Helium radii & proton analyzing powers in the NCSM

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Outline

- Motivation
- *Ab initio* no-core shell model (NCSM)
- $^{4,6,8}He$ binding energy and radius calculation
- ^{6}He excitation spectrum
- ^{6}He scattering off polarized proton target
  - Comparison to $p+^{6}Li$
- Outlook & conclusions

Motivation

- Precise measurement of ^{6}He charge radius recently performed at ANL
  - Laser spectroscopy on individual ^{6}He atoms confined in magneto-optical trap
  - r_c=2.054(14) fm
    - Much larger than ^{4}He charge radius of 1.673(1) fm
      - Due to two loosely bound neutrons forming a **halo**
- Measurement of ^{8}He charge radius under way
- Challenge for *ab initio* methods to reproduce this measurement and predict the ^{8}He charge radius
- Ongoing experiment of ^{6}He scattering off polarized proton target by CNS at RIKEN
  - Challenge to understand the ^{6}He+p polarization data simultaneously with the p+^{6}Li polarization data

Ab Initio No-Core Shell Model (NCSM)

- Presently only two methods capable to describe simultaneously ^{4}He, ^{6}He, ^{8}He and ^{6}Li
  - Green's function Monte Carlo (**GFMC**) by ANL-LANL group
  - *Ab initio* no-core shell model (**NCSM**)
- Two issues
  - Convergence of the many-body method
  - Quality of the interaction (Hamiltonian)
- **NCSM:**
- Many-body Schroedinger equation
  - *A*-nucleon wave function
- Hamiltonian
  - Realistic high-precision nucleon-nucleon potentials
    - Coordinate space - Argonne, INOY …
    - Momentum space - CD-Bonn, χPT N^{3}LO …
  - Three-nucleon interaction
    - Tucson-Melbourne TM', χPT N^{2}LO
- Finite harmonic-oscillator basis
- Need to construct effective interaction appropriate to the basis truncation
  - Done by a unitary transformation in a cluster approximation
- By construction convergent to exact solution with basis enlargement and/or increase in cluster size

$$H|\Psi\rangle = E|\Psi\rangle$$

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Six-Nucleon Spectroscopy from a Realistic Nonlocal Hamiltonian

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***Ab Initio* Shell Model Calculations with Three-Body Effective Interactions for *p*-Shell Nuclei**

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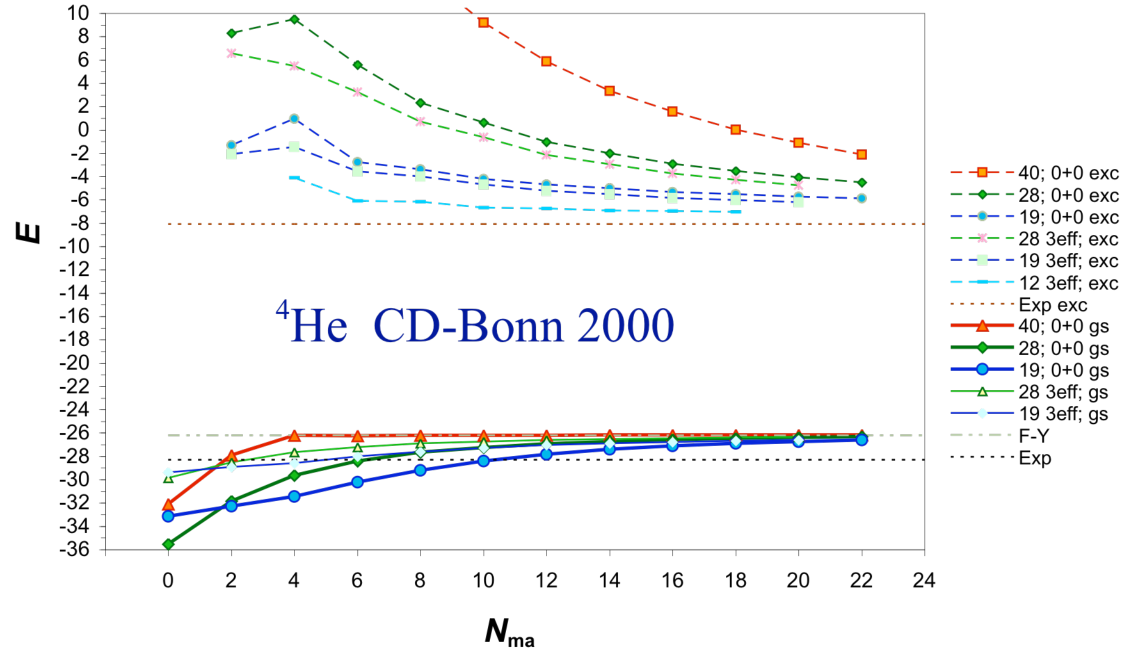
Convergent to exact solution

Nucleon-nucleon (NN) interactions

- Description of halo nuclei requires a large HO basis expansion of the wave function
  - Limit ourselves to NN interaction only and to the two-body cluster approximation
  - Shell model code Antoine (E. Caurier)
    - ^{6}He up to N_{max}=16 (dimension 700 million)
    - ^{8}He up to N_{max}=12 (dimension 500 million)
      - Dimensions smaller than possible in a standard shell model calculation (≈ billion)
      - Still more challenging due to large number of *nljm* states and an asymmetry of the proton and neutron numbers
- CD-Bonn 2000 (R. Machleidt, Phys. Rev. C **63**, 024001 (2001))
  - One-boson exchange - π, ρ, ω + phenomenological σ mesons
  - provide an accurate fit to NN data with χ^2=1.02
- INOY (P. Doleschall *et al*., Phys. Rev. C **67**, 064005 (2003))
  - Inside nonlocal, outside Yukawa
    - Multi-nucleon forces absorbed by short-range nonlocal terms in the NN interaction
    - In addition to the fit of NN phase shifts and deuteron properties A=3 binding energies fitted as well
    - Small modification of the *P*-waves to improve description of NNN analyzing powers
  - Very good convergence with NCSM for both *s*-shell and *p*-shell nuclei

