Dynamical eikonal approximation of breakup reactions

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Daniel Baye

Université Libre de Bruxelles, Belgium

with Pierre Capel and Gerald Goldstein

Dynamical eikonal approximation

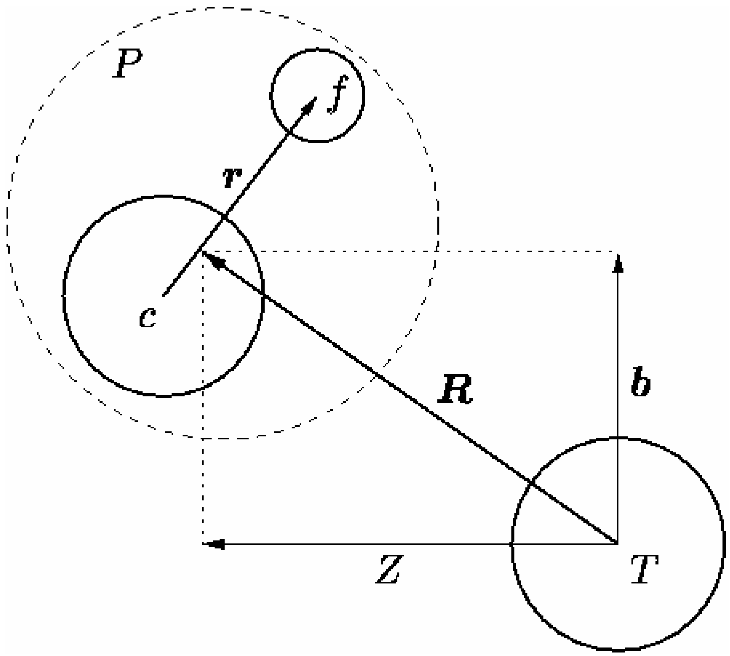
- Quantal method
- Makes use of numerical resolution of time-dependent Schrödinger equation
- Partly as usual eikonal approximation, but without adiabatic approximation
- Differential cross sections with interference effects
- Inspired by inelastic scattering in atomic physics

  F.W. Byron Jr., Phys. Rev. A 4 (1971) 1907

D. Baye, P. Capel, G. Goldstein, Phys. Rev. Lett. 95 (2005) 082502
G.Goldstein, D. Baye, P. Capel, Phys. Rev. C 73 (2006) 024602

Three-body Schrödinger equation

Target T — $m_T,\ Z_T$
Core c of projectile — $m_c,\ Z_c$
Fragment f of projectile — $m_f,\ Z_f$



Hamiltonian of projectile P

$$H_0 = \frac{p^2}{2\mu_{cf}} + V_{cf}(\boldsymbol{r})$$

Three-body equation in Jacobi coordinates

$$\left[\frac{P^2}{2\mu} + H_0 + V_{cT}(r_{cT}) + V_{fT}(r_{fT})\right]\Psi(\boldsymbol{R},\boldsymbol{r}) = E_T\Psi(\boldsymbol{R},\boldsymbol{r})$$

Core-target and fragment-target optical + Coulomb potentials

Principle of dynamical eikonal approximation

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})$$

$$\left(\frac{P^2}{2\mu} + vP_Z + H_0 - E_0 + V_{cT} + V_{fT}\right)\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}) = 0$$

Eikonal approximation: neglect $P^2\widehat{\Psi}/2\mu$

[Adiabatic approximation (not performed): neglect $H_0 - E_0$]

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r}) = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})$$

Formal equivalence with time-dependent Schrödinger equation for straight-line trajectories

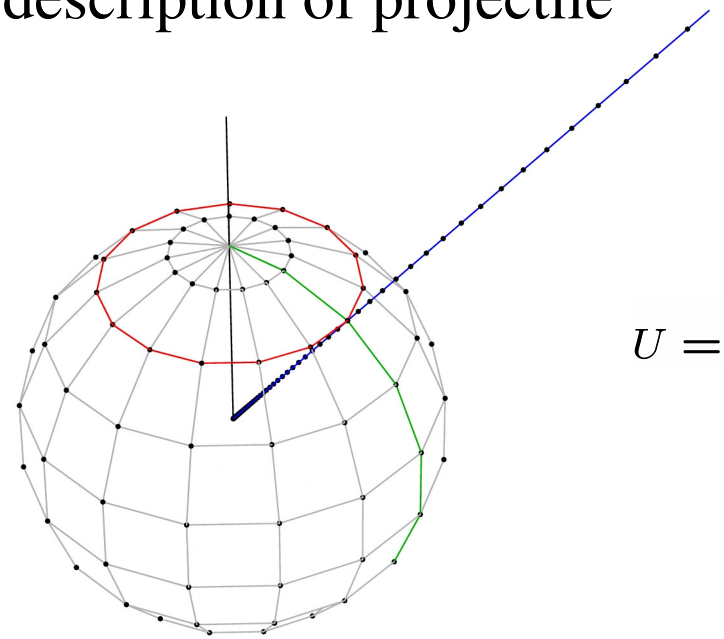
$$t = Z/v$$

$$i\hbar\frac{\partial}{\partial t}\widetilde{\Psi}(\boldsymbol{r},\boldsymbol{b},t) = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\widetilde{\Psi}(\boldsymbol{r},\boldsymbol{b},t)$$

Numerical resolution of time-dependent Schrödinger equation

Projectile frame: moving target

Spherical + radial mesh
for description of projectile



Propagation in time along
semi-classical trajectory

$$\psi(t+\Delta t) = U(t+\Delta t, t)\psi(t)$$

$$U = e^{-i\frac{1}{2}\Delta t V(t+\Delta t)} e^{-i\Delta t H_0} e^{-i\frac{1}{2}\Delta t V(t)} + O(\Delta t^3)$$

No multipole expansion

No partial wave decomposition

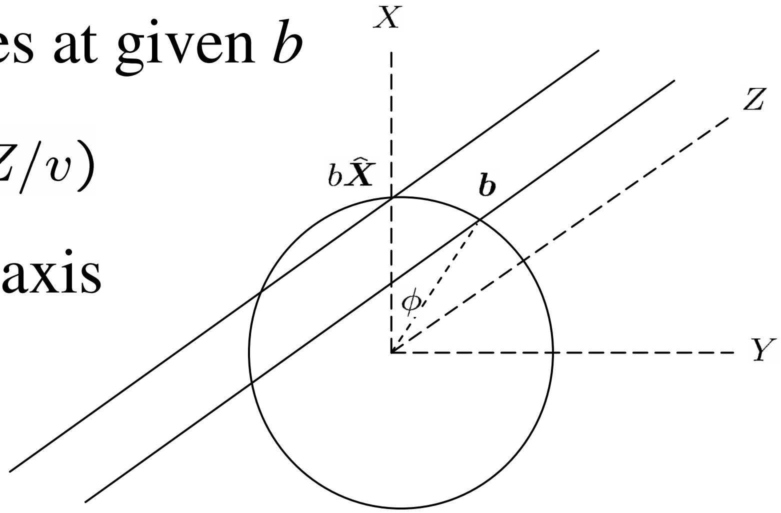
V.S. Melezhik, Phys. Lett. A 230 (1997) 203
P. Capel et al, Phys. Rev. C 68 (2003) 014612

Incoherent and coherent eikonal approximations

Incoherent assumption:

- same wave function for all trajectories at given b

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\tilde{\Psi}(\boldsymbol{r}, b\widehat{\mathbf{X}}, Z/v)$$



- violates rotational symmetry along z axis

Coherent assumption:

- wave functions at given b obtained by rotation

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}e^{-i\phi j_z}\tilde{\Psi}(\boldsymbol{r}, b\widehat{\mathbf{X}}, Z/v)$$

- ϕ : azimutal angle of $\boldsymbol{R}$ (or $\boldsymbol{b}$)
- rotationally symmetric along z axis

Also for usual eikonal approximation !

