Three-body continuum states

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Introduction

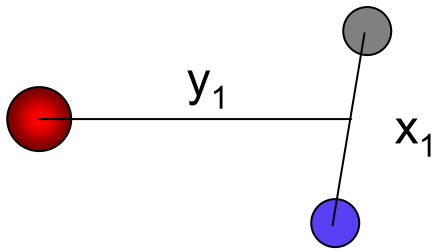
Many applications of 3-body models:

- Halo nuclei: $^{6}He=\alpha+n+n$, $^{14}Be=^{12}Be+n+n$, etc.
- Unbound nuclei: $^{5}H=^{3}H+n+n$, $^{10}He=^{8}He+n+n$, etc.
- Cluster states: $^{12}C=\alpha+\alpha+\alpha$
- Reactions: $^{7}Be(p,\gamma)^{8}B$, with $^{7}Be=\alpha+^{3}He$
- Hypernuclei

• Exotic nuclei: low separation energies (or <0) ➔ continuum states are fundamental
breakup processes need **3-body continuum** wave functions

• Astrophysics: some 3-body processes are important
$\alpha+\alpha+n$
$\alpha+\alpha+\alpha$
others?

The three-body equation



Jacobi coordinates *x_1, y_1*

3 sets *(x_i, y_i), i=1,2,3*

Hyperspherical coordinates:

$$\rho^2 = x_1^2 + y_1^2 = x_2^2 + y_2^2 = x_3^2 + y_3^2$$

$$\alpha_i = \arctan \frac{y_i}{x_i}$$

$$\Omega_{x_i},\ \Omega_{y_i}$$

6 coordinates

Hamiltonian:

$H = \sum_{i=1}^{3} T_i \ + \ \sum_{i<j} V_{ij}$ to be expressed in $(\rho, \Omega_{x_i}, \Omega_{y_i}, \alpha_i)$

Kinetic energy

$$T = T_1 + T_2 + T_3 = -\frac{\hbar^2}{2m_N}\left(\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{K^2(\Omega_{5i})}{\rho^2}\right),$$

With $K^2(\Omega)$ = angular operator (equivalent to L^2 in two-body systems)

- Eigenfunctions: $\mathcal{Y}^{\ell_x\ell_y}_{KLM_L}(\Omega_5) = \phi^{\ell_x\ell_y}_K(\alpha)\left[Y_{\ell_x}(\Omega_x)\otimes Y_{\ell_y}(\Omega_y)\right]^{LM_L}.$

  =hyperspherical harmonics

- Eigenvalues : $K(K+4)$

  With l_x, l_y = angular momenta associated with *x, y*

  K = hypermoment

  $\Phi_K(\alpha)$ = Jacobi polynomial

Spin: $\mathcal{Y}^{JM}_{\ell_x\ell_y LSK}(\Omega) = \left[\mathcal{Y}^{\ell_x\ell_y}_{KL}(\Omega)\otimes\chi^S\right]^{JM}$, with S = total spin

Schrödinger equation $H\Psi^{JM\pi} = E\Psi^{JM\pi}$

$\Psi^{JM\pi}$ is expanded over the hyperspherical harmonics

$$\Psi^{JM\pi}(\rho,\Omega_5) = \rho^{-5/2} \sum_{\ell_x \ell_y LSK} \underbrace{\chi^{J\pi}_{\ell_x\ell_y LSK}(\rho)}_{\text{To be determined}} \times \underbrace{\mathcal{Y}^{JM}_{\ell_x\ell_y LSK}(\Omega_5)}_{\text{Known functions}}$$

→ Set of equations for $\chi^{J\pi}_{\ell_x\ell_y LSK}(\rho)$

$$\left[-\frac{\hbar^2}{2m_N}\left(\frac{d^2}{d\rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2}\right) - E\right]\chi^{J\pi}_{\gamma K}(\rho) + \sum_{K'\gamma'} V^{J\pi}_{K'\gamma',K\gamma}(\rho)\,\chi^{J\pi}_{\gamma' K'}(\rho) = 0,$$

$\gamma = (\ell_x \ell_y L S)$

- Potential determined numerically
- This system is truncated at $K = K_{max}$
- *Common to bound and continuum states*

Size of the system:

Example: $^{6}He = \alpha+n+n$: $S=0$ or 1

	$J = 0^+$		$J = 2^+$	
K_{max}	$S = 0$	$S = 0,1$	$S = 0$	$S = 0,1$
8	9	15	16	46
12	16	28	33	99
16	25	45	56	172
20	36	66	85	265
24	49	91	120	378

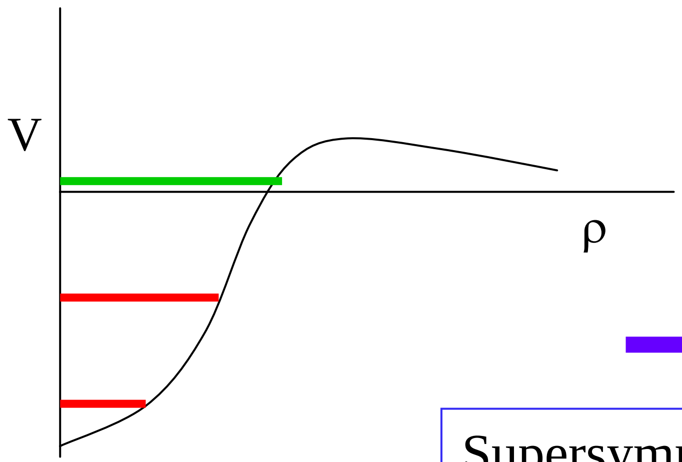
To be multiplied by the number of basis functions (~ 20 - 30)

