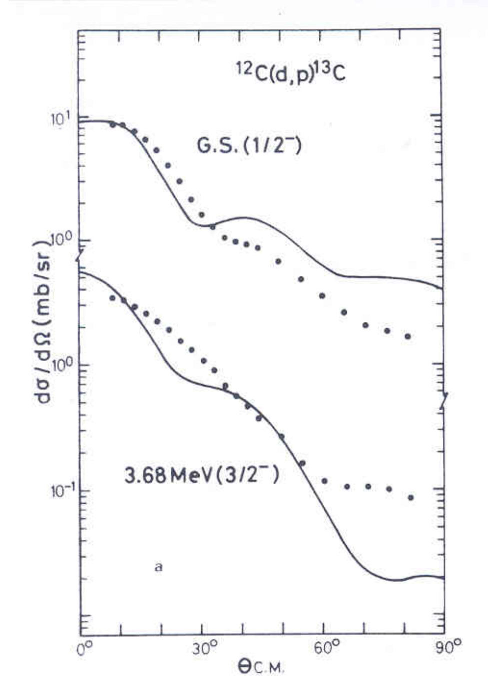
$p + {}^{13}\mathrm{C} \rightarrow d + {}^{12}\mathrm{C}$ transfer at $E_p = 35$ MeV

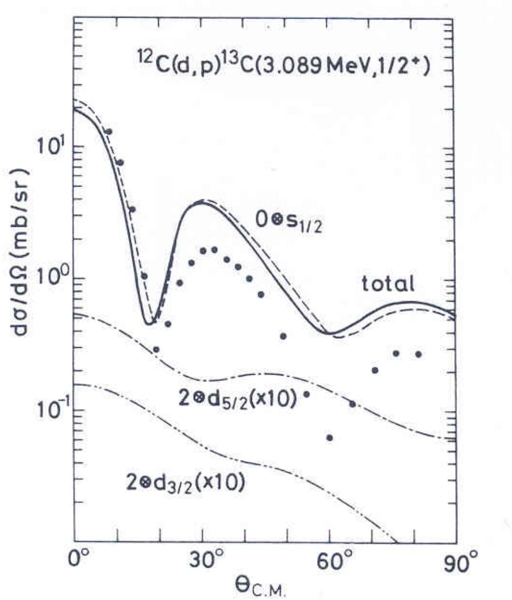


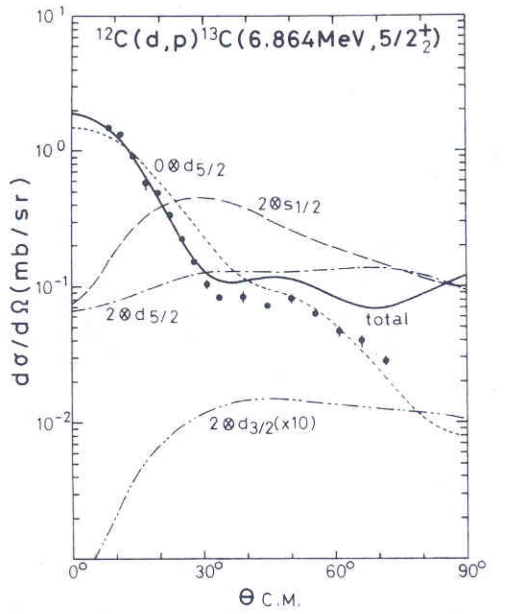


$d + {}^{12}\mathrm{C} \rightarrow p + {}^{13}\mathrm{C}$ transfer at $E_d = 30$ MeV