Elastic transition matrix element $[\boldsymbol{K} = (K, \theta, \varphi)]$

$$T_{fi} = \langle e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_0(\boldsymbol{r})|V_{cT} + V_{fT}|\Psi(\boldsymbol{R},\boldsymbol{r})\rangle$$

Dynamical eikonal approximation $\boldsymbol{K}\cdot\boldsymbol{R} - KZ \approx \boldsymbol{q}\cdot\boldsymbol{b}$

$$\begin{aligned} T_{fi} &\approx i\hbar v \int d\boldsymbol{R}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{\partial}{\partial Z}\langle\phi_0(\boldsymbol{r})|\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})\rangle \\ &= i\hbar v \lim_{Z\to+\infty} \int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\langle\phi_0(\boldsymbol{r}|\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})\rangle \end{aligned}$$

with

$$\boldsymbol{q} = \boldsymbol{K} - K\widehat{\boldsymbol{Z}} \qquad (q = 2K\sin\tfrac{1}{2}\theta)$$

Elastic amplitude from solution of TDSE

$$S_0(b) = \langle\phi_0(\boldsymbol{r})|\widetilde{\Psi}(\boldsymbol{r}, b, +\infty)\rangle - 1$$

$$T_{fi} = i2\pi\hbar v \int_0^\infty b\,db\,J_0(qb)S_0(b)$$

Partial-wave expansion $\tilde{\Psi}(\boldsymbol{r}, b, +\infty) = \frac{1}{r} \sum_{lm} \psi_{lm}(r) Y_l^m(\Omega_r)$

$$S_0(b) = \int_0^\infty u_0(r) \psi_{00}(r) dr - 1$$

Elastic cross section

$$\frac{d\sigma}{d\Omega} = \left(\frac{\mu}{2\pi\hbar^2}\right)^2 \left|T_{fi}\right|^2 = K^2 \left|\int_0^\infty b db J_0(qb) S_0(b)\right|^2$$

Also valid for usual eikonal approximation with

$$S_0^{\text{eik.}}(b) = \langle \phi_0(\boldsymbol{r})| \exp\left[-\frac{i}{\hbar v} \int_{-\infty}^{Z} (V_{cT} + V_{fT}) dZ'\right] |\phi_0(\boldsymbol{r})\rangle - 1$$

Semi-classical cross section (with model for $b_{\text{cl.}}$!)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_R}{d\Omega} |1 + S_0(b_{\text{cl.}})|^2$$

Breakup transition matrix element

$$T_{fi} = \langle e^{i\boldsymbol{K'}\cdot\boldsymbol{R}}\chi^{(-)}_{\boldsymbol{k}}(\boldsymbol{r})|V_{cT} + V_{fT}|\Psi(\boldsymbol{R},\boldsymbol{r})\rangle$$

Ingoing scattering states

$$H_0\chi^{(-)}_{\boldsymbol{k}}(\boldsymbol{r}) = E\chi^{(-)}_{\boldsymbol{k}}(\boldsymbol{r})$$

$$E = \frac{\hbar^2k^2}{2\mu_{cf}}$$

$$T_{fi} \approx i\hbar v \lim_{Z\to+\infty}\int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}\langle\chi^{(-)}_{\boldsymbol{k}}(\boldsymbol{r})|\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})\rangle$$

Breakup amplitude

$$S(\boldsymbol{k},\boldsymbol{b}) = \langle\chi^{(-)}_{\boldsymbol{k}}(\boldsymbol{r})|\widehat{\Psi}(\boldsymbol{R},\boldsymbol{r})\rangle_{Z=+\infty}$$

$$T_{fi} = i\hbar v\int d\boldsymbol{b}\, e^{-i\boldsymbol{q}\cdot\boldsymbol{b}}S(\boldsymbol{k},\boldsymbol{b})$$

Partial-wave expansion of scattering state

$$u_{kl}(r) \xrightarrow[r\to\infty]{} \cos\delta_l\, F_l(kr) + \sin\delta_l\, G_l(kr)$$

Partial amplitudes (coherent)

$$S(\boldsymbol{k},\boldsymbol{b}) = \frac{4\pi}{k}\sum_{lm} Y_l^m(\Omega_k)e^{-im\varphi}S_{klm}(b)$$

$$S_{klm}(b) = e^{i(\sigma_l+\delta_l-l\pi/2)}\int_0^\infty u_{kl}(r)\hat{\Psi}_{lm}(r)dr$$

Transition matrix element (coherent)

$$T_{fi} = i8\pi^2\frac{\hbar v}{k}\sum_{lm} Y_l^m(\Omega_k)e^{-im\varphi}\int_0^\infty bdbJ_m(qb)S_{klm}(b)$$

Incoherent: $m \to 0$

Breakup cross sections

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{1}{(2\pi)^5}\frac{\mu K'}{\hbar^3 v}|T_{fi}|^2$$

Incoherent approximation
(and usual eikonal approximation?)

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{2KK'}{\pi k^2}\left|\sum_{lm} Y_l^m(\Omega_k)\int_0^\infty bdbJ_0(qb)S_{klm}(b)\right|^2$$

• not rotationally invariant around z axis

Coherent approximation

$$\frac{d\sigma}{d\boldsymbol{k}d\Omega} = \frac{2KK'}{\pi k^2}\left|\sum_{lm} i^{|m|}Y_l^m(\Omega_k)e^{-im\varphi}\int_0^\infty bdbJ_{|m|}(qb)S_{klm}(b)\right|^2$$

• depends on $\varphi_k - \varphi$
$\rightarrow$ rotationally invariant around z axis

Applications

- $^{11}Be + {}^{12}C \rightarrow {}^{11}Be + {}^{12}C$
- $^{11}Be + {}^{12}C \rightarrow {}^{10}Be + n + {}^{12}C$
- $^{11}Be + {}^{208}Pb \rightarrow {}^{10}Be + n + {}^{208}Pb$
- $^{8}B + {}^{208}Pb \rightarrow {}^{7}Be + p + {}^{208}Pb$

- energies of GANIL, RIKEN, NSCL experiments

N. Fukuda, T. Nakamura, et al, Phys. Rev. C 70 (2004) 054606
T. Kikuchi et al, Phys. Lett. B 391 (1997) 261
B. Davids et al, Phys. Rev. Lett. 81 (1998) 2209

^{11}Be

Bound states

$1/2^+$ ($l = 0$) at –0.503 MeV

$1/2^-$ ($l = 1$) at –0.183 MeV

Resonance

$5/2^+$ ($l = 2$) at 1.27 MeV ($\Gamma = 0.10 \pm 0.02$ MeV)