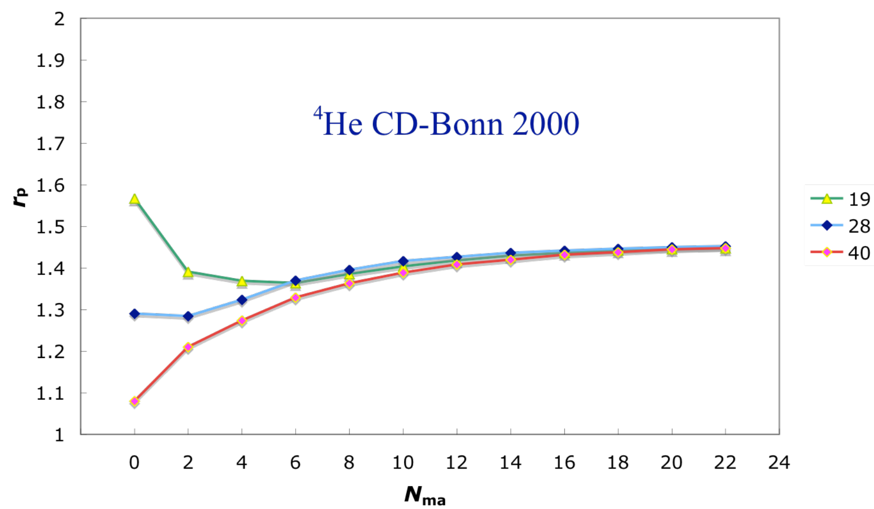
^{4}He calculation

- Ground-state & the first excited 0^+ 0 state energy
  - CD-Bonn 2000



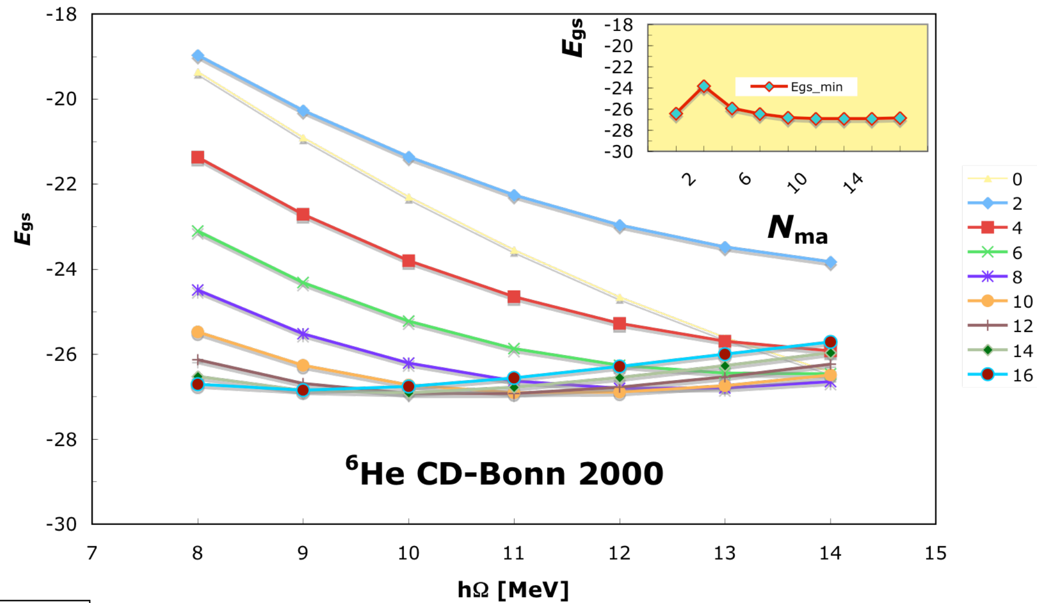
- Point-proton rms radius
  - CD-Bonn 2000
- Comparison to other methods
  - CD-Bonn 2000 & INOY

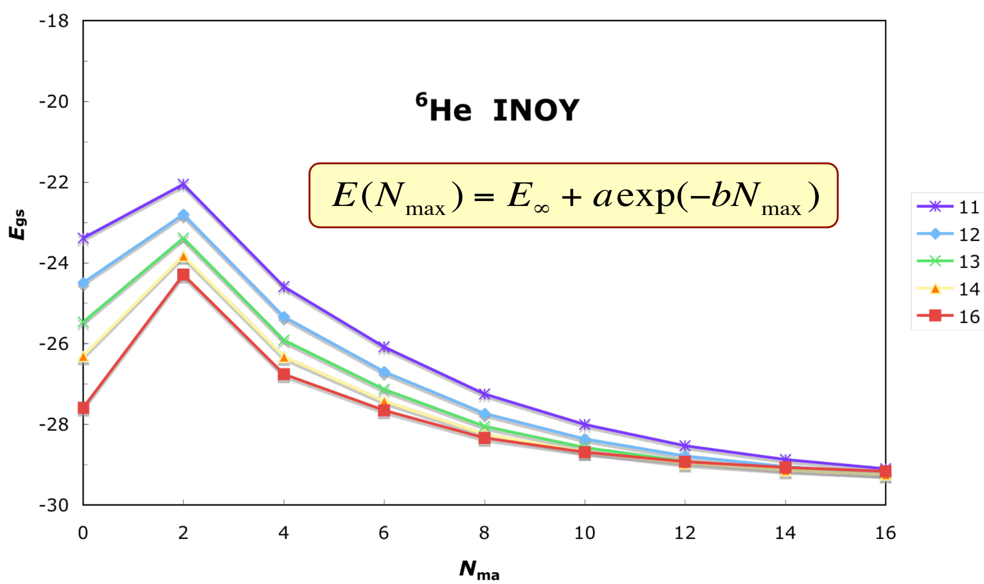
^{4}He	Exp	CD-Bonn NCSM	CD-Bonn FY-p	CD-Bonn HH	INOY NCSM	INOY FY-r
E_B	28.296	26.16(6)	26.16	26.13	29.10(5)	29.09
r_p	1.455(1)	1.45(1)	-	1.454	1.37(1)	1.376



^{6}He calculation

- Ground-state energy
  - CD-Bonn 2000
    - Stronger Ω dependence
    - Weak N_{max} dependence of minima
  - INOY
    - Weak Ω dependence
    - Stronger N_{max} dependence
    - Extrapolation possible
  - ^{6}He bound