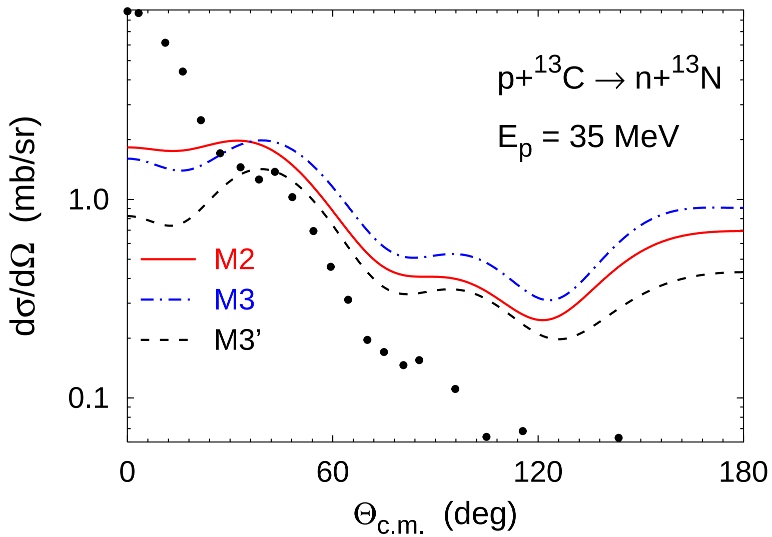


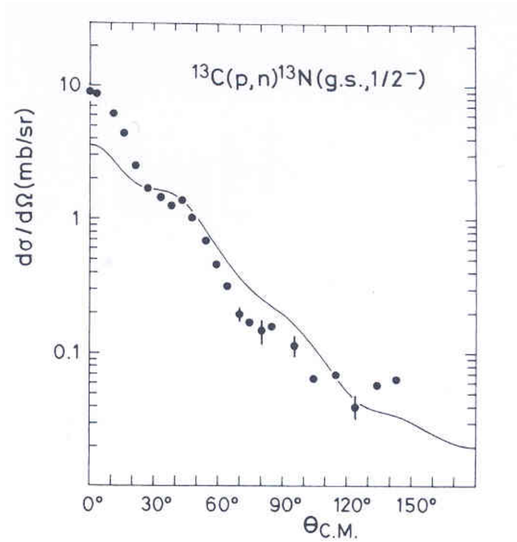




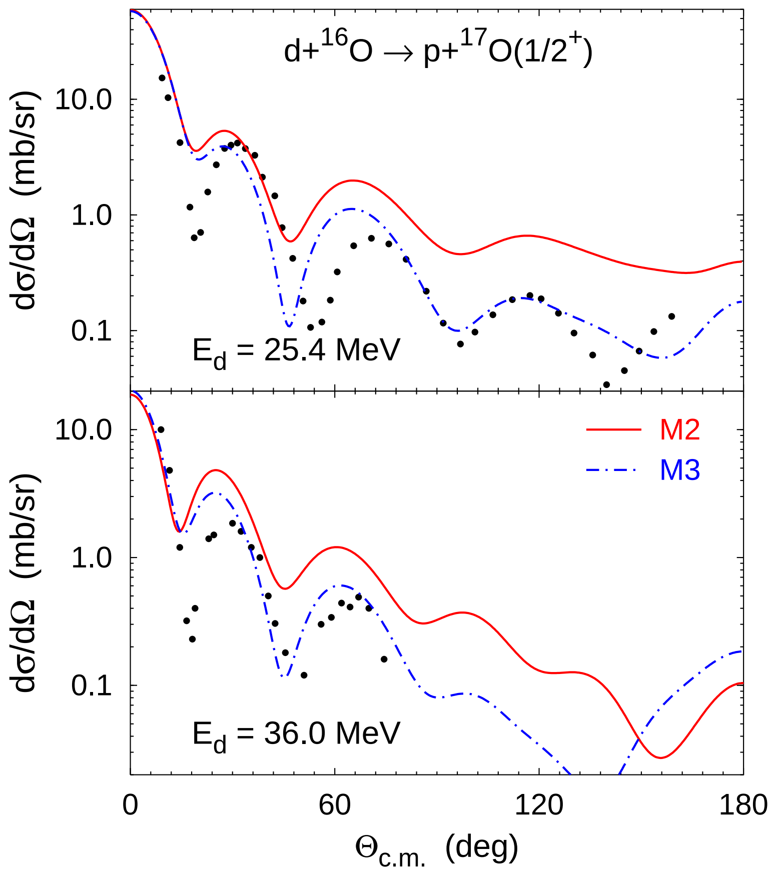


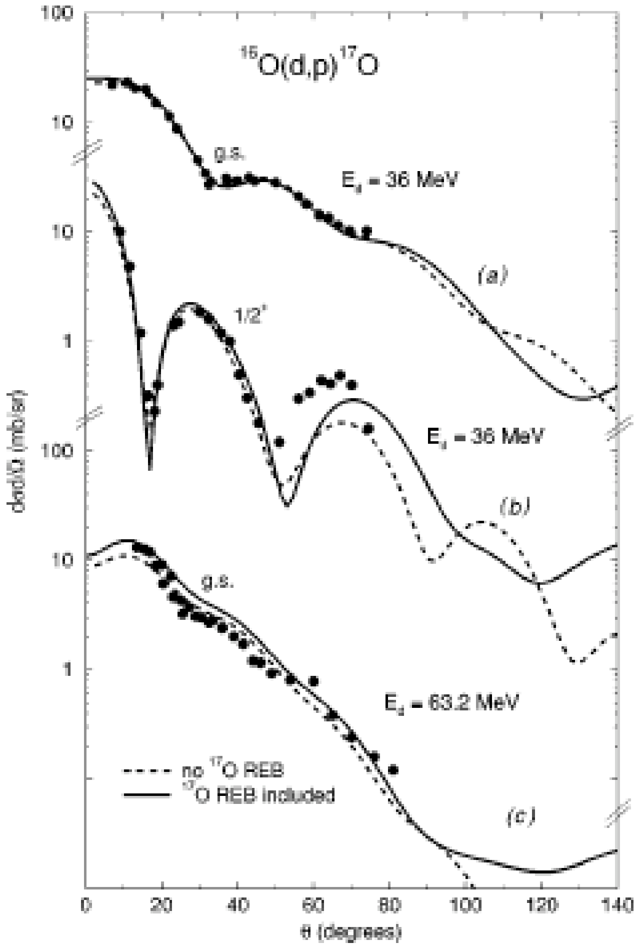
$p + {}^{13}\mathrm{C} \to n + {}^{13}\mathrm{N}$ charge exchange at $E_p = 35$ MeV