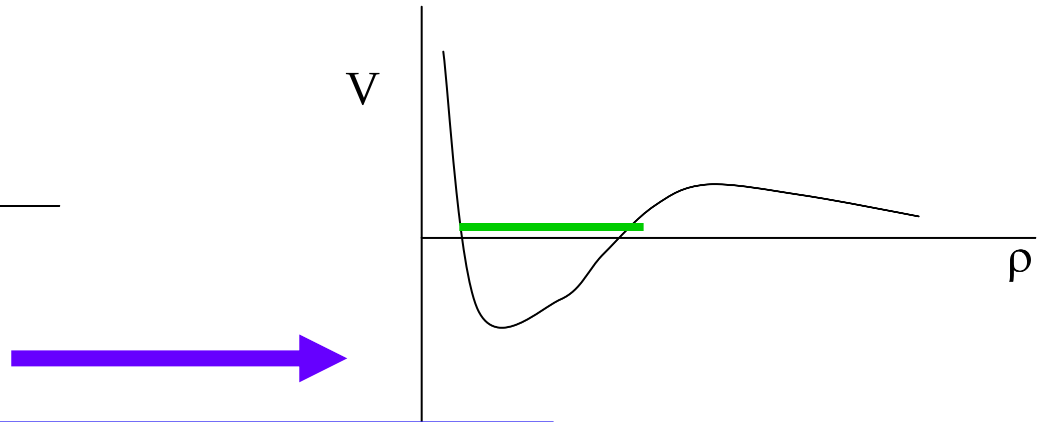
→ large systems for high *J* values

Removal of 2-body forbidden states: 2 possibilities

- Projection: Replace V_{ij} by $V_{ij}+\Lambda P_{ij}$ with
  - P_{ij}=two-body projector (non local)
  - Λ=large energy (typically ~10^6 to 10^8 MeV)
  - $\Rightarrow$ 2-body forbidden states are pushed at high energy

- Supersymmetry: replace the potential by its supersymmetric partner





Supersymmetric transformation

D. Baye, PRL58 (1987) 2738

α+n: 1 f.s. for ℓ=0
$\alpha+\alpha$: 2 f.s. for ℓ=0, 1 f.s. for ℓ=2

3-body continuum states

P. D., E. Tursunov, D. Baye, Nucl. Phys. A 765 (2006) 370

- Finite bases → use of the R-matrix method
- Long range of the potential

  → large bases
  → propagation techniques are necessary

- Three-body coulomb phase shifts

- Eigenphases

Principle of the R matrix

The space is divided in two regions:

$r < a$: $\chi_{i,\text{int}}^{J\pi}(\rho) = \sum_{j=1}^{N} C_{ij} f_j(\rho/a)$ (Legendre functions)

$r \geq a$: $X_{i,ext}(\rho)$ given by $(T - E)\chi_{i,\text{ext}}^{J\pi}(\rho) = 0$ (V negligible)

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{5}{\rho}\frac{\partial}{\partial\rho} - \frac{K(K+4)}{\rho^2} + \kappa^2\right]\chi_{i,\text{ext}}(\rho) = 0$$

Exact (asymptotic) solution: $\chi_{i\,\text{ext}}^{J\pi}(\rho) = A_i(\kappa\rho)^{1/2}(I_i(\kappa\rho) - U_{ij}O_j(\kappa\rho))$

With A_i = amplitude

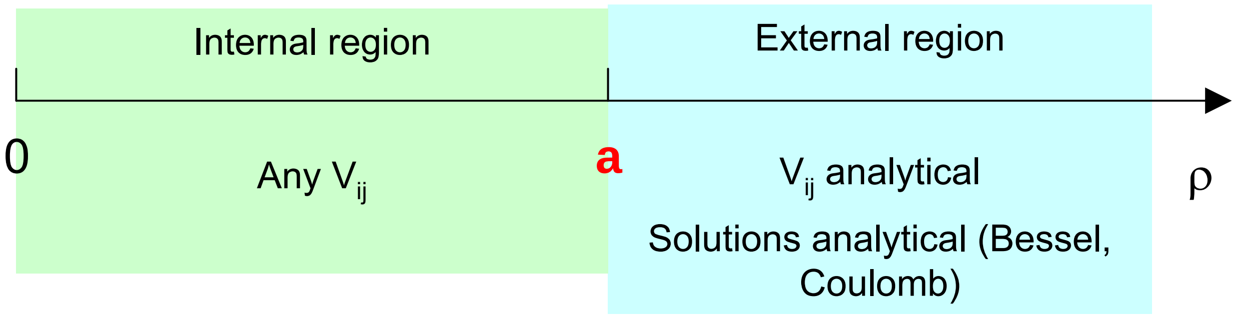
I_i, O_i = ingoing and outgoing functions (depend on Bessel functions Y_{K+2}, J_{K+2})

K = wave number = $(2m_N E/\hbar^2)^{1/2}$

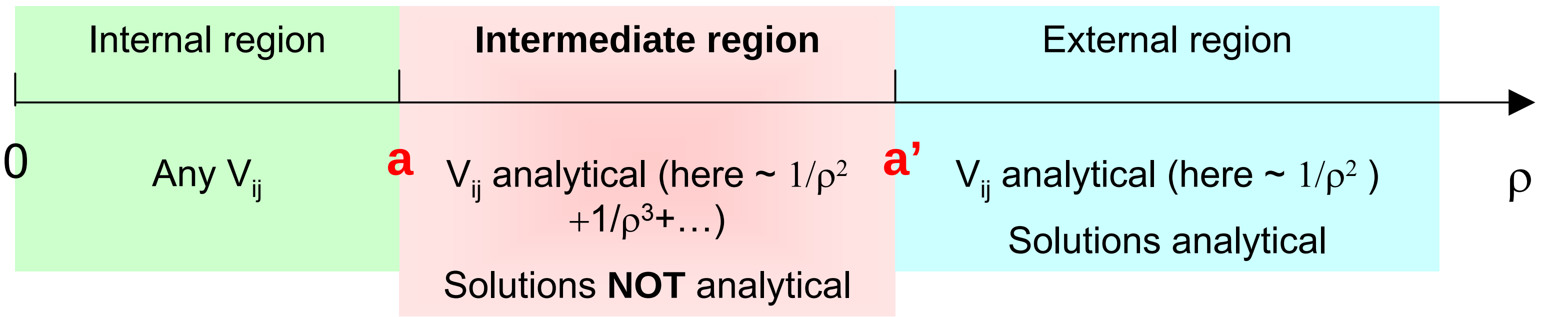
U_{ij} = collision matrix : provides the information about the collision should not depend on a