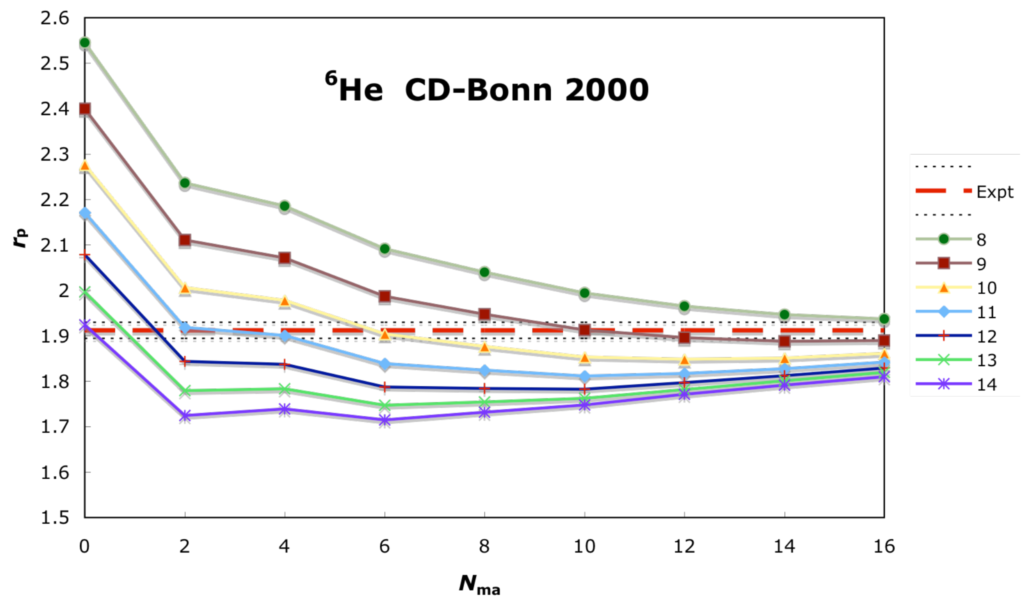


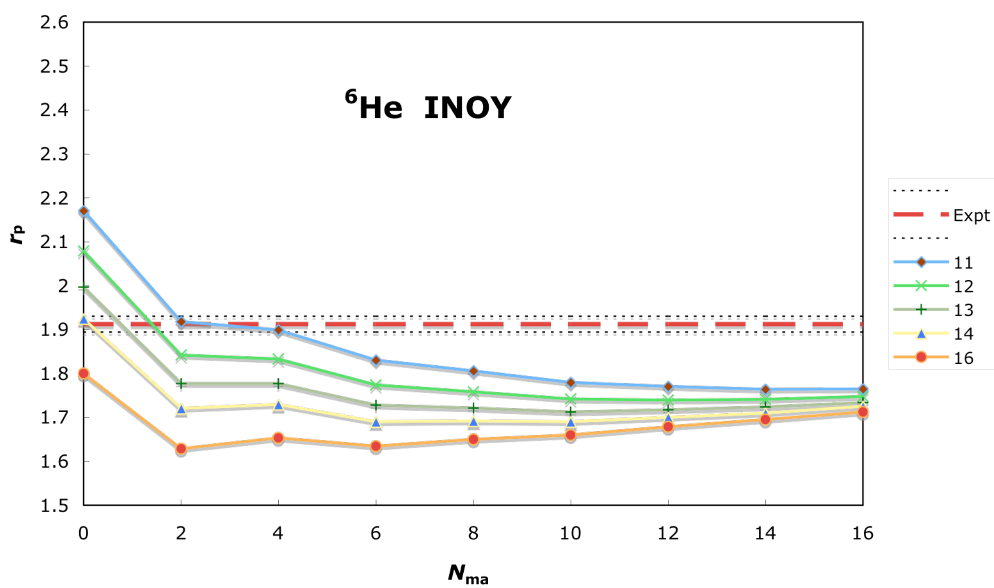


r_p [fm]	Expt.	CD-Bonn 2000	INOY
^{4}He	1.455 (1)	1.45 (1)	1.37 (1)
^{6}He	1.912 (18)		
^{8}He			
r_n [fm]	Expt.	CD-Bonn 2000	INOY
^{6}He	2.59–2.85		
^{8}He	2.69 (4)		
E_B [MeV]	Expt.	CD-Bonn 2000	INOY
^{4}He	28.296	26.16 (6)	29.10 (5)
^{6}He	29.269	26.9 (3)	29.38 (10)
^{8}He	31.408 (7)		

^{6}He calculation

- Point-proton rms radius
  - CD-Bonn 2000
    - Agreement with experiment
  - INOY
    - Faster convergence
    - Underestimates experiment