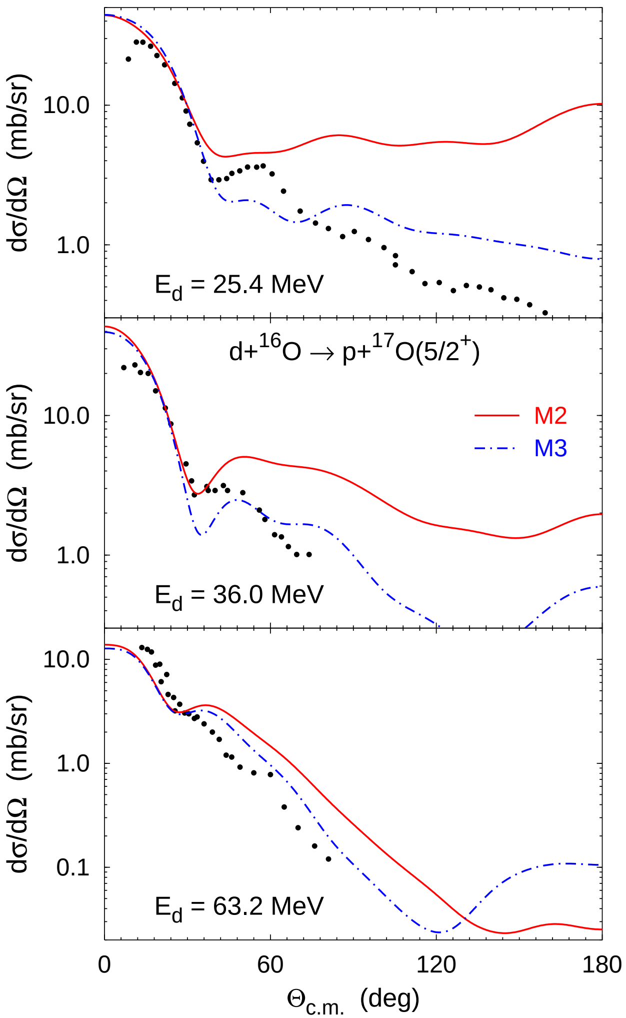


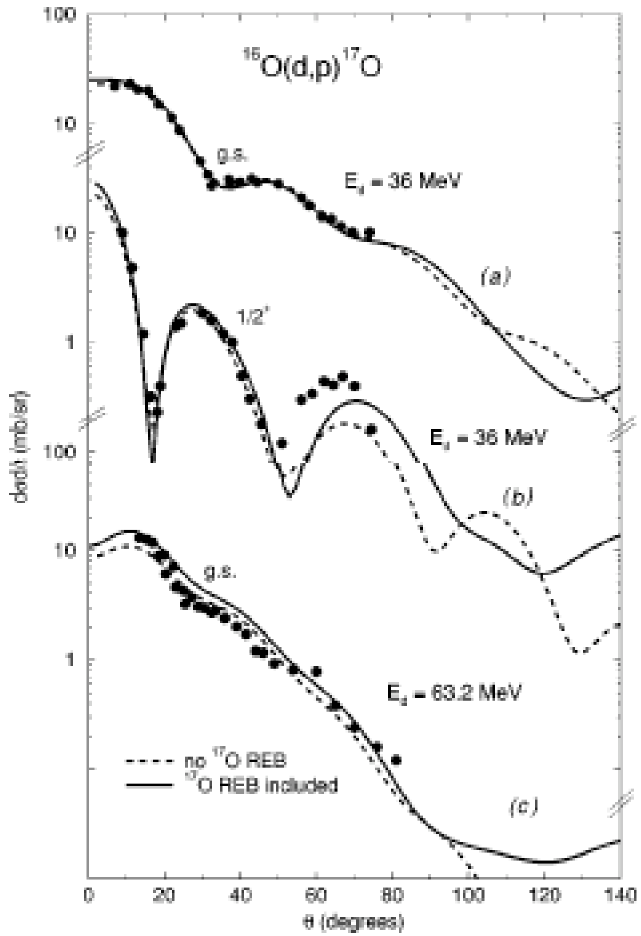
$d + {}^{16}\mathrm{O} \rightarrow p + {}^{17}\mathrm{O}$ transfer at $E_d = 25.4$ and 36.0 MeV