Potential $V_0(r)$ for ^{10}Be + n

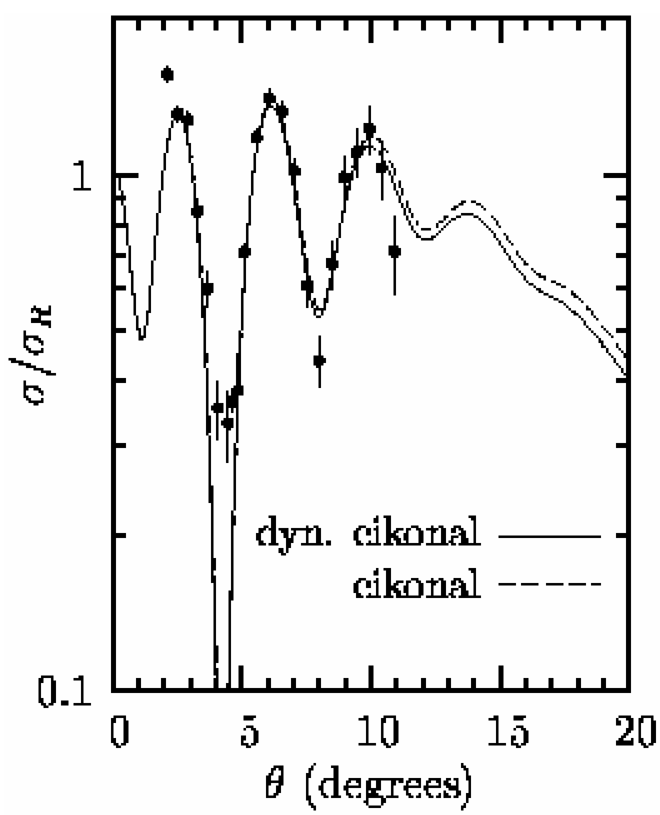
- Woods-Saxon + spin-orbit (l and j dependent)
- non-physical bound states 0s1/2 and 0p3/2
- potentials with physical 0d5/2 resonance

Optical potentials for ^{208}Pb + n and ^{208}Pb + ^{10}Be
for ^{12}C + n and ^{12}C + ^{10}Be