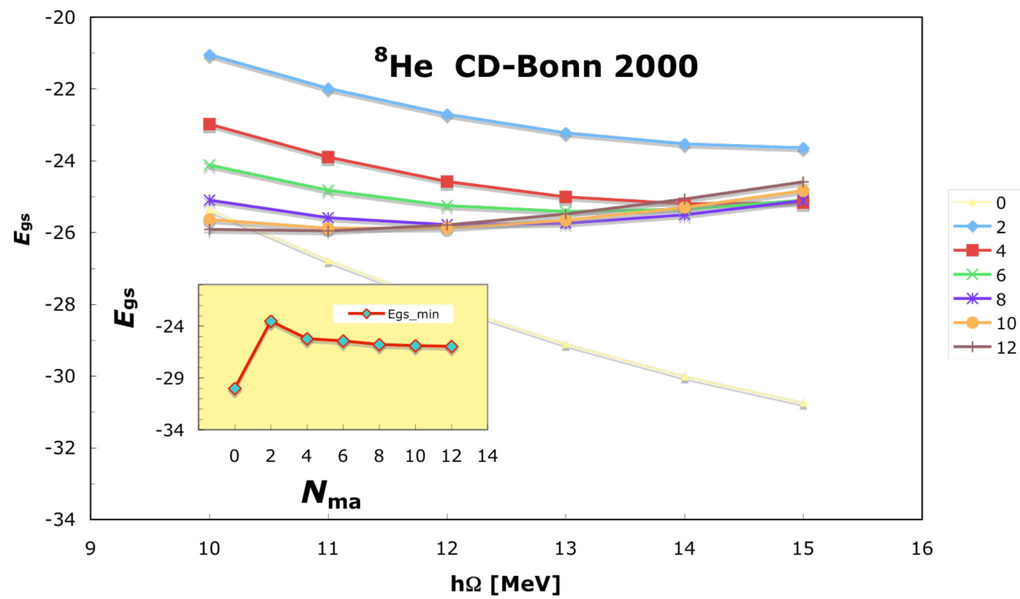


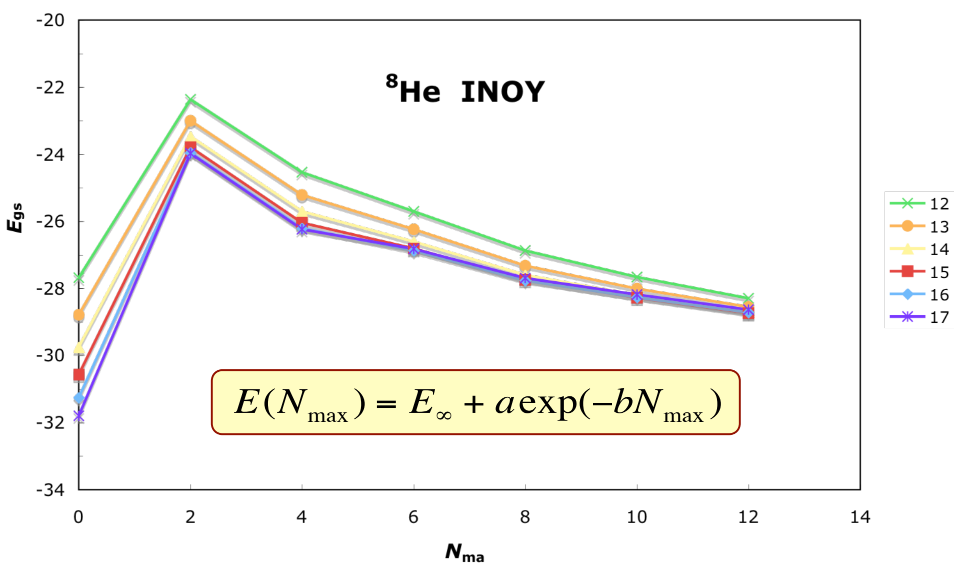


r_p [fm]	Expt.	CD-Bonn 2000	INOY
^{4}He	1.455 (1)	1.45 (1)	1.37 (1)
^{6}He	1.912 (18)	1.89 (4)	1.76 (3)
^{8}He			
r_n [fm]	Expt.	CD-Bonn 2000	INOY
^{6}He	2.59–2.85	2.67 (5)	2.55(10)
^{8}He	2.69 (4)		
E_B [MeV]	Expt.	CD-Bonn 2000	INOY
^{4}He	28.296	26.16 (6)	29.10 (5)
^{6}He	29.269	26.9 (3)	29.38 (10)
^{8}He	31.408 (7)		

^{8}He calculation

- Ground-state energy
  - CD-Bonn 2000
    - ^{8}He (likely) unbound
  - INOY
    - ^{8}He bound
  - Isospin dependence incorrect