Peculiarities of 3-body problems

2-body



3-body



a ~ 20 - 30 fm

a' ~ 1000 fm → propagation techniques are necessary

Two propagation methods

1. General properties

- The R matrix is determined at radius a from the basis functions (typically a ~25 fm , N ~ 25)
- The R-matrix is a link between the wave function and its derivative at $\rho=a$

$$\chi_i^{J\pi}(a) = \sum_j R_{ij} \left(\frac{d\chi_j^{J\pi}}{d\rho} \right)_{\rho=a}$$

2. Propagation of the wave function

•The Schrödinger equation is integrated from $\rho=a$ to $\rho=a'$ with the Numerov algorithm

$$(T-E)\chi_i^{J\pi}(\rho) + \sum_j V_{ij}(\rho)\chi_j^{J\pi}(\rho) = 0$$

with $V_{ij}(\rho)$ known analytically: $$V_{ij}(\rho) \approx \sum_{k=1}^{N_0} \frac{\alpha_{ij}^k}{\rho^{3+2k}}$$

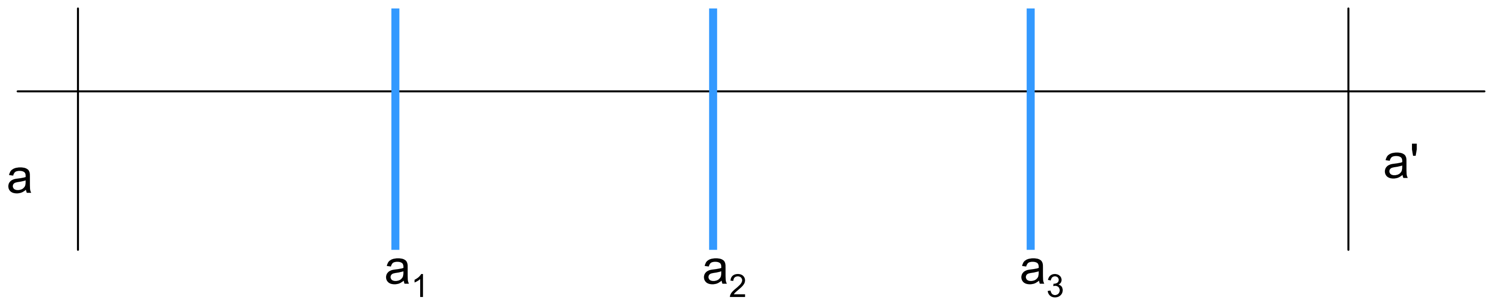
•should converge with N_0 (typically N_0 ~ 5-10)

•coefficients α_{ij} are determined from the potential

•Not possible with projection on pfs (non local potential)

3. Propagation of the R matrix

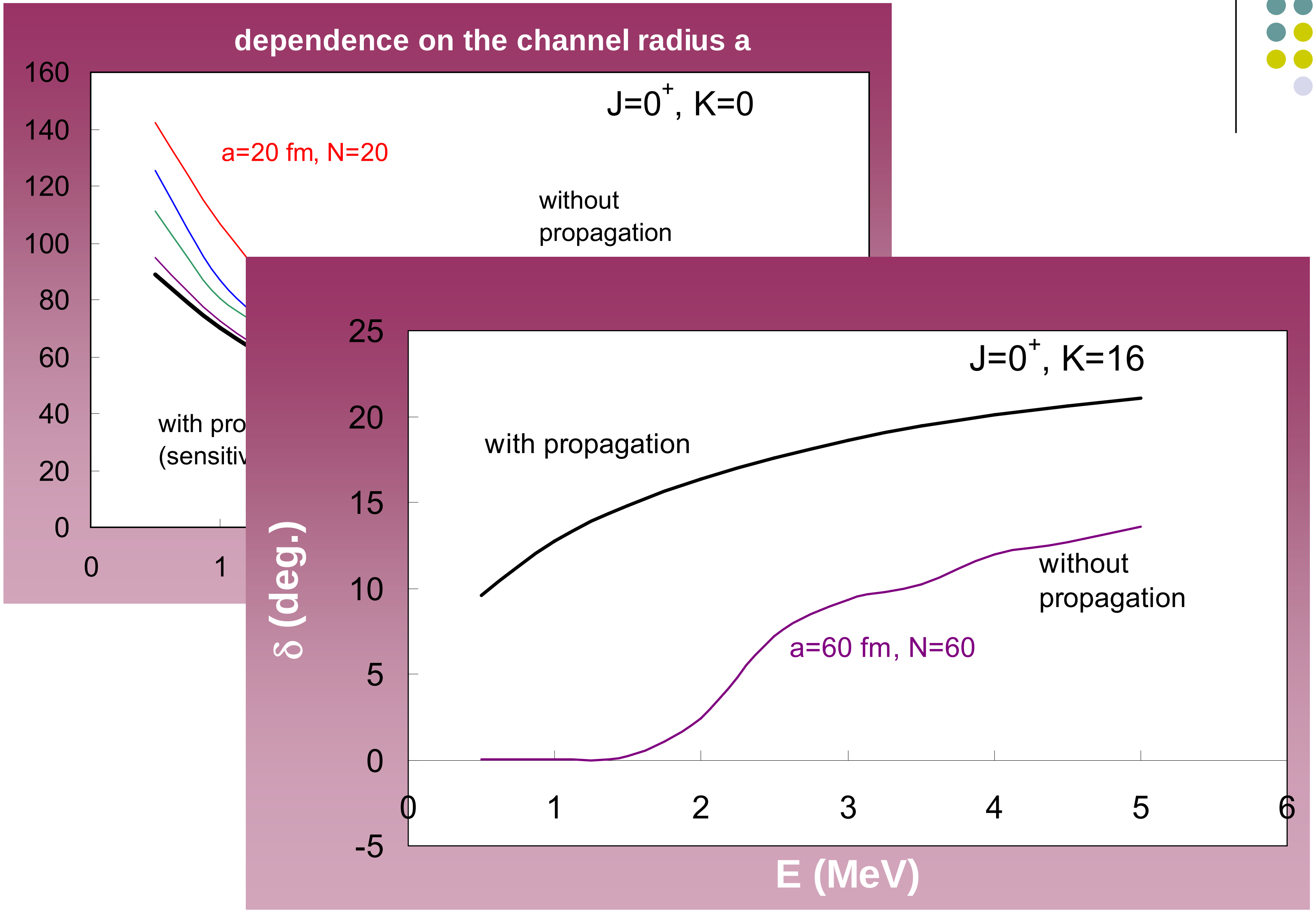
- Burke et al., Comp. Phys. Com. 27 (1982) 299
- Common to supersymmetry and projection techniques
- Idea: to split the interval [a,a'] in N pieces