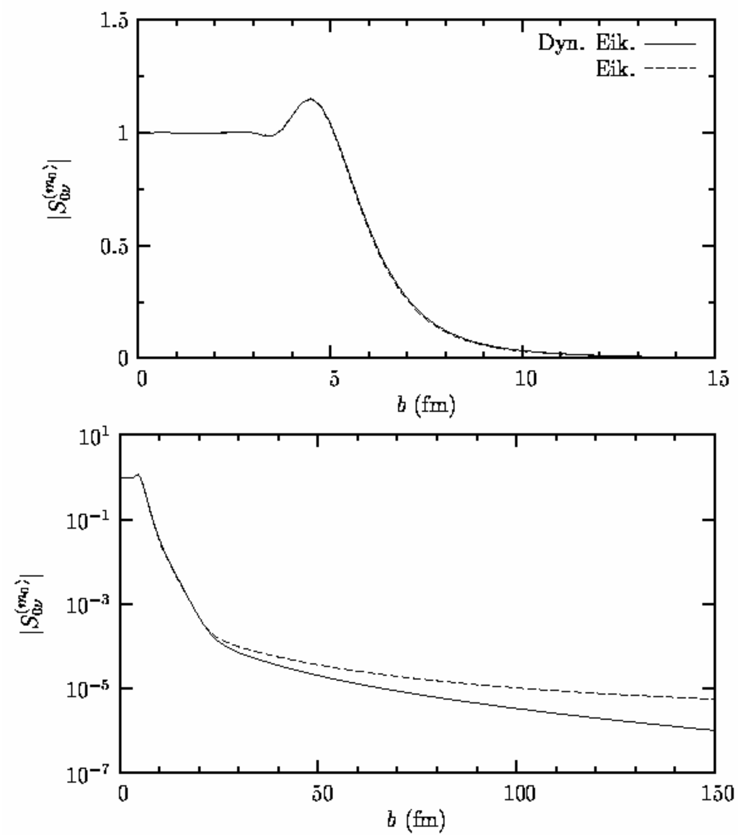
No free parameter

^{11}Be + ^{12}C elastic scattering at 49.3 MeV/nucleon

σ/σ_R



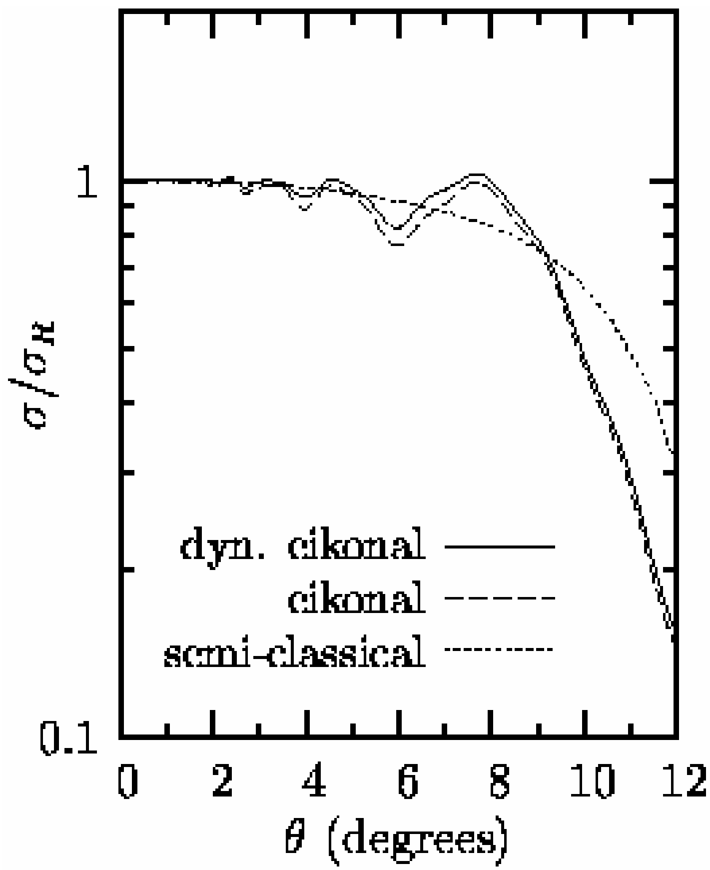
Amplitude S_0



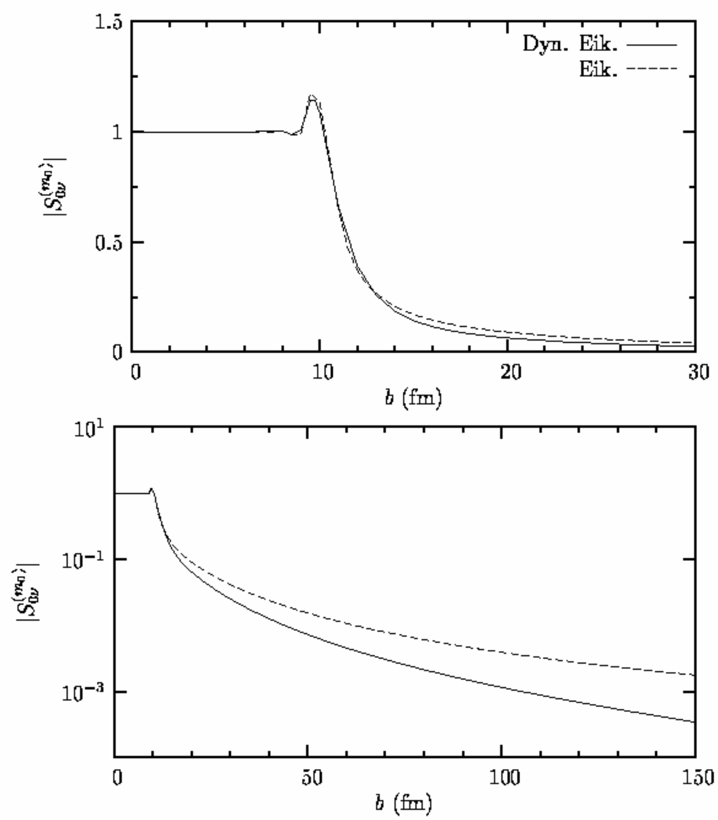
Exp: M.D. Cortina-Gil, PhD thesis, Université de Caen (1996)

^{11}Be + ^{208}Pb elastic scattering at 20 MeV/nucleon

σ/σ_R



Amplitude S_0

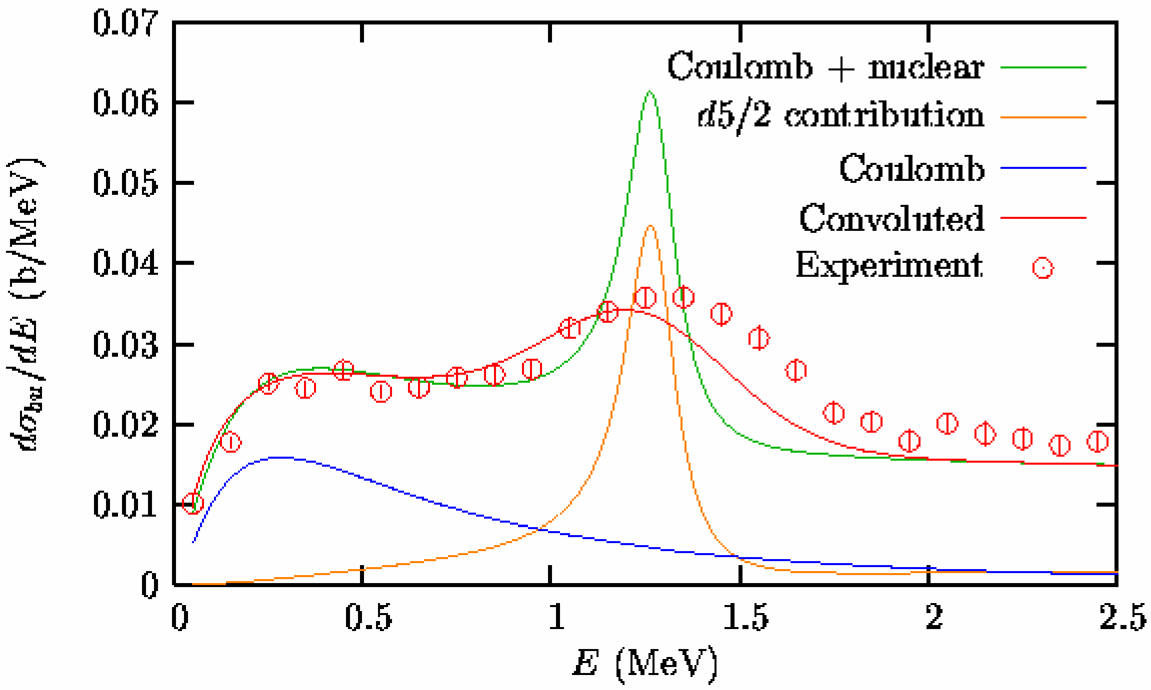


D. Baye, P. Capel, G. Goldstein, Phys. Rev. Lett. 95 (2005) 082502

$^{11}Be + ^{12}C \rightarrow ^{10}Be + n + ^{12}C$

Semi-classical calculation

67 MeV/nucleon



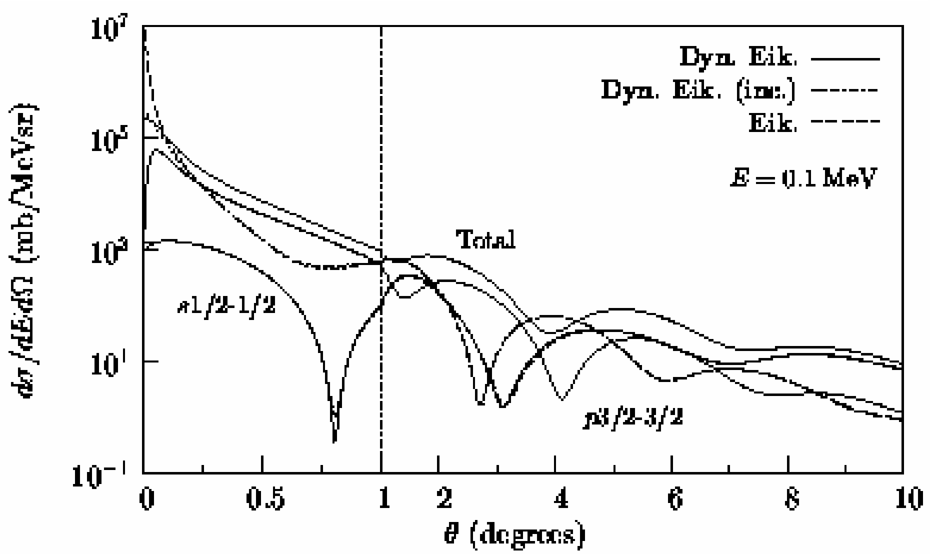
Th:: P. Capel et al, Phys. Rev. C 70 (2004) 064605