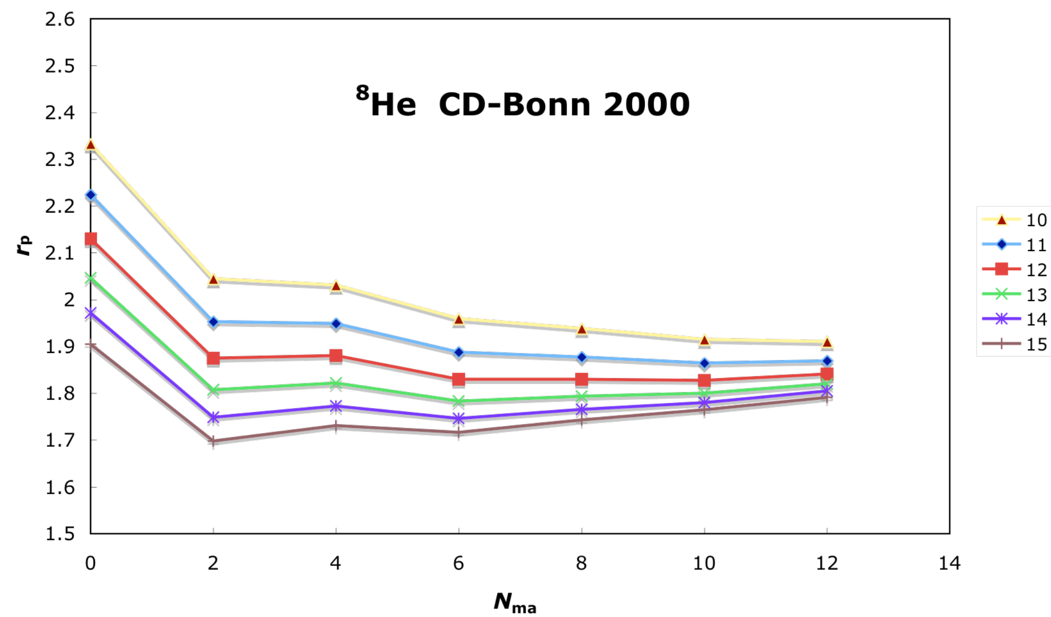


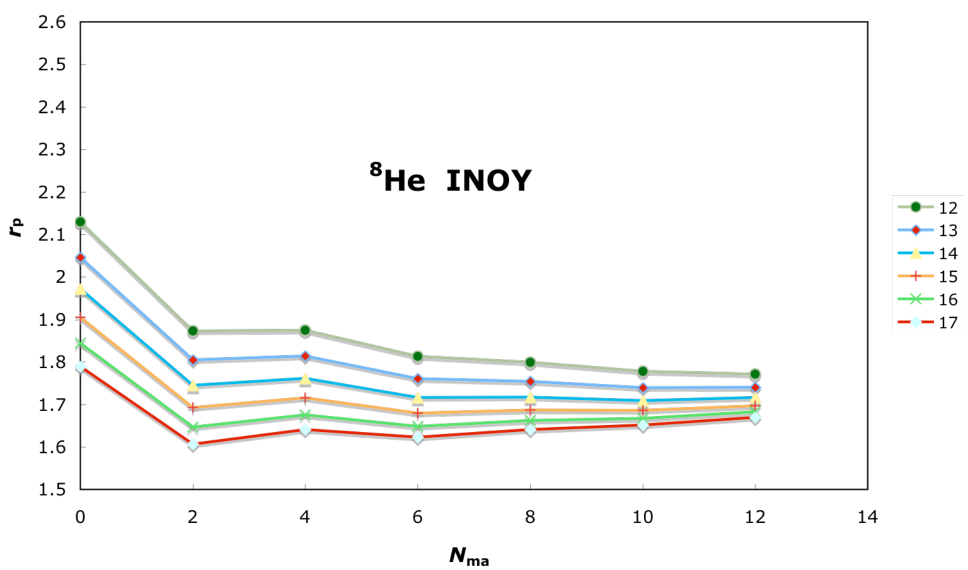


r_p [fm]	Expt.	CD-Bonn 2000	INOY
^{4}He	1.455 (1)	1.45 (1)	1.37 (1)
^{6}He	1.912 (18)	1.89 (4)	1.76 (3)
^{8}He			
r_n [fm]	Expt.	CD-Bonn 2000	INOY
^{6}He	2.59–2.85	2.67 (5)	2.55(10)
^{8}He	2.69 (4)		
E_B [MeV]	Expt.	CD-Bonn 2000	INOY
^{4}He	28.296	26.16 (6)	29.10 (5)
^{6}He	29.269	26.9 (3)	29.38 (10)
^{8}He	31.408 (7)	26.0 (4)	30.30 (30)

^{8}He calculation

- Point-proton rms radius
  - CD-Bonn 2000
  - INOY
  - Slightly smaller than ^{6}He r_p
  - Much larger than ^{4}He r_p
  - CD-Bonn 2000 result is a more realistic prediction





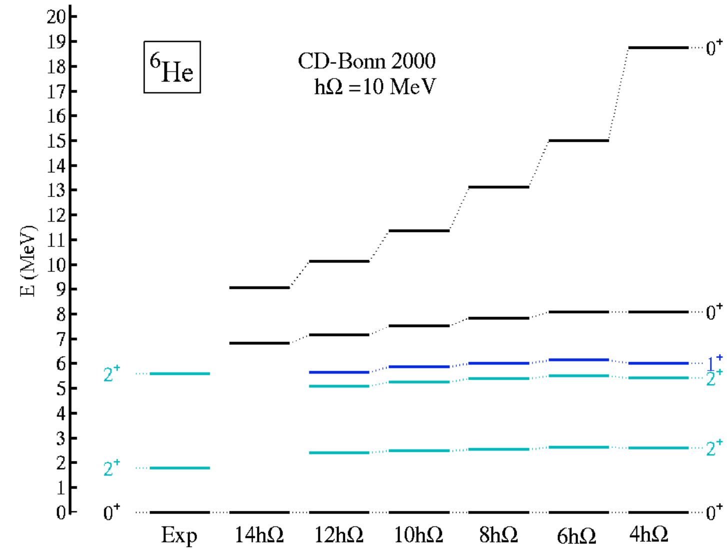
r_p [fm]	Expt.	CD-Bonn 2000	INOY
^{4}He	1.455 (1)	1.45 (1)	1.37 (1)
^{6}He	1.912 (18)	1.89 (4)	1.76 (3)
^{8}He		1.88 (6)	1.74 (6)
r_n [fm]	Expt.	CD-Bonn 2000	INOY
^{6}He	2.59–2.85	2.67 (5)	2.55(10)
^{8}He	2.69 (4)	2.80 (10)	2.60 (10)
E_B [MeV]	Expt.	CD-Bonn 2000	INOY
^{4}He	28.296	26.16 (6)	29.10 (5)
^{6}He	29.269	26.9 (3)	29.38 (10)
^{8}He	31.408 (7)	26.0 (4)	30.30 (30)

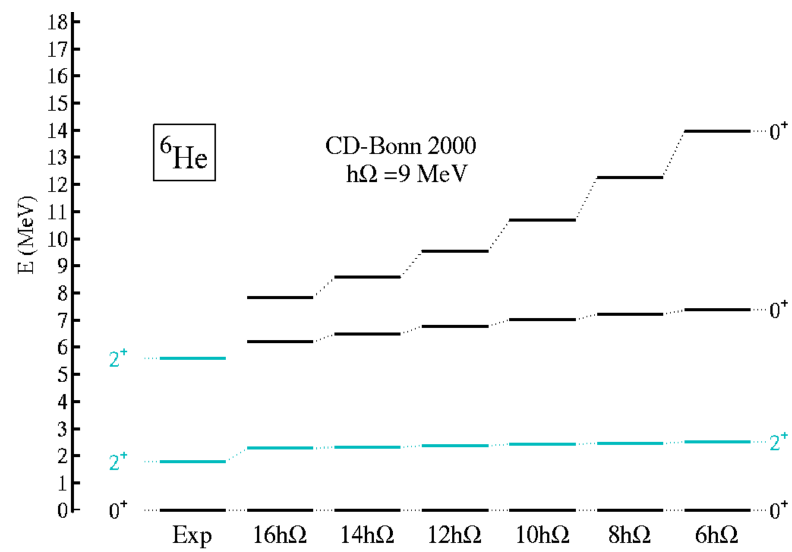
^{6}He excitation spectrum

- 5 *p*-shell states
- 0^+_3 higher-ħΩ-dominated state
- Configurations at N_{max}=16 (ħΩ=9 MeV):

^{6}He	0 ħΩ	2 ħΩ	4 ħΩ	6 ħΩ	8 ħΩ	10 ħΩ	12 ħΩ	14 ħΩ	16 ħΩ
0^+_1	0.43	0.18	0.16	0.09	0.06	0.04	0.02	0.01	0.01
0^+_2	0.28	0.19	0.20	0.12	0.08	0.05	0.03	0.02	0.02
0^+_3	0.02	0.27	0.20	0.20	0.13	0.09	0.05	0.030	0.02