- In sub-interval i, define basis functions
- Here: Lagrange functions
- Determine the R matrix R_i from R_{i-1}
- Typical values: a~40 fm, a'~1000 fm
  N~30 with 20-30 basis functions

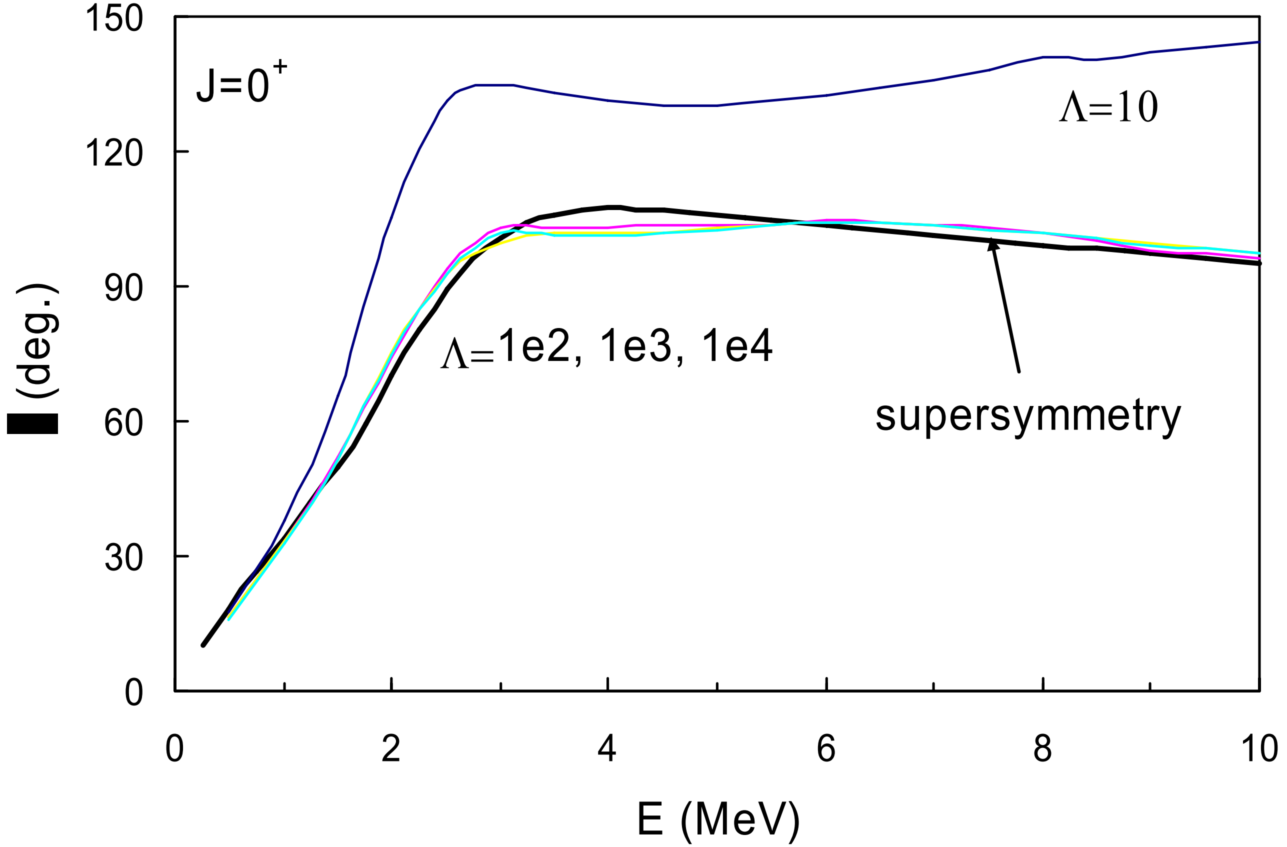
Main difficulty: matrix elements of the projector on forbidden states

Example: $^{6}He=\alpha+n+n$



Comparison: projection to pfs ←→ supersymmetry

Done on ^{6}He, $J=0^+$, $K_{max}=16$



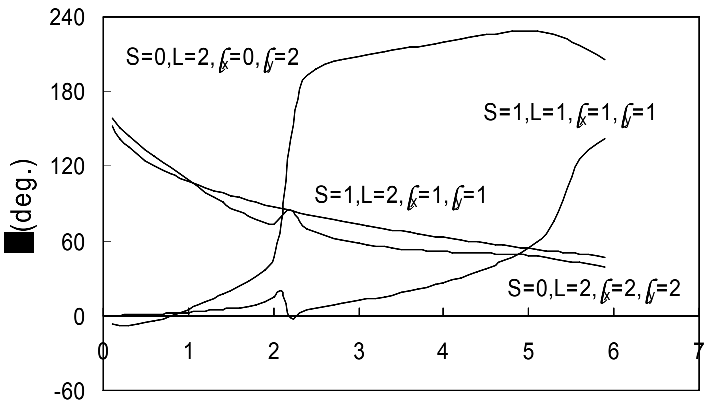
➔ very close to each other for ^{6}He

Eigenphases

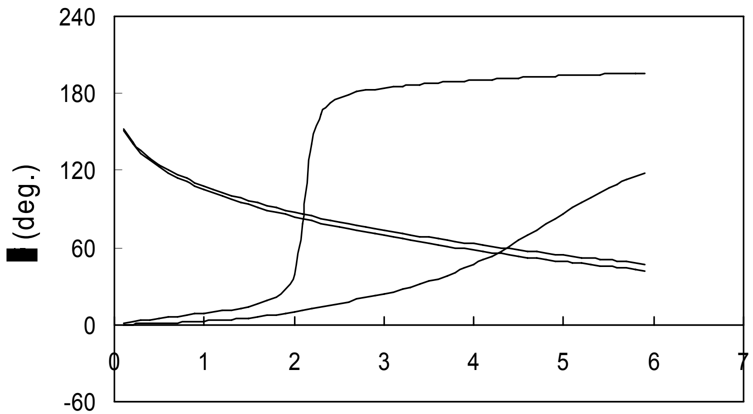
For large K values the number of channels is quite large: ~100-200
→ eigenphases of the collision matrix