Exp.: N. Fukuda et al., Phys. Rev. C 70 (2004) 054606

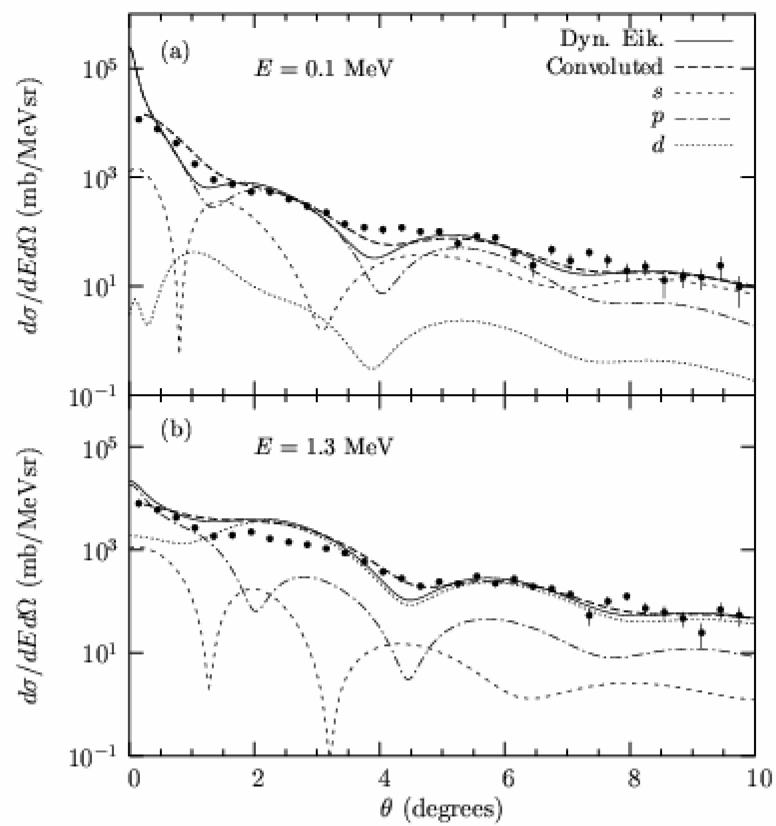
Convolution of theory with experimental energy resolution

^{11}Be + ^{12}C elastic breakup at 67 MeV/nucleon

ljm decomposition

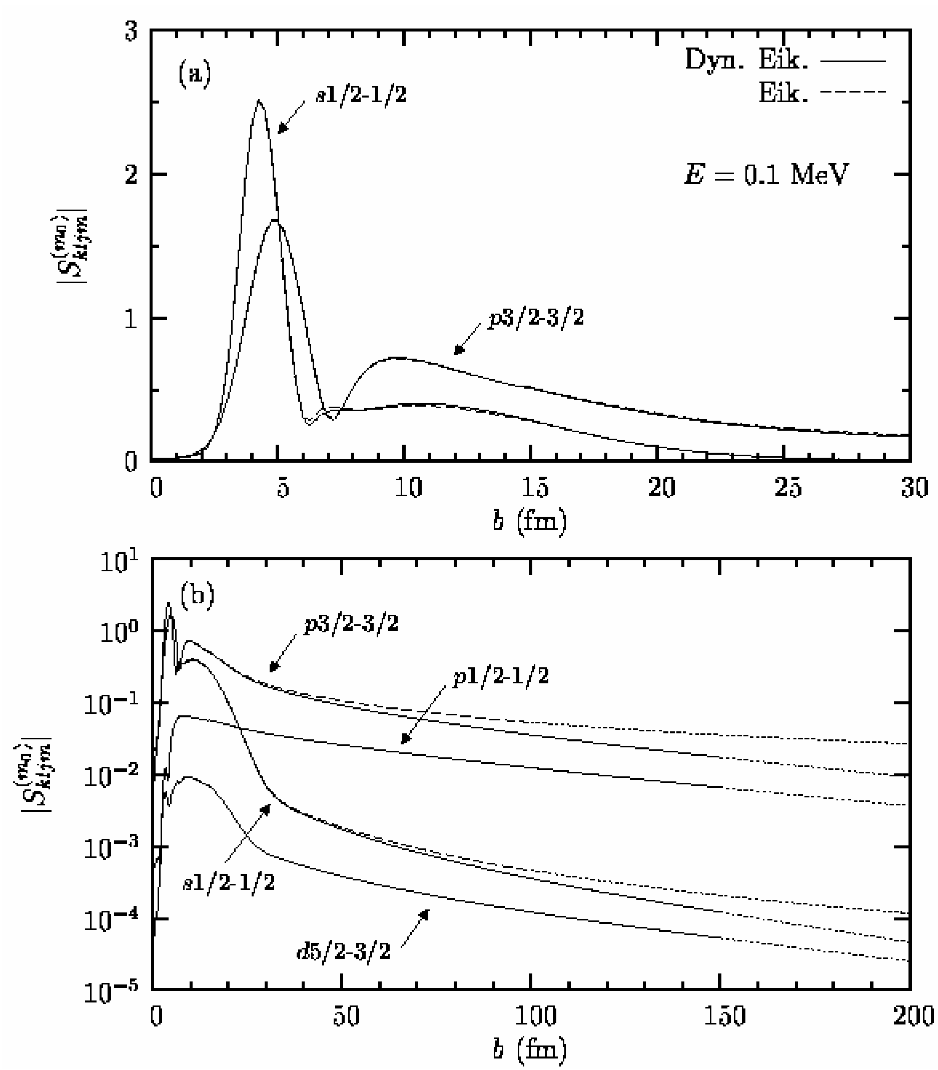


l decomposition



Convolution of theory
with experimental angular resolution

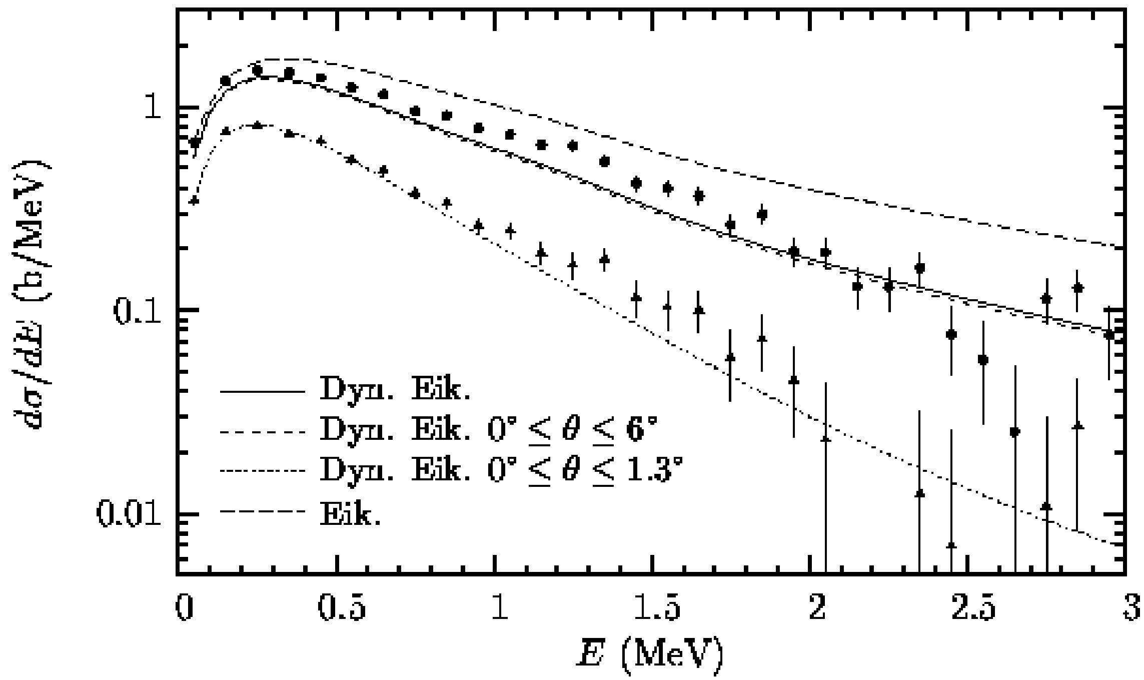
Amplitudes for dynamical and usual eikonal approximations
^{11}Be + ^{12}C elastic breakup at 67 MeV/nucleon