- GFMC ^{6}He calculations with the AV8’ NN potential place the 0^+_2 state at 4.96(9) MeV





JLM folding optical potential

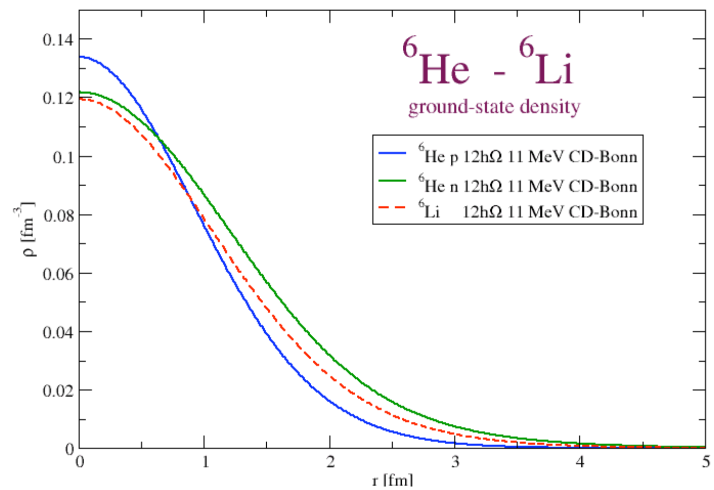
- Starting point: ^{6}He and ^{6}Li ground-state and transition translationally-invariant densities
  - ^{6}Li point-proton rms radius from CD-Bonn 2000 in agreement with experiment
    - Validation of our ^{6}He point-proton rms radius calculation
- Spin-orbit interaction

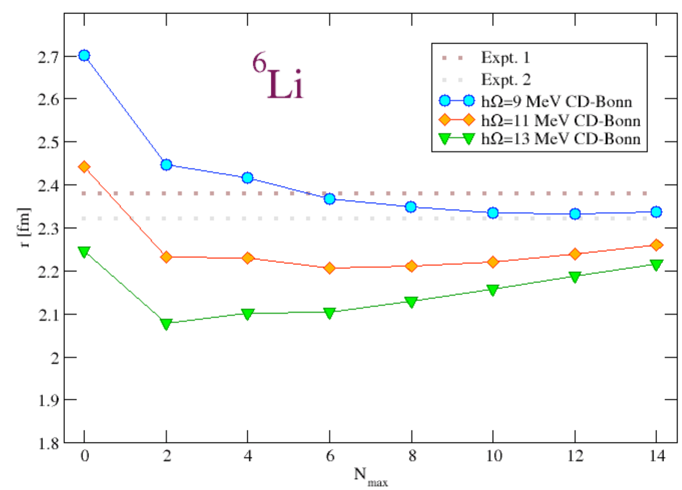
$$V_p^{so}(r) = \lambda_{v_{SO}} \tfrac{1}{r}\tfrac{d}{dr}\left(\tfrac{2}{3}\rho_n[r] + \tfrac{1}{3}\rho_p[r]\right)\vec{l}\cdot\vec{\sigma}$$

- JLM central optical model potential

$$U(r,E) = (t\sqrt{\pi})^{-3}\int \frac{U_{NM}\left(\rho[\tfrac{1}{2}(r+r')],E\right)}{\rho[\tfrac{1}{2}(r+r')]}\exp\left(-|\vec{r}-\vec{r}'|/t^2\right)\rho[r']d\vec{r}'$$

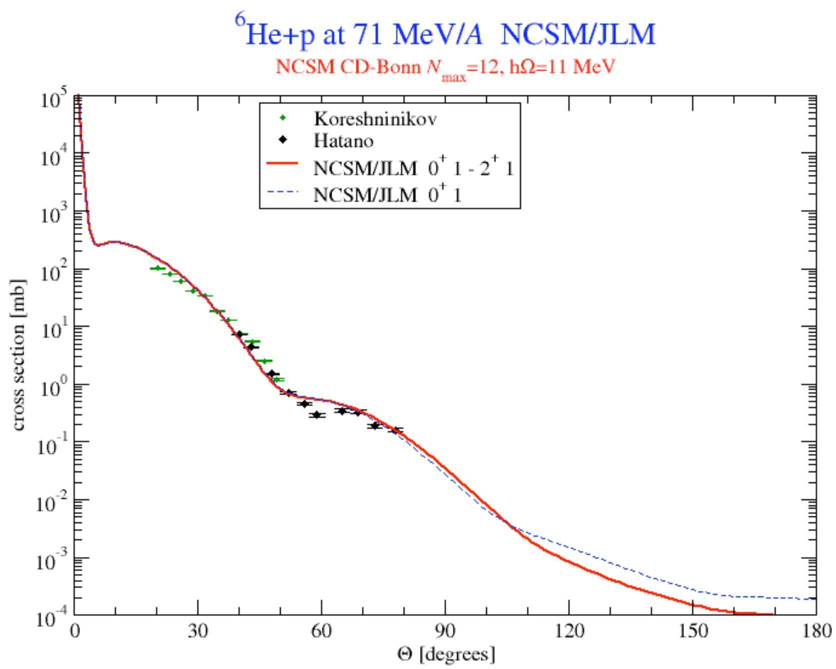
- Parameterization from E. Bauge *et al.*, PRC **58**, 1118 (1998)