A simple example: α+n+n, J=2$^+$, Kmax=2
→ 4 channels

before diagonalisation



after diagonalisation



Coulomb phase shifts

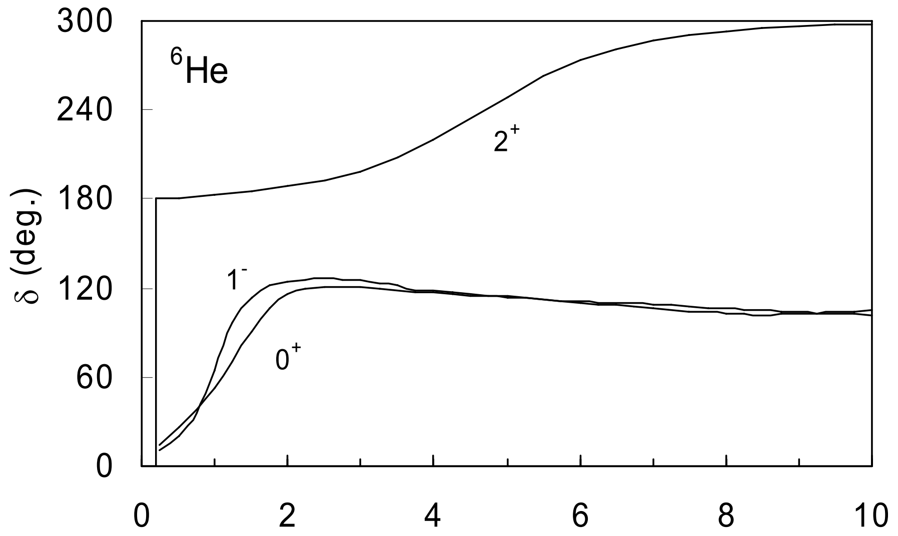
For 2-body: $\delta=\delta_N+\delta_C$

For 3-body: more complicated since the coulomb phase shifts are not diagonal

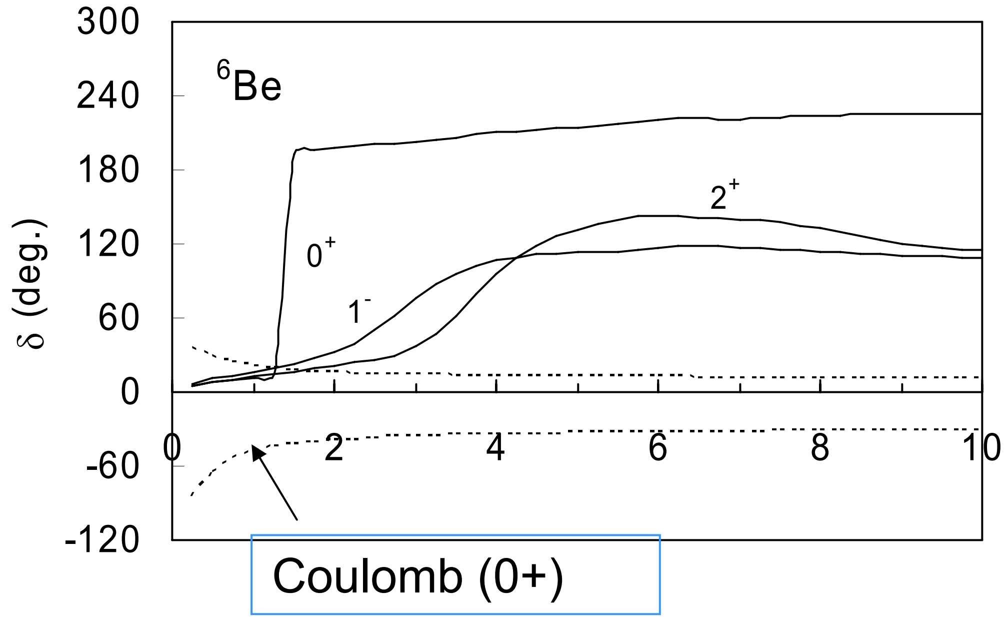
➔ $$\boldsymbol{U} = \boldsymbol{U_C}^{1/2}\boldsymbol{U_N}\boldsymbol{U_C}^{1/2}.$$

where U_C is obtained from the Coulomb potential only

^{6}He and ^{6}Be phase shifts (eigenphases): similar to I. Thompson et al, PRC61 (2000) 024318

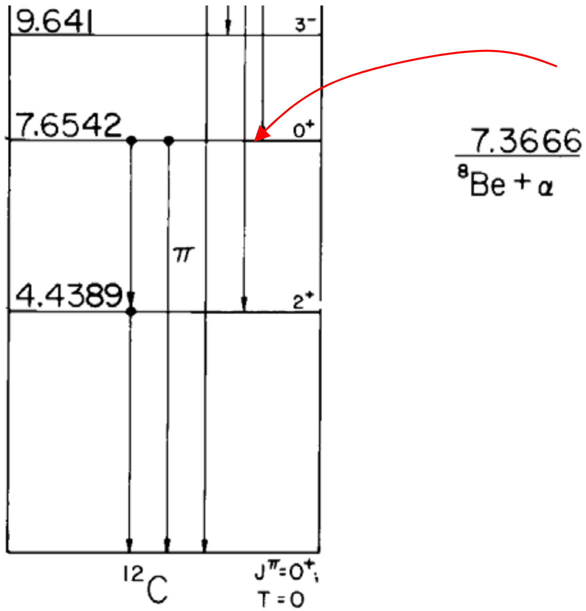


Kmax= 24 (0^+)
19 (1^-)
16 (2^+)