Integrated cross sections

^{11}Be + ^{208}Pb elastic breakup at 69 MeV/nucleon

$$\frac{d\sigma}{dE}(\theta_{\mathsf{max}}) = 2\pi \int_0^{\theta_{\mathsf{max}}} \frac{d\sigma}{dE d\Omega}$$

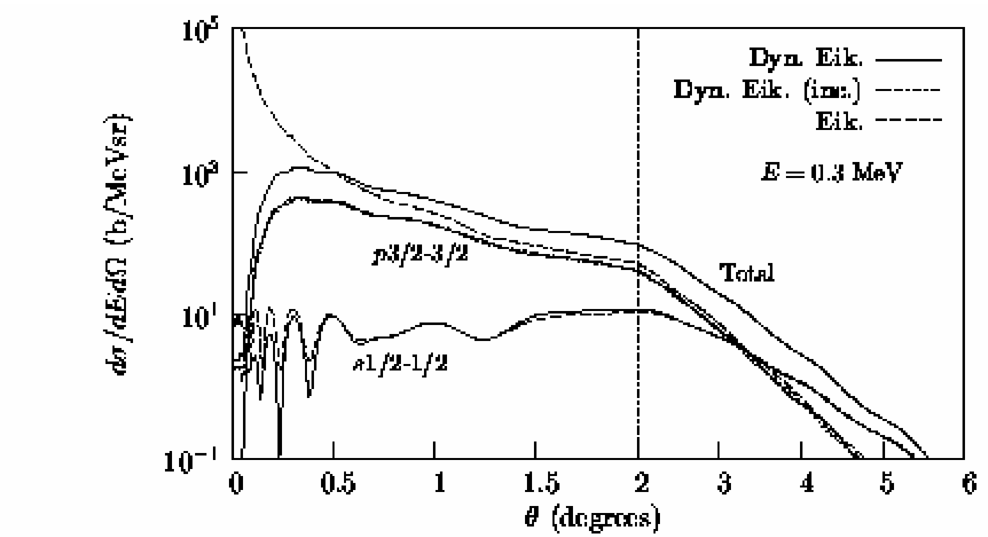


$$\frac{d\sigma}{dE} = \frac{4\mu_{cf}}{\hbar^2 k} \sum_{lm} \int_0^{\infty} bdb\,|S_{klm}(b)|^2$$

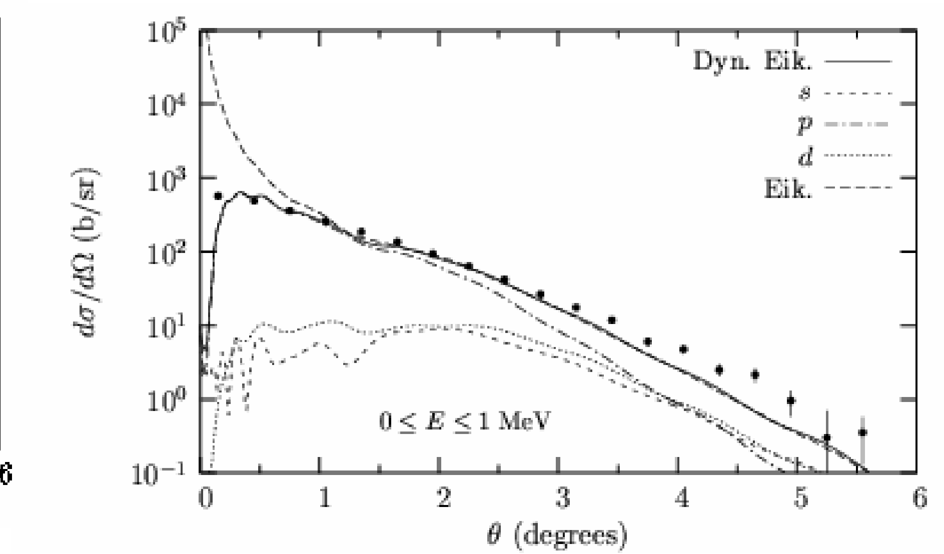
$^{11}Be + ^{208}Pb$ elastic breakup at 69 MeV/nucleon

dynamical and usual eikonal approximations

ljm decomposition

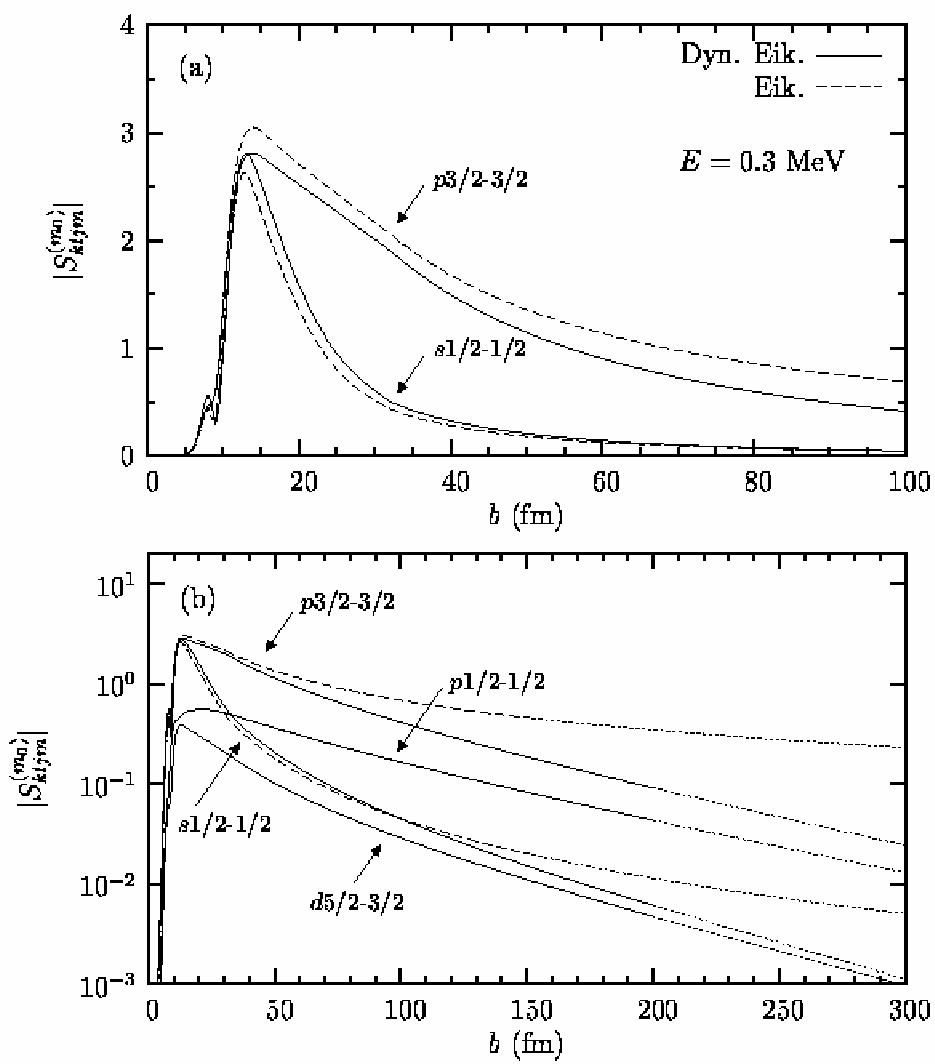


l decomposition



Exp: N. Fukuda, et al,
Phys. Rev. C 70 (2004) 054606

Amplitudes for dynamical and usual eikonal approximations
^{11}Be + ^{208}Pb elastic breakup at 69 MeV/nucleon