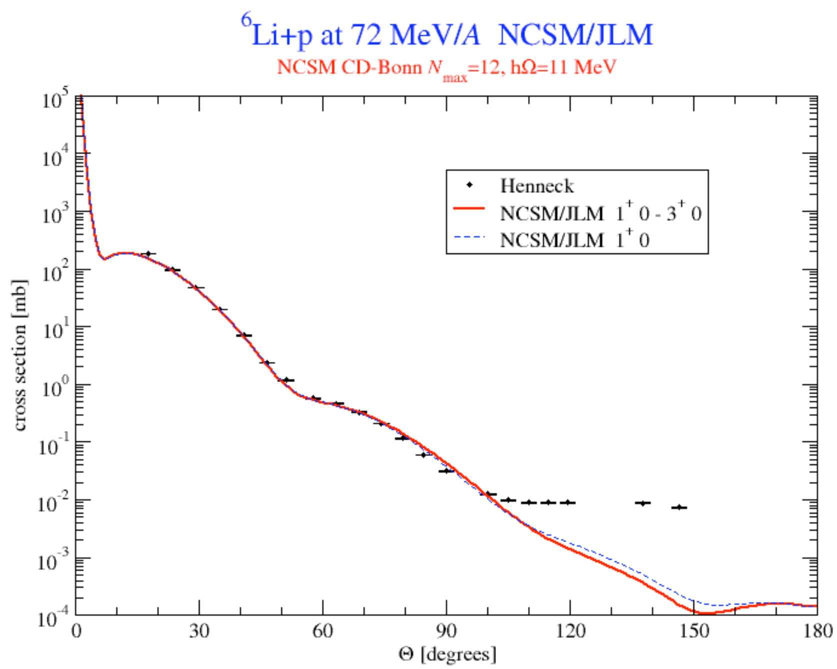




^{6}He scattering off polarized proton target at 71 MeV/A

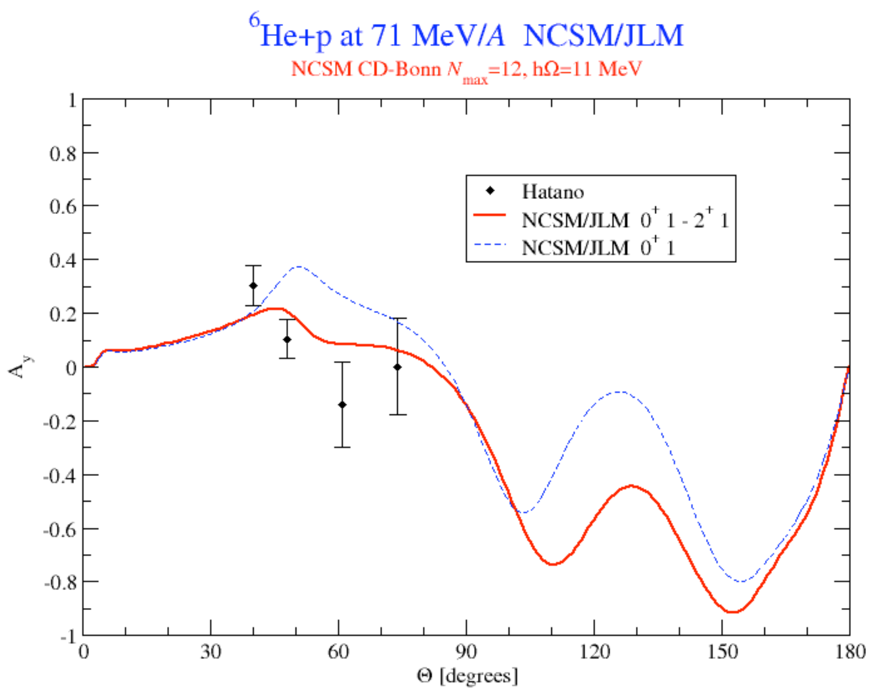
- Simultaneous χ^2-fit to ^{6}He+p and ^{6}Li+p elastic scattering and analyzing power data
  - CD-Bonn 2000, 12hΩ, hΩ=11 MeV NCSM density
  - JLM microscopic optical potential + spin-orbit potential
  - Fresco coupled-channel calculations
  - Four scaling parameters fitted
    - λ_V=0.90, λ_W=1.00, λ_{Vso}=0.81, λ_{Wso}=0.98

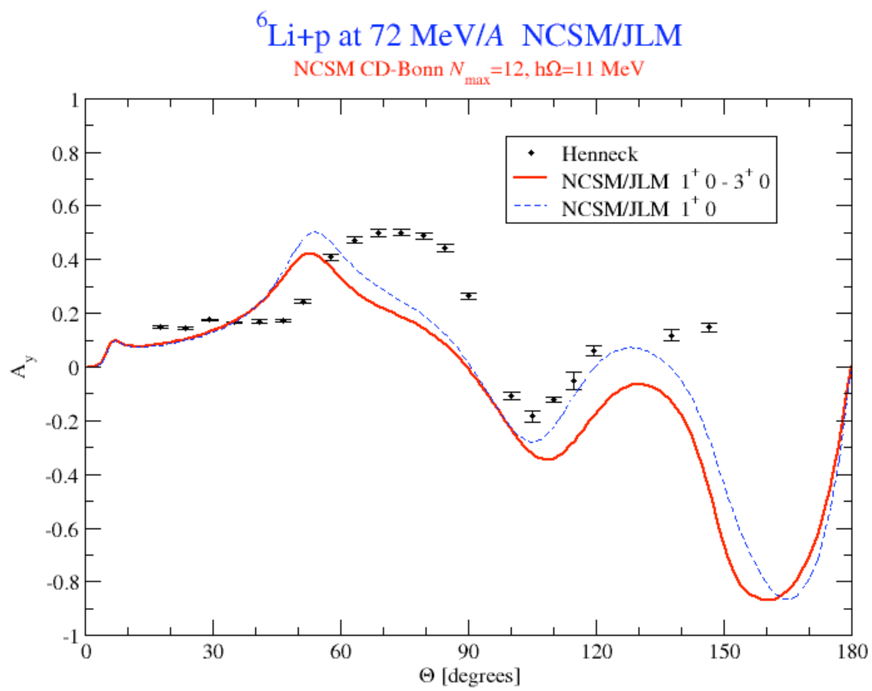




^{6}He scattering off polarized proton target at 71 MeV/A

- Analyzing powers not fitted well
- Calculated ^{6}Li and ^{6}He A_y tends to have the same sign
- Coupling of the ^{6}He 2^+ 1 excited state appears important
- Additional terms in the p+^{6}Li optical potential likely required





Outlook: ^{6}He+p in an *ab initio* approach?