3α continuum states

- Many works on ^{12}C: recently above the 3α threshold



"Hoyle state": astrophysics

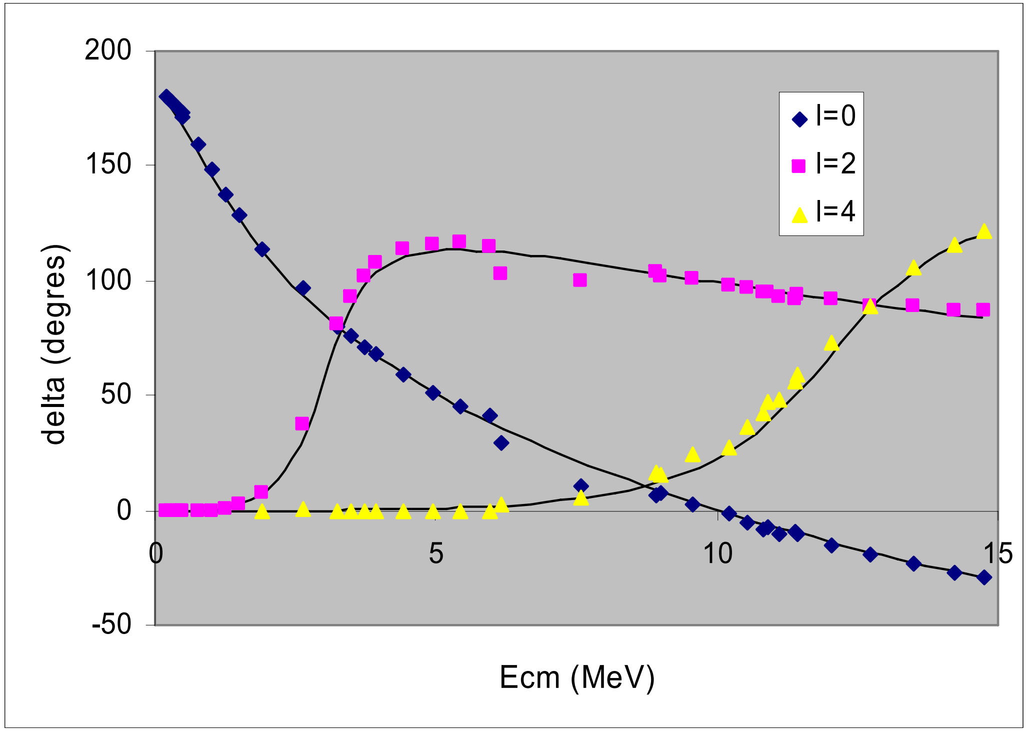
<u>Important issues:</u>

- 0^+_2 = Bose-Einstein condensate?
- 2^+ partner of 0^+_2 (exists? where?)
- Fynbo et al.: no 2^+_2 but a 0^+_3

- Broad resonances → importance of a 3-body continuum model
- α+α scattering well described by different potentials
  - deep potentials (Buck potential)
  - shallow potentials (Ali-Bodmer potentials)