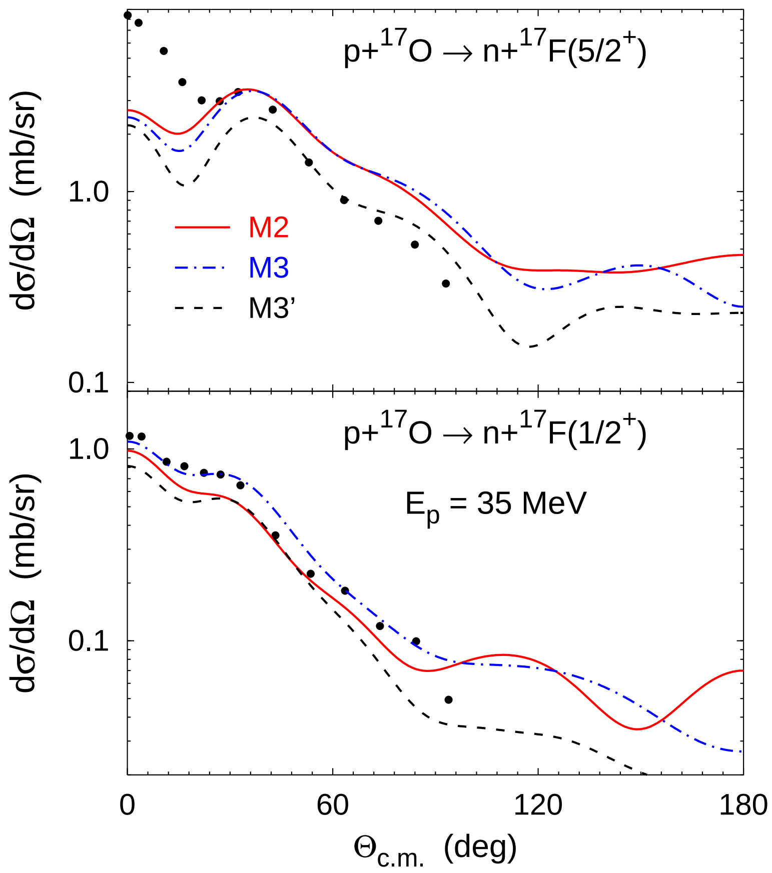


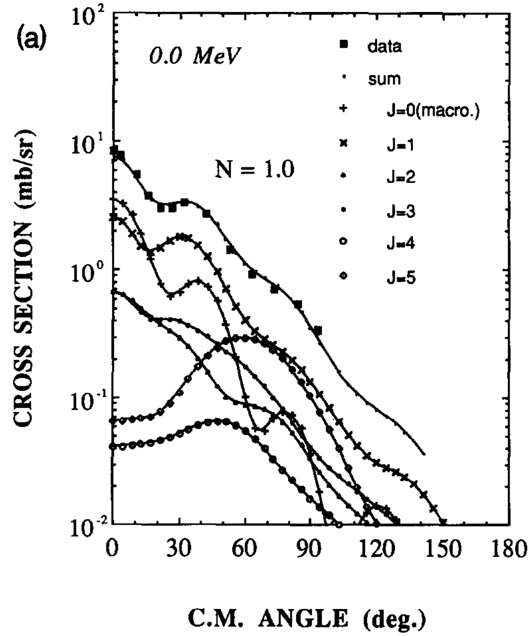
$d + {}^{16}\mathrm{O} \to p + {}^{17}\mathrm{O}$ transfer at $E_d = 25.4$, 36.0, and 63.2 MeV