^{8}B

Bound state

2^{+} (p3/2) at –0.137 MeV

Potential $V_0(r)$ for $^{7}Be + p$

-Woods-Saxon + spin-orbit (l and j dependent)

H. Esbensen, G.F. Bertsch, Nucl. Phys. A600 (1996) 37

- non-physical 0s1/2 bound state and 0p1/2 resonance

Optical potentials for $^{208}Pb + p$ and $^{208}Pb + {}^{7}Be$

No free parameter

Laboratory frame

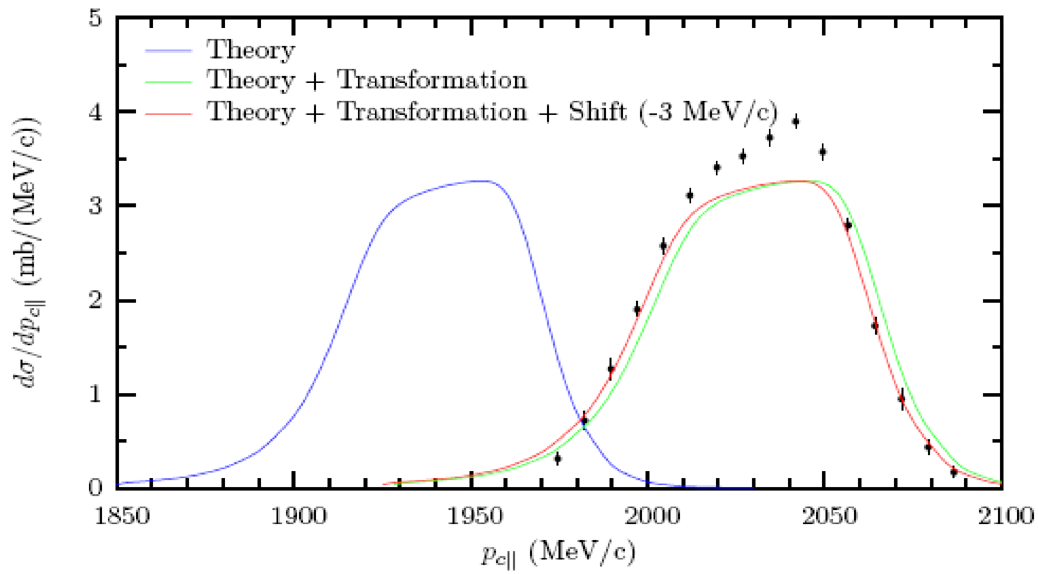
Parallel momentum distribution of ^{7}Be

$$\frac{d\sigma}{dp_{c||}} = \frac{2\pi}{m_c}\int_0^{p_c^{\max}} dp_c \int_0^{\pi} d\theta_f \sin\theta_f \int_0^{2\pi} d\Delta\varphi \frac{d\sigma}{dE_c d\Omega_c d\Omega_f}$$

$$p_c^{\max} = p_{c||}/\cos\theta_c^{\max} \qquad \Delta\varphi = \varphi_c - \varphi_f$$

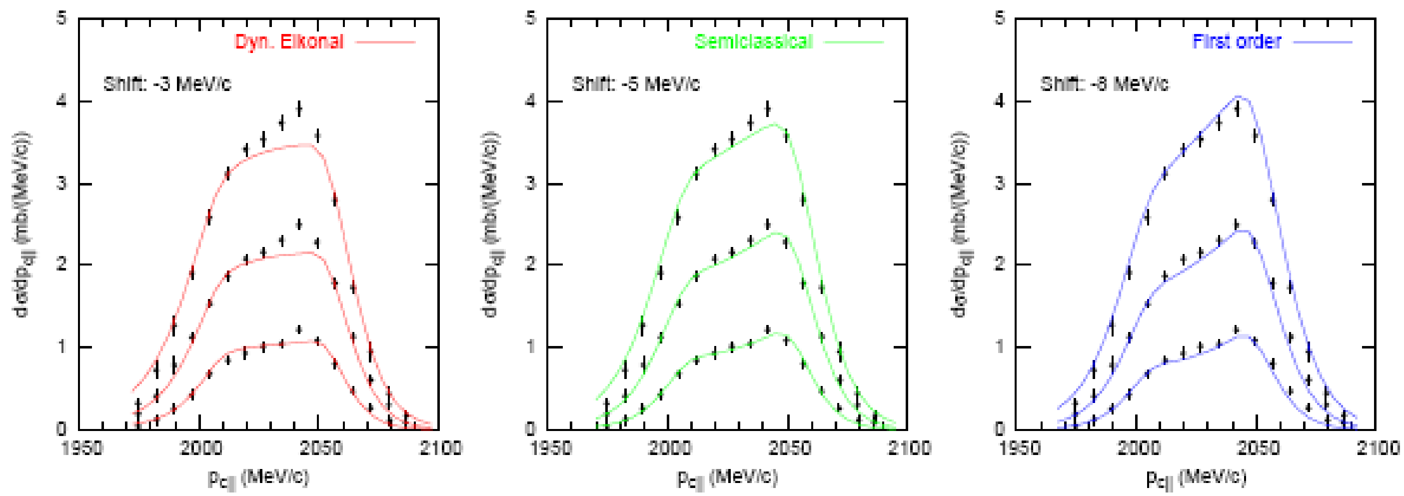
Relativistic momentum transformation

$$p_i \xrightarrow{\text{R}} v \xrightarrow{\text{NR}} \text{dynamical eikonal calculation} \xrightarrow{\text{NR}} m_c\, v_{c\parallel} \xrightarrow{\text{R}} p_{c\parallel}$$