- RGM with *ab initio* NCSM cluster wave functions and effective interactions

$$\mathrm{H}\varphi(r) = E\,\mathrm{N}\varphi(r)$$

$$\mathrm{N}\varphi(r) \equiv \int \mathrm{d}r' N(r,r')\varphi(r')$$

$$N(r,r') = \left\langle \Phi_r^{(A)} \middle| \Phi_{r'}^{(A)} \right\rangle$$

$$H(r,r') = \left\langle \Phi_r^{(A)} \middle| H \middle| \Phi_{r'}^{(A)} \right\rangle$$

- First step: Norm kernel (*a*=1)
  - Jacobi coordinates (Sofia Quaglioni)
    - $N_{\max}$=60 for $(A\text{-}1)$=3
    - $N_{\max}$=20 for $(A\text{-}1)$=4
  - Single-particle coordinates, Slater determinants
    - Good for A>4
    - $N_{\max}$=16 for $(A\text{-}1)$=6
    - Two-step calculation
  - Tested to give the same answer when both choices feasible (A=5)

$$\langle\Phi^{(A-1,1)JT}_{(\alpha' I_1' T_1',\,\frac12\frac12)s'l'};n'l'|P_{A,A-1}|\Phi^{(A-1,1)JT}_{(\alpha I_1 T_1,\,\frac12\frac12)sl};nl\rangle =$$
$$\sum\langle A-1\alpha' I_1' T_1'|(N_{A-2}i_{A-2}I_{A-2}T_{A-2},n'_{A-1}l'_{A-1}\mathcal{J}'_{A-1}\tfrac12)I_1'T_1'\rangle$$
$$\langle(N_{A-2}i_{A-2}I_{A-2}T_{A-2},n_{A-1}l_{A-1}\mathcal{J}_{A-1}\tfrac12)I_1T_1|A-1\alpha I_1T_1\rangle$$
$$\langle n'l'n'_{A-1}l'_{A-1}\lambda|n_{A-1}l_{A-1}nl\lambda\rangle_{A(A-2)}\left\{\begin{matrix}\frac12 & T_{A-2} & T_1'\\ \frac12 & T & T_1\end{matrix}\right\}\hat{T}_1\hat{T}_1'(-1)^{T_1+T_1'+1}$$
$$\hat{s}\hat{s}'\hat{\lambda}^2\hat{Y}^2\hat{I}_1\hat{I}_1'\hat{\mathcal{J}}_{A-1}\hat{\mathcal{J}}'_{A-1}(-1)^{s+s'+l'_{A-1}+l}\left\{\begin{matrix}\mathcal{J}'_{A-1} & I_1' & I_{A-2}\\ \mathcal{J}_{A-1} & I_1 & Y\end{matrix}\right\}$$
$$\left\{\begin{matrix}l'_{A-1} & \frac12 & \mathcal{J}'_{A-1}\\ I_1 & Y & s\end{matrix}\right\}\left\{\begin{matrix}I_1' & \mathcal{J}_{A-1} & Y\\ l_{A-1} & s' & \frac12\end{matrix}\right\}\left\{\begin{matrix}s' & J & l'\\ l_{A-1} & l & \lambda\\ Y & s & l'_{A-1}\end{matrix}\right\}.$$

$${}_{\rm SD}\langle\Phi^{(A-a,a)JT}_{(\alpha' I_1'T_1',\beta' I_2'T_2')s'l'};n'l'|P_{(A-a,a)}|\Phi^{(A-a,a)JT}_{(\alpha I_1T_1,\beta I_2T_2)sl};nl\rangle_{\rm SD} =$$
$$\sum_{n_rl_rn_r'l_r'J_r}\langle\Phi^{(A-a,a)J_rT}_{(\alpha' I_1'T_1',\beta' I_2'T_2')s'l_r'};n_r'l_r'|P_{(A-a,a)}|\Phi^{(A-a,a)J_rT}_{(\alpha I_1T_1,\beta I_2T_2)sl_r};n_rl_r\rangle$$
$$\sum_{NL}\hat{l}\hat{l}'\hat{J}_r^2(-1)^{s+l_r-s-l_r'}\left\{\begin{matrix}s & l_r & J_r\\ L & J & l\end{matrix}\right\}\left\{\begin{matrix}s' & l_r' & J_r\\ L & J & l'\end{matrix}\right\}\langle n_rl_rNLl|00nll\rangle_{\frac{a}{A-a}}\langle n_r'l_r'NLl'|00n'l'l'\rangle_{\frac{a}{A-a}}$$

$${}_{\rm SD}\langle\Phi^{(A-1,1)JT}_{(\alpha' I_1'T_1',\,\frac12\frac12)s'l'};n'l'|P_{A,A-1}|\Phi^{(A-1,1)JT}_{(\alpha I_1T_1,\,\frac12\frac12)sl};nl\rangle_{\rm SD} =$$
$$\frac{1}{A-1}\sum_{jj'K\tau}\left\{\begin{matrix}I_1 & \frac12 & s\\ l & J & j\end{matrix}\right\}\left\{\begin{matrix}I_1' & \frac12 & s'\\ l' & J & j'\end{matrix}\right\}\left\{\begin{matrix}I_1 & K & I_1'\\ j' & J & j\end{matrix}\right\}\left\{\begin{matrix}T_1 & \tau & T_1'\\ \frac12 & T & \frac12\end{matrix}\right\}$$
$$\hat{s}\hat{s}'\hat{j}\hat{j}'\hat{K}\hat{\tau}(-1)^{I_1'+j'+J}(-1)^{T_1+\frac12+T}\;{}_{\rm SD}\langle A-1\alpha' I_1'T_1'|||(a^\dagger_{nlj\frac12}\tilde{a}_{n'l'j'\frac12})^{(K\tau)}|||A-1\alpha I_1T_1\rangle_{\rm SD}$$

Conclusions

- *Ab initio* NCSM capable to describe loosely bound systems
  - Very challenging problem
  - Large HO basis expansion of the wave function
  - Large dimensions
  - Asymmetry in proton-neutron number adds to technical difficulty (Antoine code)
- Convergence of the ^{4,6,8}He charge radius
  - CD-Bonn 2000 ^{4,6}He charge radii in agreement with experiment
    - ^{8}He point-proton rms radius prediction r_p=1.88(6) fm
  - INOY NN potential underestimates He charge radii
- ^{6}He elastic scattering calculations
  - Mixing of *ab initio* and semi-microscopic approaches
  - Good description of the cross section
  - Improvements needed for the analyzing power description
- In progress:
  - RGM with *ab initio* NCSM cluster wave functions and effective interactions