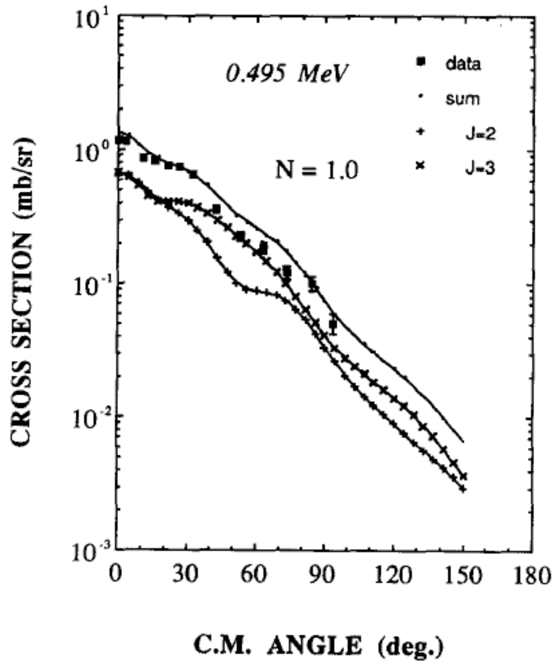




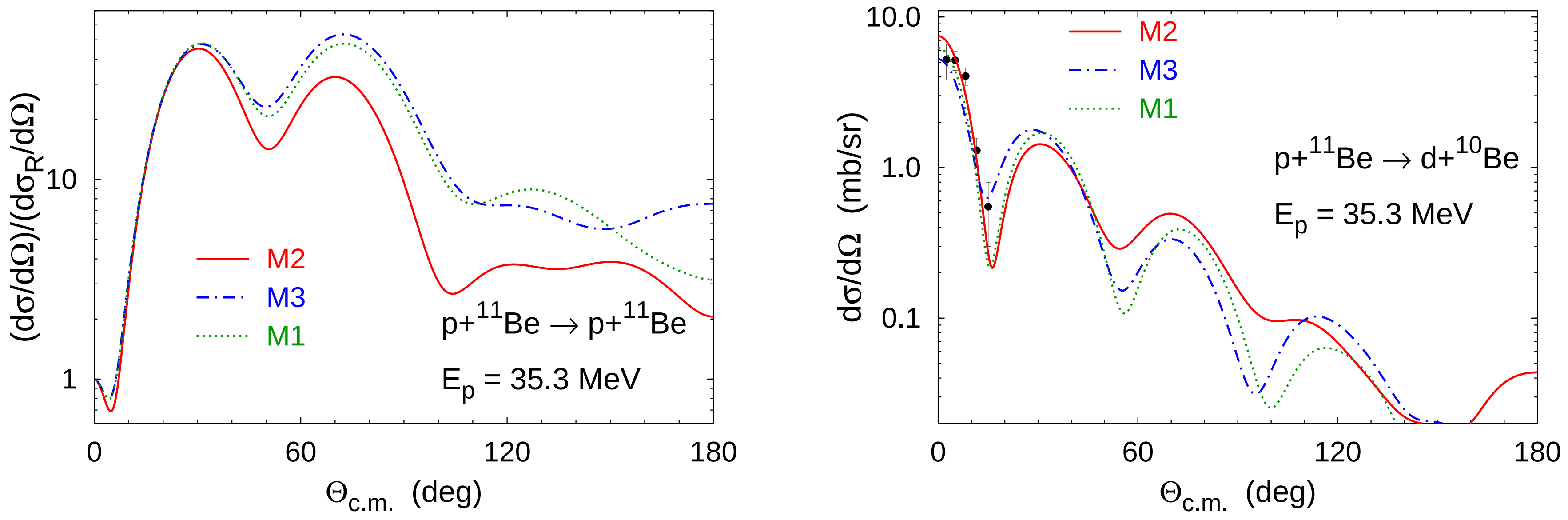
$p + {}^{17}\text{O} \rightarrow n + {}^{17}\text{F}$ charge exchange at $E_p = 35$ MeV