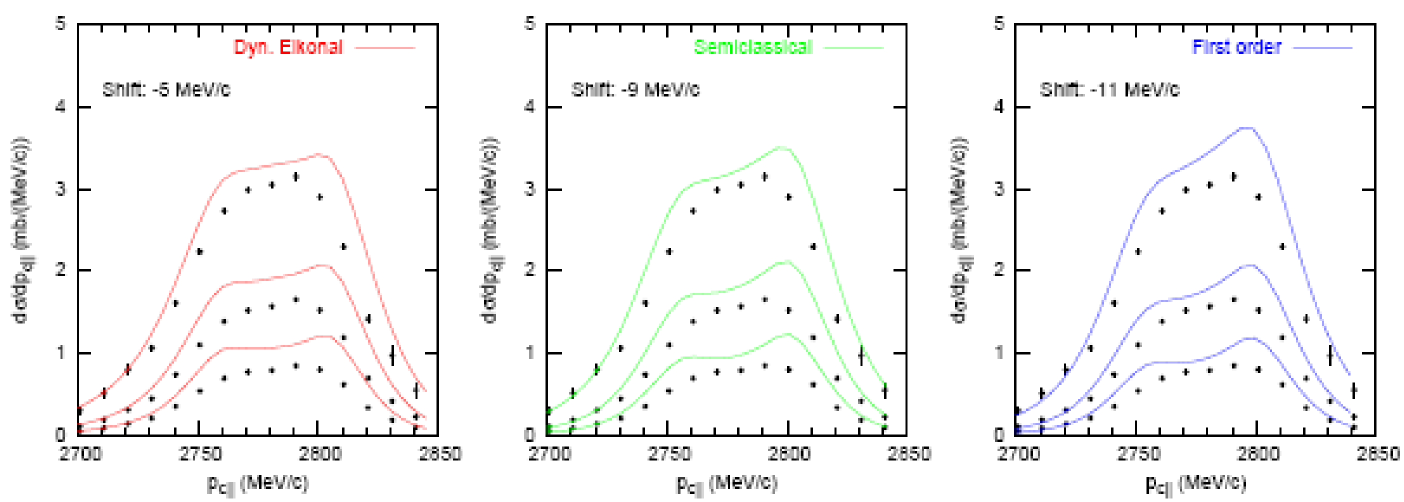


44 MeV/nucleon

$^{8}B + ^{208}Pb \rightarrow ^{7}Be + p + ^{208}Pb$



44 MeV/nucleon : θ < 1.5, 2.4, 3.5 º

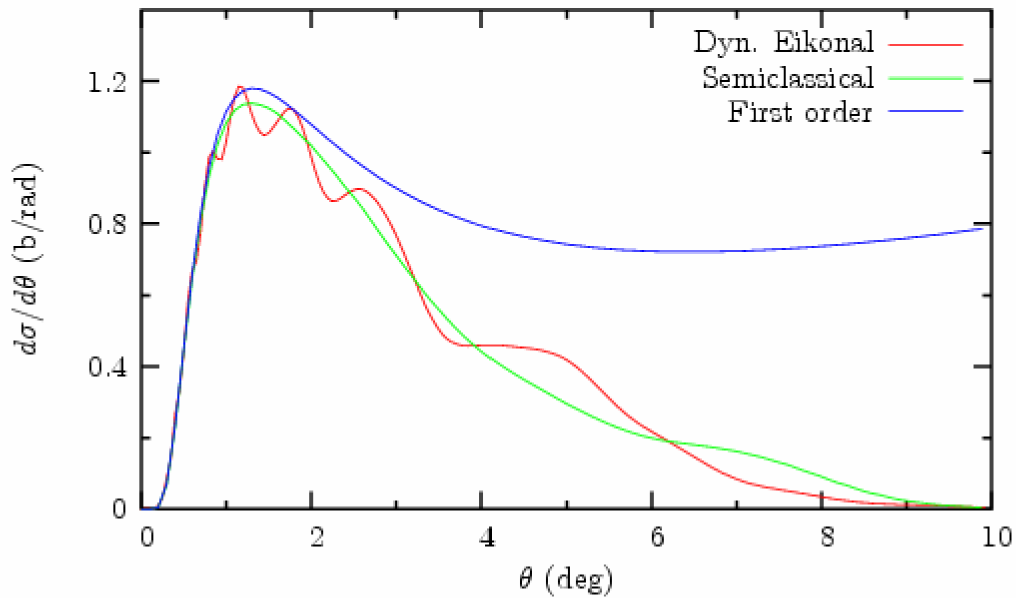


81 MeV/nucleon : θ < 1.0, 1.5, 2.5 º

B. Davids et al, Phys. Rev. Lett. 81 (1998) 2209

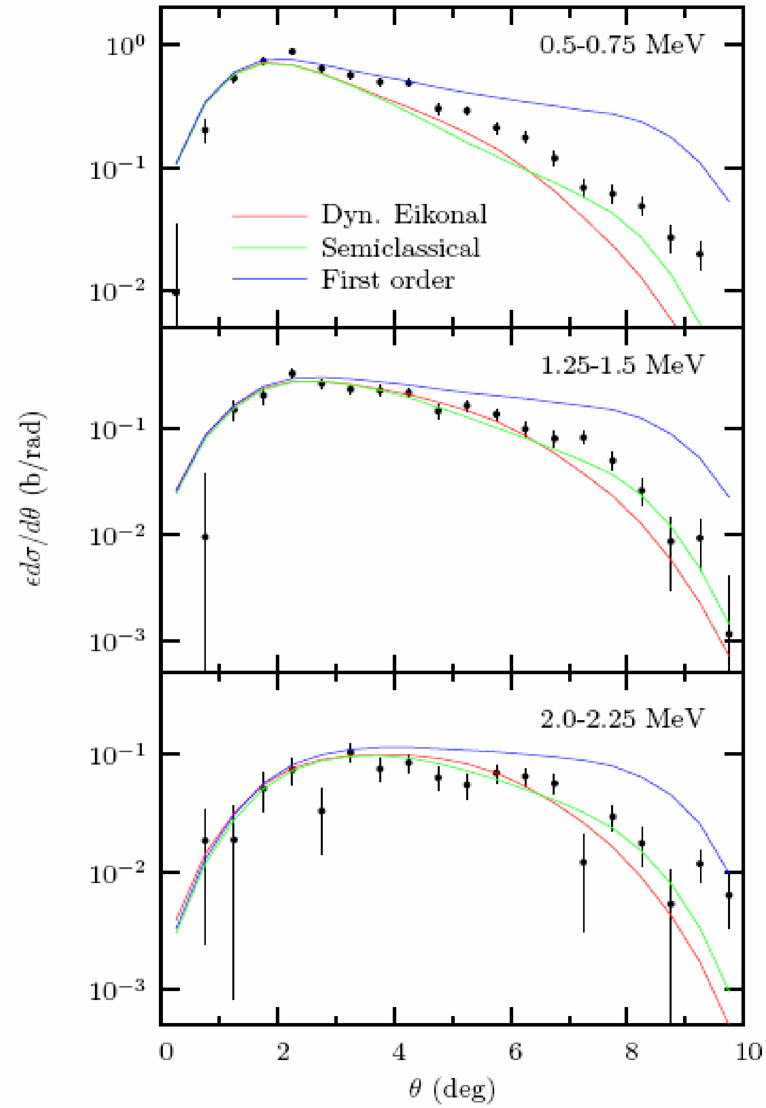
Angular distributions of ^{8}B on ^{208}Pb

Comparison of approximations



52 MeV/nucleon

$^{8}B + {}^{208}Pb \rightarrow {}^{7}Be + p + {}^{208}Pb$



52 MeV/nucleon

T. Kikuchi et al, Phys. Lett. B 391 (1997) 261

Conclusion

- Quantal approximation based on semi-classical resolution of the time-dependent Schrödinger equation
- Two variants: incoherent and coherent

  coherent more physical (respects symmetry)
- Interference effects taken into account
- Differential cross sections for elastic scattering and breakup of halo nuclei
- Fair agreement with experiment (no parameter)
- Lack of asymmetry for ^{8}B

D. Baye, P. Capel, G. Goldstein, Phys. Rev. Lett. 95 (2005) 082502
G.Goldstein, D. Baye, P. Capel, Phys. Rev. C 73 (2006) 024602