➔ we may expect a good description of the 3α system

- $\alpha+\alpha$ phase shifts

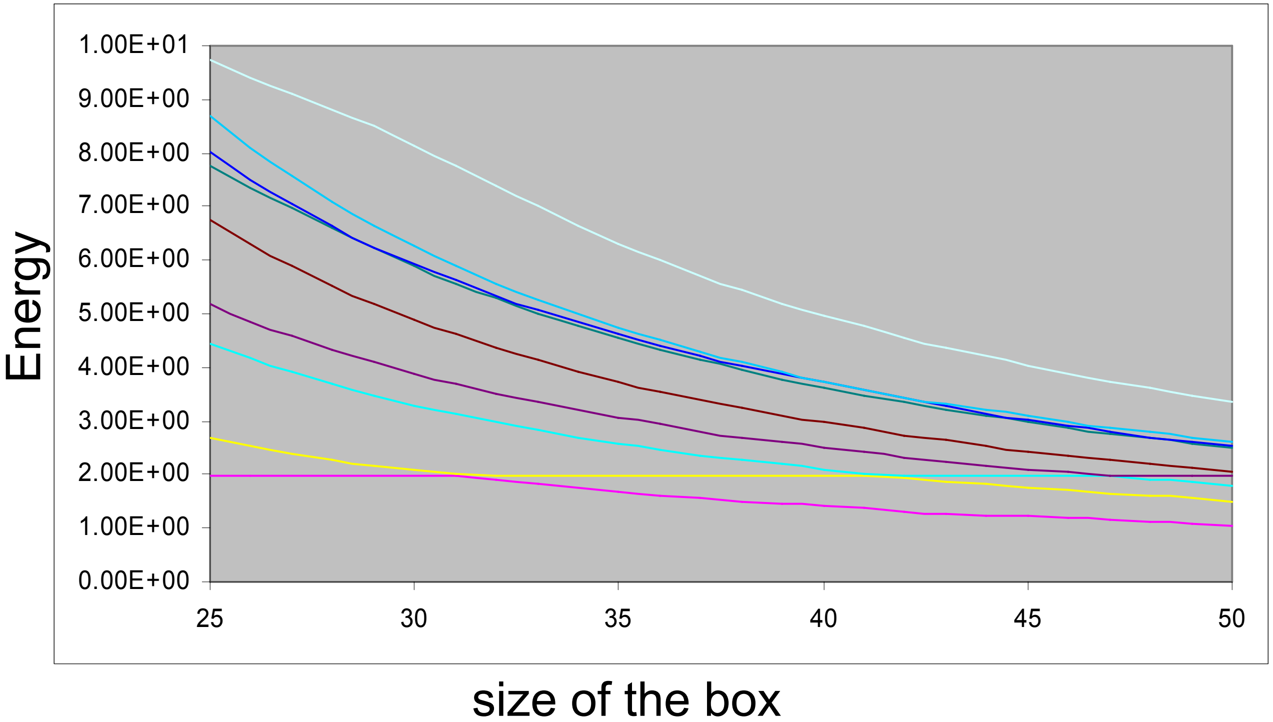


Buck potential

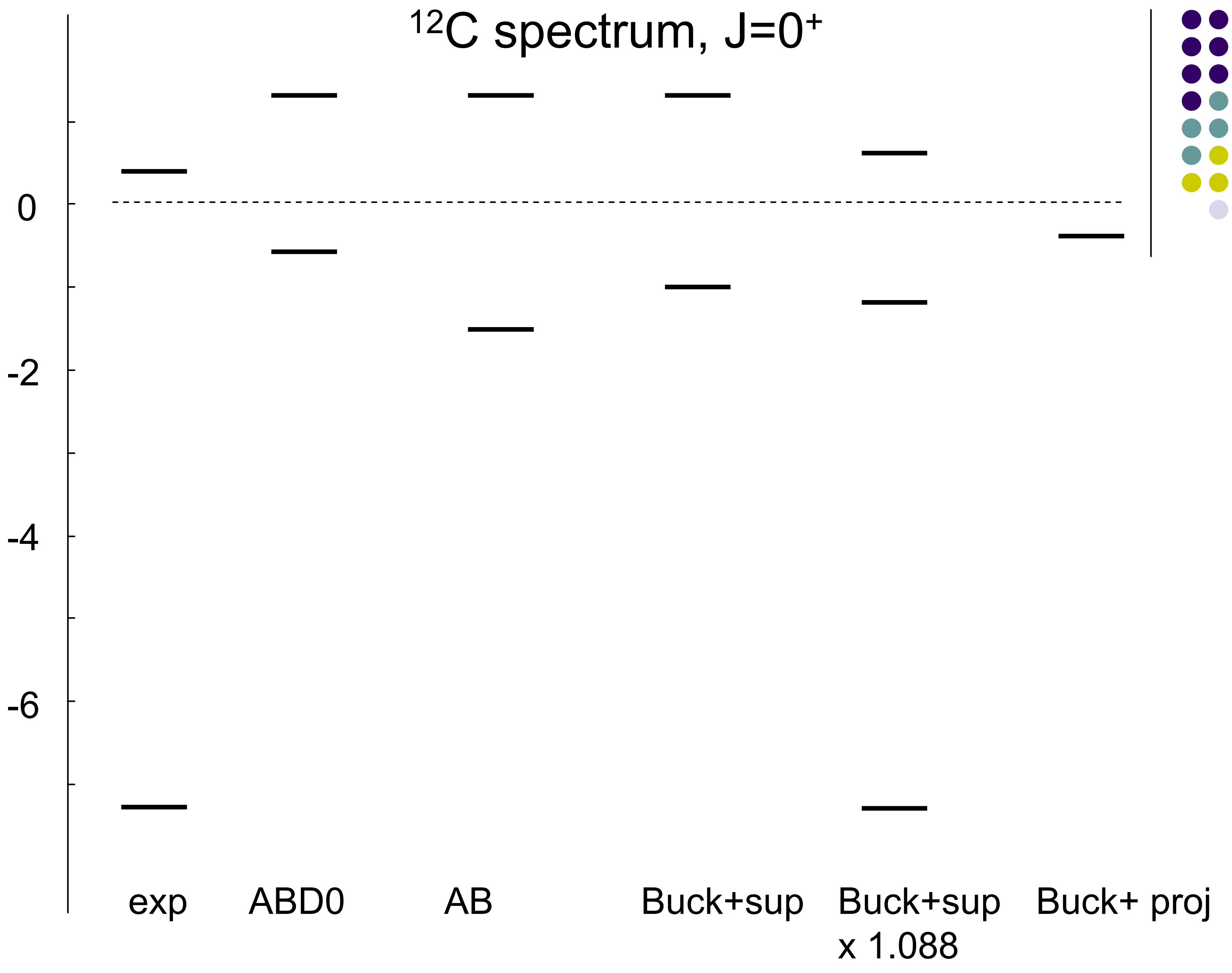
- $V=-122.6\ \exp(-(r/2.13)^2)$
- deep
- ℓ independent

Others: similar quality

- 3α spectroscopy
  - Bound states: simple diagonalization
  - Resonances: box method (Maier et al. JPB 13 (1980) L119)