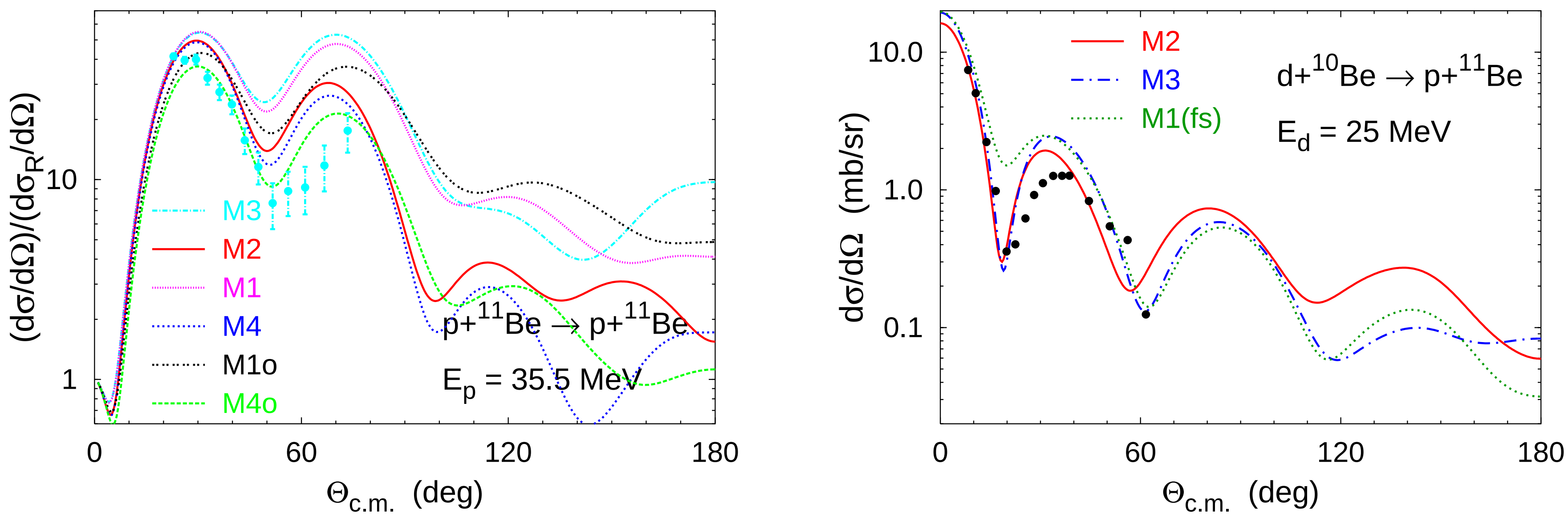




$p + {}^{11}\mathrm{Be}$ reactions



$p + {}^{11}$Be and $d + {}^{10}$Be reactions



3.5 CONCLUSIONS

1. Elastic results are strongly dependent on the chosen model for the interactions.

2. The model dependence is a result of greater diffraction in M2 and M3 due to the potential becoming real at negative energies and a decreased imaginary potential at positive energies.

3. Transfer and Charge Exchange reactions at small angles are not sensitive to how well elastic reactions are described over the whole angular region.

4. Comparison with DWBA, CCBA and adiabatic calculations indicate that, qualitatively, our results are similar, but the fitting to data less perfect given that our calculations have less

parameters to play with.

5. Non orthogonally does not seem to play a major role.