➔ narrow resonance near 2 MeV

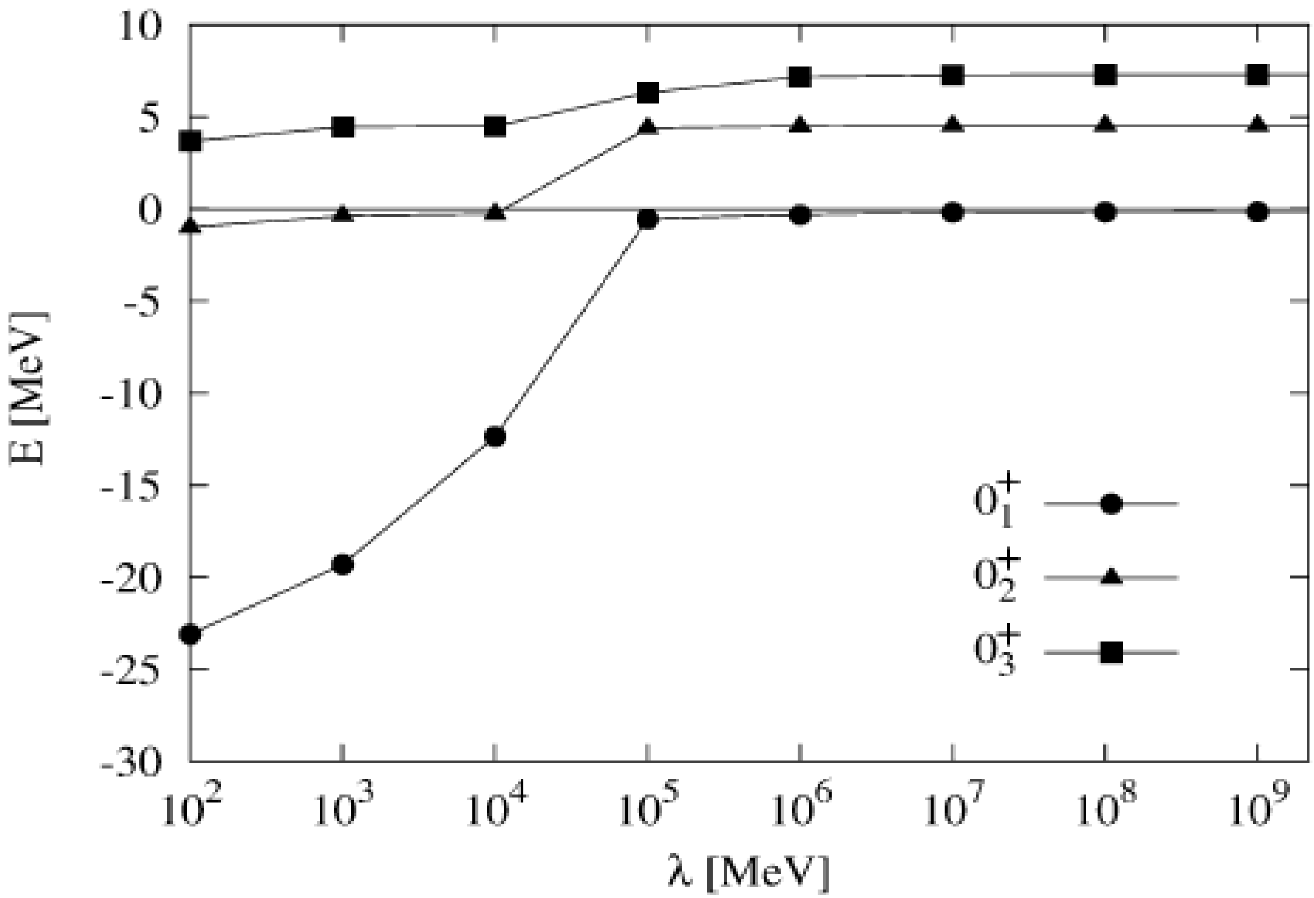


➔ **no satisfactory potential!!**

Projection technique: E vs Λ

H. Matsumura et al., Nucl. Phys. A776 (2006) 1

No ground state if Λ too large

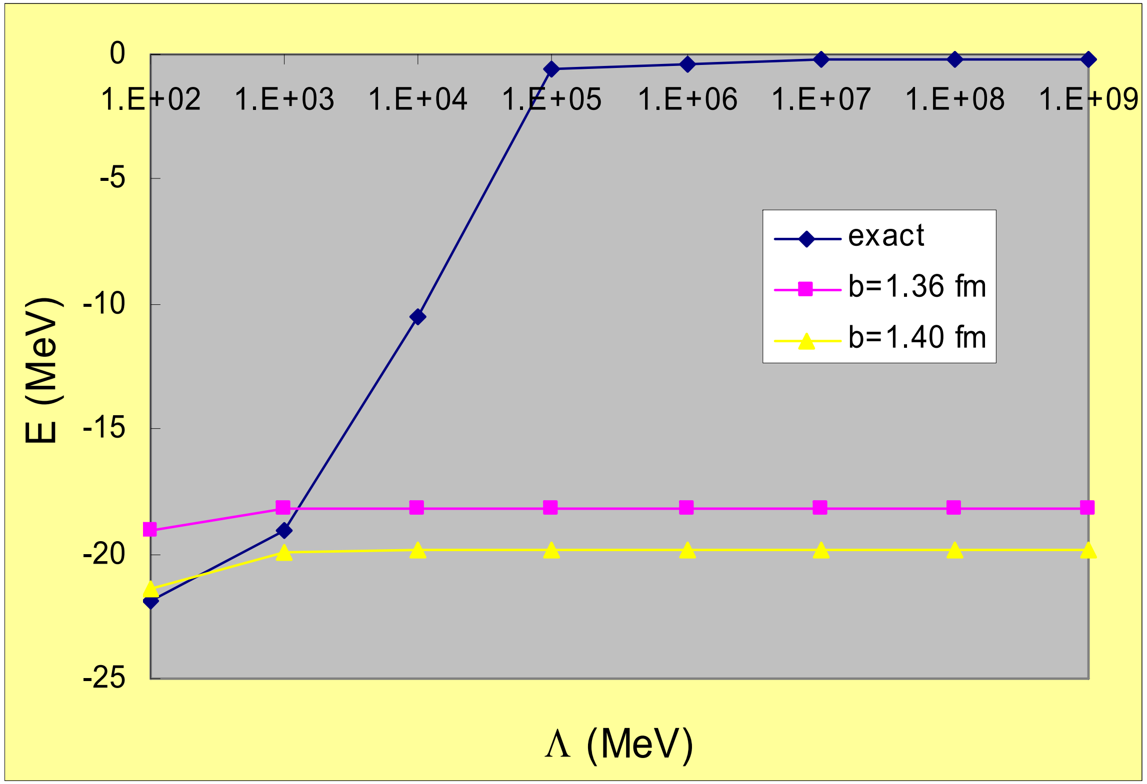


Further problem: definition of the forbidden states

Projector: $P = \sum_f |\psi_f(\boldsymbol{x})\rangle\langle\psi_f(\boldsymbol{x})|,$

Two possible definitions:

1. $\psi_f(x)$ is the exact solution associated with the potential
2. $\psi_f(x)$ is consistent with the cluster theory → h.o. orbitals



→strong difference

→ sensitivity with b

→"almost" forbidden states

Conclusion

- 3-body continuum much more complicated than 2-body continuum (hyperspherical coordinates)
  → propagation method is necessary to achieve a good convergence
- Recent developments: 2 propagation methods (necessary to use the projection technique)
- Use of the Lagrange-mesh technique for bound and scattering states:
  - fast (no integral for the matrix elements)
  - accurate (stability with the channel radius *a*)
  - Coulomb easily included
- ^{6}He: relatively simple, good test case
- ^{12}C:
  - ► more complicated (no suitable $\alpha+\alpha$ potential)→ in progress
  - ► Matsumura et al: BFW potential should be adapted (range)
  - ► 3-body forces?
  - ► Non-local potentials (based on RGM kernel)?
  - ► Microscopic approach? (for the moment: spectroscopy only)